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# APPLICATIONS OF DRIFT SPECTRA IN SEISMIC DESIGN

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## SUMMARY

The seismic performance of building structures is a direct function of the maximum interstorey drifts which occur during strong seismic ground motions. Maximum drifts can be estimated using the concept of drift spectra. This paper presents a simple technique for deriving drift spectra for frame structures which is based on elementary shear-beam vibration theory. These spectra, together with recently proposed natural period bound estimates for framed building structures, are then incorporated into a simple performance-based seismic design procedure. A numerical example illustrates the proposed approach.

## INTRODUCTION

As is known, structural and nonstructural damage due to earthquake is a result of excessive strains. However, in performance based design it is more convenient to use displacement or drift as the controlling criterion of structural performance. This is because these parameters are familiar and easy to compute; also, field data and experimental research results are readily available. As noted in Vision 2000 [1], the displacement-based design approach begins with establishing target levels of displacement or drift consistent with the selected performance objectives. The target displacement or drift should conform to the anticipated ductility capacity of the planed structure.

In present design practice post-yield drifts are estimated by factoring linear drifts through displacement amplification factors. There is no agreement among seismic codes regarding the numerical values of these factors. However, it is now believed that for the medium and long period ranges, values on the order of the force reduction (or modification) factors, or somewhat smaller, rather than the UBC ones (i.e. $R_w$ = 3/8 where $R_w$ = force reduction factor at working stress level) are reasonable estimates (e.g. the $C_d$ values in NEHRP 1998 [2]).

For multistorey buildings, roof displacement per se is not a useful performance criterion because it represents only an average value of the interstorey drift. For these structures a direct measure of drift is therefore preferable. The concept of a drift spectrum is not new. Iwan [1997] developed drift spectra for uniform cantilever shear beams to predict interterstorey drift demand of frames at their base level. These spectra were obtained from the solution of the damped wave equation in a one-dimensional continuum. Iwan's purpose was to show that drift is likely to be higher for frames located in the near field of a strong earthquake. Yet, since drift is a pivotal performance criterion, such spectra may be useful tools for the seismic design of frames in general.

The purpose of this study is first to present a simple technique for deriving drift spectra for frame structures which is based on elementary shear-beam vibration theory. These spectra, together with recently proposed natural0 period bound estimates for framed building structures [Goel & Chopra 1997] are then incorporated in a simple performance-based design procedure. A numerical example illustrates the proposed approach.

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## BASIC THEORY

Modelling of regular moment-resisting frames as shear beams is a common approximation in structural design. Moreover, uniform shear-beams can simulate the behaviour of actual frames, even when the bottom storey is somewhat higher than the typical storey, which is often the case. The following equation defines the force-displacement relationship for the elastic shear beam:

$$du/dx = V/(GA) \tag{1}$$

in which u = lateral displacement at height "x" above the base, V = shear force at this level, and GA = shear beam stiffness. Formulas for evaluating GA in terms of beam and column stiffnesses are readily available in the literature. One of the most common formulas for uniform frames is given below [Wilbur 1935]:

$$GA = [12\,E / h]\,[\,1/\Sigma k_c + 1/\Sigma k_g\,]^{-1} \tag{2}$$

In which E = modulus of elasticity, h = storey height, $k_c = I_c/h$, $k_g = I_g/l_g$ are the respective stiffnesses of the columns and beams, I = element moment of inertia and $l_g$ = beam span. The summation is over the storey columns and beams.

The maximum drift in the frame occurs at the base, and can be expressed as follows:

$$D_r(0) = du/dx(0) = V_0/(GA) \tag{3}$$

In which $D_r(0)$ = base drift ratio (normally expressed as the ratio of storey drift to storey height), and $V_0$ = base shear. For a shear beam subjected to seismic excitation, the base shear can be expressed as:

$$V_0 = (S_{af}/g)\,W \tag{4}$$

in which W = total weight of the uniform beam structure, g = acceleration of gravity, and $S_{af}$ = the shear beam spectral acceleration, which includes the effect of higher modes [Heidebrecht & Lu 1988].

The fundamental period $T_1$ of a uniform cantilever shear beam is given by:

$$T_1 = 4\sqrt{WH/(gGA)} \tag{5}$$

in which H = beam length. Combining eqns. 3, 4 and 5 yields

$$D_r(0) = T_1^2 S_{af}/(16H) \tag{6}$$

Equation 6 can be used to determine the maximum drift ratio of a uniform shear beam resulting from any earthquake time-history, provided $S_{af}$ is known.

Several approximations can be used to simplify the calculation of $D_r(0)$. The first one is to calculate the drift using only the first mode of the shear beam. This approximation is suitable for low-rise frames, say below 10 storeys, in which the response is dominated by the first mode. It can easily be shown that the following relation holds for the first mode drift $D_{r1}(0)$:

$$D_{r1}(0) = \pi/(2H)u(H) \tag{7}$$

$$u(H) = (4/\pi)S_d \tag{8}$$

in which $S_d$ = the spectral displacement at the fundamental period $T_1$ . Assuming the harmonic relationship between spectral displacement and spectral acceleration, namely

$$S_d = [T_1^2/(4\pi^2)]\,S_a \tag{9}$$

and substituting from eqns. 7 and 8 leads to:

$$D_{r1}(0) = T_1^2\,S_a/(2\pi^2 H) \tag{10}$$

Equation 10 can be used to determine the first mode drift ratio directly from the spectral acceleration of the record. Alternatively, the first mode drift can be obtained directly from the spectral displacement by substituting eqn. 8 into eqn. 7:

$$D_{r1}(0) = 2\, S_d / H \tag{11}$$

As a second approximation one can assume that $S_{af} \approx S_a$ in eqn.6, which leads to:

$$D_r(0) \approx D_{r2}(0) = T_1^2\, S_a /(16H) \tag{12}$$

## COMPUTATION OF DRIFT SPECTRA

A simple and useful parameter for characterizing the frequency content of earthquake ground motions is the ratio a/v in which a = peak ground acceleration and v = peak ground velocity of the earthquake record. Motions with high a/v generate significant response in short period structures, whereas those with low a/v generate significant response in long period ones. If a is expressed in units of g and v in m/s, then a/v ratios for actual earthquake records can range from about 0.3 to over 3; typical intermediate values (which characterize ground motions in the west coast of Canada and the U.S.) are in the neighbourhood of 1.

Several ensembles of actual earthquake records with different a/v ratios have been selected [Naumoski, Heidebrecht and Rutenberg 1993] for use in design and research. Each ensemble comprises 15 time-histories in order to ensure that the variability of amplification and duration is included. Three of these ensembles are used in this paper; their designation and mean a/v ratios are follows:

NL(low): mean a/v = 0.7; NI (intermediate): mean a/v = 1.0; NH(high): mean a/v = 2.0

Figure 1 shows the $D_r(0)$, $D_{r1}(0)$ and $D_{r2}(0)$ spectra calculated from the mean plus one standard deviation (M+SD) response spectra of the three ensembles noted above, with peak ground velocity v normalised to 1m/s, and for a reference building height H=30m. Spectra for other values of v and H can be determined by scaling these curves directly in proportion to v and H. This figure shows that $D_{r1}(0)$ generally underestimates the true drift $D_r(0)$, but that $D_{r2}(0)$ envelopes the true drift except for long period structures (T > 2.0s) subject to the NH ensemble. However, for T < 1.0s $D_{r1}(0)$ is a very good approximation of the true drift for all three ensembles. Consequently, by using eqn. 12, conservative estimates of the drift spectrum can be determined directly from the acceleration spectrum.

A comparison of the true drifts generated by each of the three ensembles shows that the NH ensemble produces the largest drifts when T < 0.5s, the NI ensemble drifts are the largest for 0.5s < T < 1.0s, and NL produces the largest drifts when T > 1.0s.

An important cautionary observation is that the curves in Fig.1 are only applicable in the realistic range of periods for frames with H = 30m. Depending upon whether the frames are of reinforced concrete (RC), or steel, and for RC frames upon the extent of cracking, natural periods for frames of this height are expected to be in the range of 1.0 to 1.8 seconds. Goel & Chopra [1997] have proposed upper and lower bound fundamental period formulas ($T_U$ and $T_L$ respectively) for both steel and RC frames; for RC frames these take the form:

$$T_L = 0.0466\, H^{0.90} \tag{13}$$

$$T_U = 0.0670\, H^{0.90} \tag{14}$$

in which H is expressed in unit of metres. Figure 2a shows the percent-drift for the NI ensemble (normalised to an excitation velocity of 1 m/s) calculated for both $T_L$ and $T_U$, that is obtained by inverting eqns. 13 and 14 and substituting into eqn.6. The M+SD spectral acceleration $S_a/g$ for this ensemble, using the same numerical scale, is also shown in this figure. For H = 30 m, eqns. 13 and 14 yield values for $T_L$ and $T_U$ of 0.995s and 1.43s respectively. As would be expected, the upper bound period $T_U$ results in the larger drift (3.2% compared with 2.5% for $T_L$), but the difference in drifts between the two period bounds is surprising small.

Figure 2b shows, for the same ensemble, the ratio of percent-drift to $S_a/g$ for the two period-bounds. While not shown in this figure, the corresponding curves for the other ensembles are very similar to those shown here, i.e. this ratio is relatively insensitive to the frequency content of the ground motion. These curves also show that drift is essentially a linear function of period, as is the displacement.

It should also be noted that since the numerical values of a and v, expressed in g and m/s respectively, are equal for the NI ensemble, the curves in Figs. 2a and 2b can also be scaled by the peak ground acceleration a. Given the above, the curves in Fig. 2b can be used to estimate upper and lower bounds of drift for any ground motion for which the spectral acceleration is known.

## SIMPLE DRIFT-BASED DESIGN

Figure 2, in combination with eqns. 13 and 14, can be used to determine the spectral acceleration which will be required to generate the range of $D_{r2}(0)$ for a reinforced concrete frame structure of a given height when subject to a ground motion with a known spectral acceleration. Alternatively, eqn. 12 can be used to estimate drift from spectral acceleration for a frame structure with known period.

Drift-based design requires that appropriate drift limits be set. Vision 2000 [SEAOC 1995] recommends the following performance limits: Operational (maximum drift = 0.5 %), Life Safety (maximum drift = 1.5 %) and Near Collapse (maximum drift = 2.5 %).

After the design as been more or less completed then an elastic eigenvalue program should be used (with appropriate stiffness reductions to take into account of cracking) to obtain a better estimate of T to ensure that it is bounded by $T_L$ and $T_U$. If this is not the case, then the proportions of the designed frame are considered exceptional and the designer should review both the design requirements and the design calculations.

## NUMERICAL EXAMPLE

The task in this example is to determine the expected drifts for a 9 storey RC frame (H = 30m) in San Francisco at two return periods, using Fig. 3 that shows the variation of median values of $S_a(1.0)/g$ with return period for a number of Canadian and US locations [USGS 1999] In accordance with the previous discussion, it is assumes that California earthquake records may be represented by the NI ensemble (i.e. a = v)

As indicated previously, $T_L$ and $T_U$ for this structure are 0.995s and 1.43s respectively, which can be approximated as 1.0s and 1.5s. Since uniform hazard spectra (UHS) are not available for T=1.5s, it is assumed here that their $S_a/g$ varies with 1/T in this period range, i.e. $S_a(1.5) \approx 0.67\, S_a(1.0)$. Drifts will be checked at both $T_L$ and $T_U$.

For a return period of 500 years, which corresponds approximately to the 10% in 50 years probability of exceedance level used in most design codes, Fig. 3 yields $S_a(1.0) = 0.6g$, and therefore $S_a(1.5) = 0.4g$. For this ensemble, at an excitation of a = 1g , Fig. 2a shows that $S_a(1.0) = 1.4g$ ; consequently the UHS is equivalent to scaling the NI ensemble to a peak ground acceleration a = 0.6/1.4 = 0.43g. This is very close to 0.4, the value applicable in San Francisco in previous editions of the UBC.

The curves in Fig. 2b can be approximated as follows:

[Drift for $T_L$]/ [$S_a/g$] = 1.75T (for a = 1g), and

[Drift for $T_U$]/ [$S_a/g$] = 2.5T (for a = 1g)

For this example, using the 500 year spectral accelerations given previously

Drift for $T_L$ = 1.75 x 1 x 0.6 = 1.05%, and

Drift for $T_U$ = 2.5 x 1.5 x 0.4 = 1.5%.

If the structure is designed at $T_U$, the maximum drift will be essentially equal to the SEAOC Vision 2000 life safe limit. Whereas these drifts represent elastic values, their post-yield counterparts will be of the same order of magnitude.

Now consider a return period of 2500 years, which corresponds approximately to the 2% in 50 years probability of exceedance level now being used in the Uniform Building Code [International Conference of Building Officials 1997](with a multiplier of 2/3). This multiplier is intended to bring design forces and displacements in the western US back to approximately the levels calculated using the 10% in 50 years values as done above.

From Fig. 3: $S_a(1.0)/g = 1.0$ and consequently, $S_a(1.5)/g = 0.67$,. Using the same approach as above, the design drifts would be as follows:

Drift for $T_L$ = 1.75 x 1.0 x (2/3 x 1.0) = 1.17%, and

Drift for $T_U$ = 2.5 x 1.5 x (2/3 x 0.67) = 1.68%.

These values are only slightly higher than those estimated directly from 10% in 50 years spectral values. Given the approximate nature of these drift estimates, the conclusions of drifts at or below life safe limits still applies.

Estimates of drifts at the 2500 return period can be made by removing the 2/3 factor used for design purposes, resulting in values ranging from 1.75% to 2.5%. Maximum drifts at this return period can approach the SEAOC Vision 2000 near collapse limit.

Several observations can be made from this example. First, the sensitivity of drift to period is somewhat larger than that indicated in Fig. 2a. This is the case because the ratio of $S_a(1.5)/S_a(1.0)$ for the NI ensemble is 0.57 rather than 0.67 for the UHS, as assumed in this example. The assumption in this example is more conservative and is also consistent with the normal simplified design spectra for which spectral velocity is constant in the intermediate period region, leading to $S_a$ varying with 1/T.

Second, as would be expected, the convolution (i.e. multiplying the 2% in 50 years hazard by 2/3 to bring forces back to 10% in 50 years levels) adopted in UBC97 produces essentially the same estimates of maximum drift.

Third, ground motions at the 2500 year return period, can result in near collapse levels of lateral drift if the period approaches $T_U$. This provides a sense that return periods for structural collapse of normally proportioned structures will be somewhat in excess of 2500, i.e. with probabilities of exceedance below the 2% in 50 years level.

## DISCUSSION AND CONCLUSIONS

The approach presented in this paper and the example using this approach have shown that maximum drifts for frame structures can be estimated quite easily using the concept of drift spectra. Upper and lower bounds for the drift of a frame structure of a given height and subject to ground motions with a specified spectral ordinates (either the spectra for actual earthquakes or the ordinates of a uniform hazard spectrum) can be determined very simply. These bounds can be used by designers to determine the sensitivity of the particular structure to lateral drift. This information can then be used in proportioning the structure to meet drift performance criteria.

For the particular example considered, which is for one of the most seismic locations in the U.S., life safe drift performance can easily be met by normally proportioned RC frame structures; drifts are well below that limit if structures are on the stiff end of the normal range. RC frame structures of the considered height (30m) at locations with lower spectral ordinates would be even less sensitive to drift considerations. Fig. 2a shows that structures with periods in the range 0.5 to 1.0s would have slightly larger drifts, with maximum values (i.e. for $T_U$ = 0.5 to 0.7s) approximately 30% larger than those indicated in this example.

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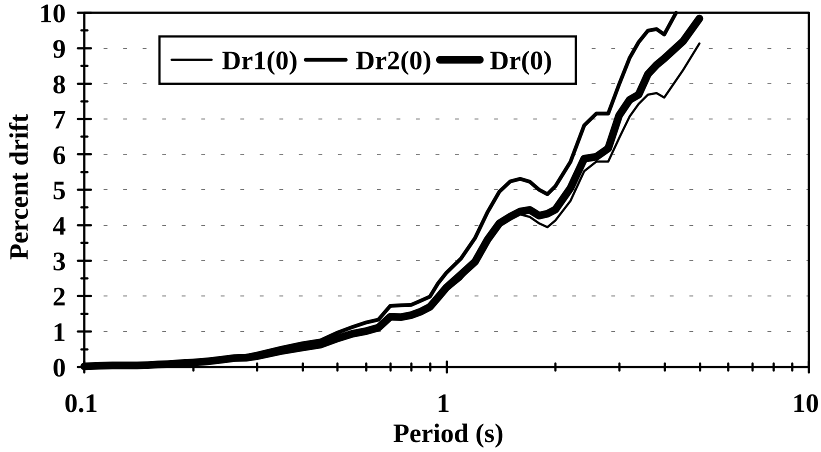
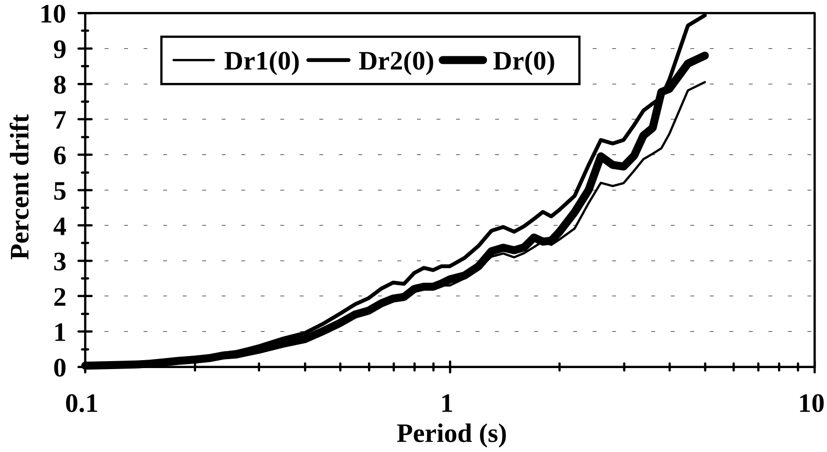
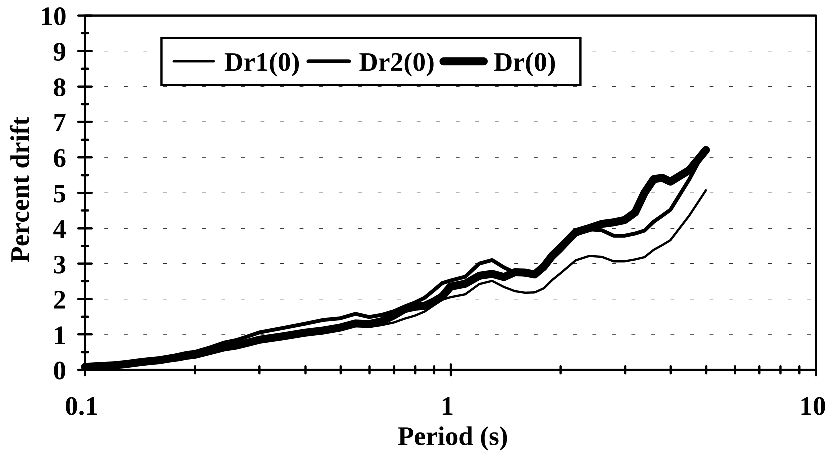



(a) NL Ensemble (low a/v)

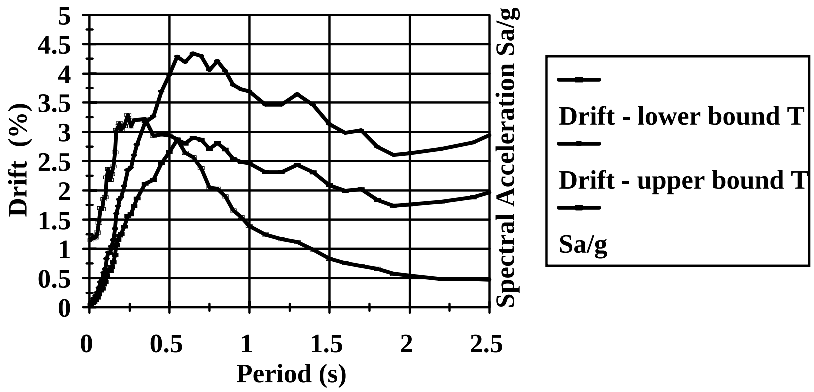



(b) NI Ensemble (intermediate a/v)

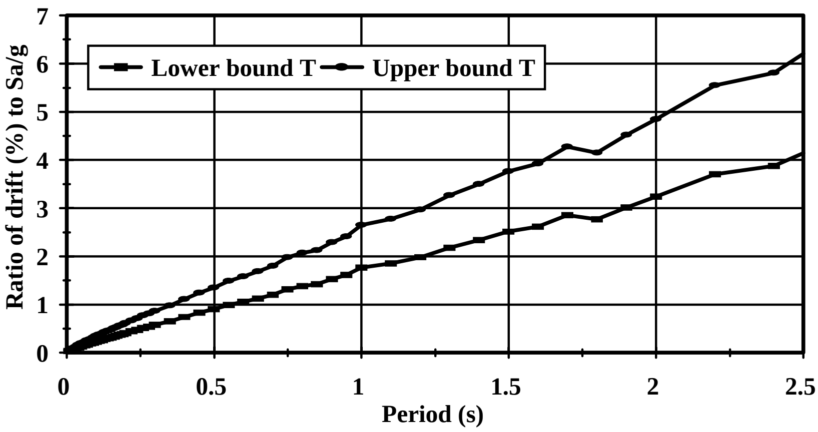



(c) NH Ensemble (high a/v)

Figure 1 Mean Plus One Standard Deviation (M+SD) Drift Spectra for H = 30 m




(a) M+SD Spectra for NI Ensemble Scaled to v = 1 m/s




(b) M+SD Response Ratios for NI Ensemble

Figure 2 Response of Reinforced Concrete Frames Using Goel & Chopra Period Formulae

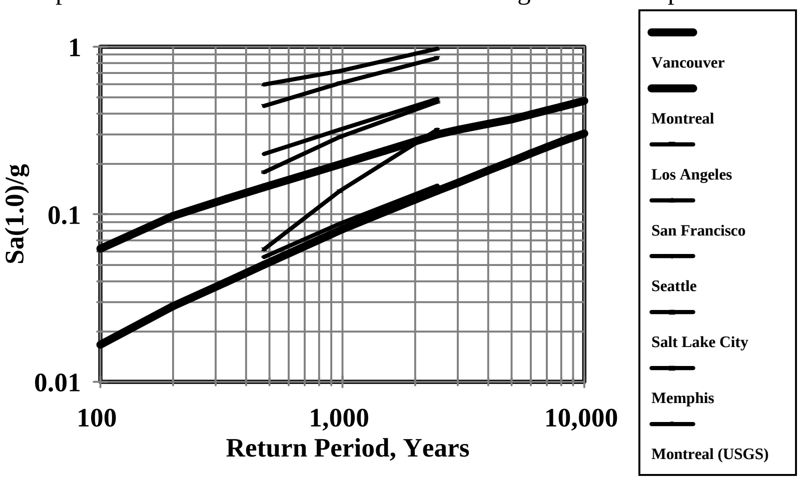



Figure 3 Seismic Hazard at Period of 1.0s, Selected Canadian and US Locations