COMPATIBLE ACCELERATION AND DISPLACEMENT SPECTRA FOR SEISMIC DESIGN CODES

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SUMMARY

The development of displacement-based approaches to assessment and design has created the need for reliable displacement response spectra for a wide range of response periods and damping ratios. The displacement spectra obtained from conversion of the acceleration spectra in most codes are unrealistic in both shape and amplitude. Recent work has led to the development of simple procedures to construct displacement response spectra for specific design earthquake scenarios. In this paper, such an approach is developed for the construction of compatible acceleration and displacement response spectra for inclusion in seismic design codes, on the basis of three ground-motion parameters for which hazard maps can be easily developed. The results reflect the influence of magnitude, distance and site classification on both the amplitude and shape of the spectra.

THE NEED FOR DISPLACEMENT RESPONSE SPECTRA

Deformations are increasingly being considered as more appropriate measures of seismic performance than forces. Procedures for a complete reversal of the conventional force-based design approach have been made alongside methods of accounting for member deformations in seismic design. Such procedures require the rigorous definition of displacement spectra for seismic design. Nevertheless, displacement spectra are also required for other applications. For structural response periods beyond about 2 seconds, displacement is much more relevant than force. Examples of structures with periods well beyond the limits of acceleration or velocity sensitivity are long-span bridges (cable-stayed, suspension), high-rise buildings and isolated structures, both buildings (base-isolated) and bridges (pier-head isolated). Therefore, the spectra derived below are of relevance to a wide range of structural forms and applications.

ACCELERATION AND DISPLACEMENT RESPONSE SPECTRA

The introduction of displacement-based design creates the need for reliable response spectra of relative displacement (SD) for several damping levels and as wide a range of response periods as possible. The simplest way to obtain such spectra would be to convert the absolute acceleration response spectra (SA) from existing seismic design codes using the relationship:

$$SD = SA\left(\frac{T}{2\pi}\right)^2 \qquad (1)$$

where T is the response period in seconds. The displacement spectra obtained in this way from most seismic codes have ordinates that increase indefinitely with period, either linearly, as in the case of the codes from Japan and New Zealand, or even parabolically, as in the case of UBC (Bommer & Elnashai, 1999). There are a few exceptions to these trends, such as the French seismic code, Eurocode 8 (EC8) and the NEHRP provisions (FEMA, 1997), in which the ordinate of the displacement spectrum is constant beyond a certain period. In the NEHRP provisions, the period is 4 seconds, in EC8 it is 3 seconds and in the French code it is a function of the site classification, ranging from 2.7 seconds for rock sites to 4.4 seconds for soft soil sites. However, even the spectral shapes in these codes result in more realistic displacement spectra, code acceleration spectra tend to be

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conservative at longer periods with the result that the long-period ordinates of the displacement spectra are unnecessarily high. This has been shown to be the case for EC8 (Tolis & Faccioli, 1999), which is consistent with the fact that the EC8 acceleration spectrum provides a good fit to average European acceleration spectra for larger events (M_s 7) but is increasingly conservative for smaller earthquakes.

Since none of the existing code acceleration spectra seem to provide suitable displacement response spectra, regression analyses were performed to derive attenuation relationships directly for ordinates of spectral displacement (Bommer *et al.*, 1998). The data set employed for these regression was essentially the same as that employed in the derivation of attenuation relationships for ordinates of spectral acceleration in Europe by Ambraseys *et al.* (1996), with the exception that all data from earthquakes of small magnitude (Ms < 5.5) were removed and a two records, from the 1976 Friuli and 1995 Aegion earthquakes, were added to the data set. Each record was individually processed using an elliptical filter, gradually increasing the low cut-off frequency from 0.1 Hz until the displacement time-history found from double integration appeared physically reasonable and little improvement was obtained from further decrease of the cut-off period. The regressions were performed using the same attenuation model as that employed in the study of Ambraseys *et al.* (1996):

$$log[\,SD(\xi,T\,)] = C_1 + C_2 M_s + C_4\,log(\,r\,) + C_A S_A + C_S S_S \qquad (2)$$

where ξ is the damping (as a percentage of critical), S_A is binary variables taking a value of 1 for stiff soil sites and 0 otherwise, S_S being similarly defined for soft soil sites. The site classification scheme is also the same as that employed by Ambraseys *et al.* (1996), adopted from Boore *et al.* (1993), based on the average shear wave velocity, V_s, over the upper 30 m at the site: rock sites are those with V_s values greater than 750 m/s, soft sites those with V_s less than 360 m/s and stiff soil sites are those with intermediate values. The term *r* is defined by:

$$r = \sqrt{d^2 + h_0{}^2} \qquad (3)$$

where *d* is the shortest distance from the surface projection of the fault rupture in km and h_0 is a regression coefficient determined together with C_1, C_2, C_4, C_A and C_S. The regressions were performed for damping ratios of 5, 10, 15, 20, 25 and 30% of critical and for periods up to 3.0 seconds, beyond which the data was insufficient, since each record was only employed in regressions for periods up to the cut-off period applied in the filtering.

A number of important observations could be made from the resulting attenuation relationships, which allowed considerable simplification of their presentation. The influence of the damping ratio is almost constant across the period range and hence it is only necessary to provide the relationships for a single damping level. Using the 5% damped spectrum as the standard, the ordinates for other damping levels can be found by applying the factor η given by the following expression:

$$\eta = \sqrt{\frac{10}{5+\xi}} \qquad (4)$$

These damping correction factors lie between those presented in EC8 for the acceleration spectrum and those proposed by Tolis & Faccioli (1999) for displacement response spectra.

The second important observation was that the influence of magnitude on the shape of the displacement spectrum is very strong, even more so than on the shape of the acceleration spectrum, and the site conditions also have a pronounced effect on the spectral shape. However, the influence of the distance on the shape of the displacement spectra is small, with very little period-to-period variation of the coefficient C_4 in Eq.(2). As a result of this observation, it was possible to develop a simplified parametric form for the displacement spectra to be defined for damping levels up to 30% of critical and periods up to 3 seconds (Bommer & Elnashai, 1999). The displacement spectrum can be fully defined with only two control periods and their corresponding ordinates, which are presented as functions of magnitude and site classification, and a single factor to account for the decay of spectral amplitude with distance.

A limitation of the simplified parametric presentation of the displacement spectrum is that if converted to SA using Eq.(1) it results in very unrealistic acceleration ordinates at short periods, where displacement ordinates are always very small. This does not in any way prevent the results of Bommer & Elnashai (1999) being employed for displacement-based design where an earthquake scenario can be defined in terms of magnitude and distance. The lack of compatibility between SD and SA was not regarded as a problem since in that study the particular method of displacement-based design presented employs a substitute elastic structure to represent the inelastically deformed structure. Hence, the horizontal axis of the displacement spectrum refers to a period, which is entirely different from the period defined on the horizontal axis of the acceleration spectrum, which is the initial, elastic period of the structure. However, for seismic design codes it is preferable to have acceleration and displacement that are fully compatible. Furthermore, for code applications, it is necessary to map one or more simple parameters that can be used to construct the spectrum, thus rendering the influence of magnitude and distance invisible (except perhaps for the case of the near-source factors in UBC97) to the user.

NEW MAPPING PROCEDURES FOR CODE SPECTRA

Most current seismic design codes present only a single zonation map from which a factor is read and used to anchor the elastic acceleration spectrum. Since the shape of acceleration spectrum is strongly influenced by magnitude and also varies with distance, the use of a constant shape anchored to a zonation factor that is effectively the peak ground acceleration will inevitably result in non-uniform hazard levels across the period range (McGuire, 1977). Some codes have adopted alternative approaches so that the variation is spectral shape can be reflected in the design spectrum. The most explicit of these is the Canadian code, which presents zonation maps for peak ground acceleration (PGA) and peak ground velocity (PGV), and both are used to construct the acceleration spectrum (Basham *et al*., 1985). The zonation factor related to PGV controls the spectral amplitudes at intermediate periods and the short-period amplitudes are dependent on the ratio of PGA/PGV. A similar approach has been adopted in the 1984 seismic code of Colombia. Other seismic codes have adopted alternative approaches to take account of the different spectral shapes resulting from nearby earthquakes of moderate magnitude and distant earthquakes of large magnitude. The Spanish code of 1992 achieves this through contours of a factor K on the zonation map, superimposed on the basic PGA-related factor, which influences the long-period ordinates of the reponse spectrum. The seismic codes of China and Portugal simply specify different spectra for near and distant earthquakes, the shapes reflecting the influence of magnitude and distance.

In this study a new approach is proposed that enables the simple construction of the elastic acceleration response spectrum from which a reliable displacement spectrum can be obtained using Eq.(1). In many ways the proposal is a resurrection of the method of Newmark & Hall (1969, 1982), which was based on the assumption that the short-, intermediate- and long-period portions of the spectrum can be controlled by the acceleration, velocity and displacement of the ground respectively. The influence of PGA on the short-period spectral amplitudes and PGV on intermediate-period amplitudes has been confirmed by more recent studies (Sucuoglu & Erberik, 1998) and forms the basis of the approach used in the Colombian and Canadian seismic codes. The limitation with the Newmark & Hall (1969) method, when it was originally proposed, was the difficulty in estimating the ground displacement. As a result, a linear relationship was proposed so that PGD could be estimated directly from PGA:

$$PGD = \alpha . PGA \qquad (5)$$

If both quantities are measured in units of centimetres and seconds, then the value of α is 0.093. This approach is reflected in the study of Tolis & Faccioli (1999), adapted from EC8, in which the value of α takes a value of 0.036, 0.054 and 0.065 for rock, stiff soil and soft soil sites respectively. It is interesting to note that the original expression in EC8 for the intermediate soil class gives the same value of α as proposed by Newmark & Hall (1969). The shortcoming with such relationships is the implication that PGA and PGD scale by the same amount with magnitude, distance and site classification, which is not supported by the available data. In Table 1 regression coefficients are presented for PGA, PGV and PGD, using the same attenuation model as in Eqs. (2) and (3). The PGA coefficients are from Ambraseys *et al*. (1996) and the PGD coefficients from Bommer & Elnashai (1999); the PGV coefficients have been derived using exactly the same data set as for PGD.

Table 1 Coefficients of attenuation relationships of the form of Eq.(2)

Parameter	C_1	C_2	h_0	C_4	C_A	C_S	σ[log(Y)]
PGA (cm/s^2)	1.512	0.266	3.5	-0.922	0.117	0.124	0.25
PGV (cm/s)	-0.195	0.390	4.5	-1.074	0.142	0.185	0.27
PGD (cm)	-1.757	0.526	3.5	-1.135	0.114	0.217	0.32

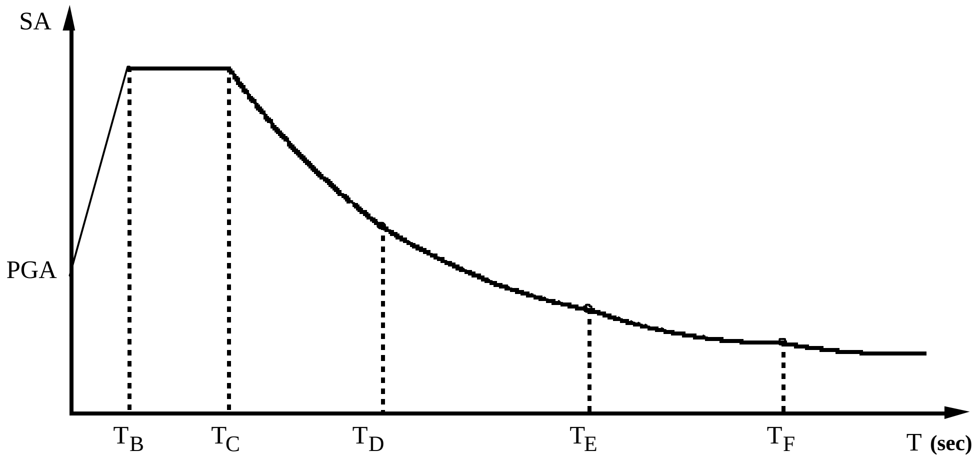
Before discussing the implications of these coefficients, it is important to make some comments regarding the equation for PGD since it is well-known that the displacement time-history is highly sensitive to the processing applied to the accelerograms. The filter parameters used for each accelerogram were selected individually and although there is undoubtedly a degree of uncertainty regarding the values of PGD, a reasonable level of confidence can be assumed; errors are as likely to be positive as negative and hence there is reason to believe that the mean values estimated are of the right order. Although direct comparison is not possible due to different definitions of the independent variables, the values of PGD predicted by the coefficients in Table 1 are comparable to those predicted by the relationships of Gregor & Bolt (1997). Another problem that exists with ground displacements is that the peaks may be due to several different influences, including the fault offset and surface waves in basins, which renders a single predictive function inappropriate. Due to the baseline correction and elliptical filtering applied to the records in the dataset used in thus study, it can be assumed that any displacement related to fault offset has been removed. The distribution of the data in magnitude-distance space also supports the contention that surface waves do not exert an appreciable influence on the calculated values of PGD. Of course, as more digital accelerograms are obtained, these equations can be further refined, but the resulting changes are not likely to be very large.

The coefficients in Table 1 reveal many interesting characteristics, the first being the increasing dependence on magnitude from acceleration to displacement. However, the influence of distance is similar for all three parameters, although it would be expected that PGA would decay more rapidly with distance being associated with higher-frequencies. The influence of soft soil sites on the amplitude of the motion also increases very significantly from PGA to PGD and this effect, relative to that of stiff soil sites, also increases. The only result that would not be expected is the fact that the influence of stiff soil sites is greater for PGV than PGD, but this may be a result of the limited size of the dataset employed.

Taking up the idea of Newmark & Hall (1969), proposed 30 years ago, it is proposed that hazard maps be drafted for 475-year values of PGA, PGV and PGD. The normal practice would be to produce these maps for bedrock motions and then accommodate the influence of different site classification through tabulated coefficients, in the case of UBC97 these tabulated values reflecting the influence of soil non-linearity. For simplicity in these illustrative examples, it may be assumed that separate maps are produced for each of the three site categories. There is an advantage in this approach, this being that if future equations, supported by expanded datasets, use a more sophisticated form that that of Eq.(2), allowing the influence of the soil to be also a function of magnitude and distance (and hence amplitude), then the effect of soil non-linearity could be directly incorporated. The proposed spectral forms are shown in Figure 1 and are directly compatible through the relationship in Eq. (1).

Following the procedure used in the majority of current seismic codes, the acceleration spectrum is anchored to the value of PGA at zero period and the maximum amplitude of the spectrum is determined assuming an amplification factor of 2.5. The next control parameter to be fixed is the corner period T_C and for this use is made of the ratio of PGV/PGA. Tso *et al.* (1992) have shown that the ratio of PGA/PGV indicates the relative frequency content of the ground motion and hence PGV/PGA reflects the period content. Since the short-period amplitudes are related to PGA and the intermediate-period amplitudes to PGV, and since the period T_C can be considered as the transition between these two sections of the spectrum, it can be assumed that T_C could be directly and linearly related to PGV/PGA. It has been found that a constant multiplier on the ratio yields values of T_C that are consistent with the acceleration spectra predicted by the relationships of Ambraseys *et al.* (1996). The value of T_B can then be determined directly from T_C, using the multiplier of 0.25 from EC8 or 0.20 from UBC97.

The next step is to fix the constant displacement plateau that begins at the control period T_D. There are two options that can be used here, the first being to follow the Newmark & Hall (1969) procedure and scale the amplitude of the SD plateau directly from PGD. It is found that a multiplier between 2.3 and 3.0, increasing with the transition of site classification from rock to soft soil, yields a good fit to the displacement spectra predicted by the parameterised format of Bommer & Elnashai (1999). The corner period T_D is then determined from the intersection of the linearly increasing section of SD beyond T_C and the amplitude of the plateau. However, a more elegant approach is proposed, following the same reasoning used to determine T_C. In the same way that the ratio PGV/PGA is an indication of the transition period from the acceleration-controlled portion of the spectrum to the velocity-controlled portion, it is reasonable to assume that the ratio PGD/PGV is an indication of the transition period to the displacement-controlled section. It has been found that a constant multiplier on the PGD/PGV ratio does indeed yield spectra that are consistent with those predicted by Bommer & Elnashai (1999) over the range of magnitudes of engineering interest, being slightly conservative in the near-field and slightly unconservative at greater distances.



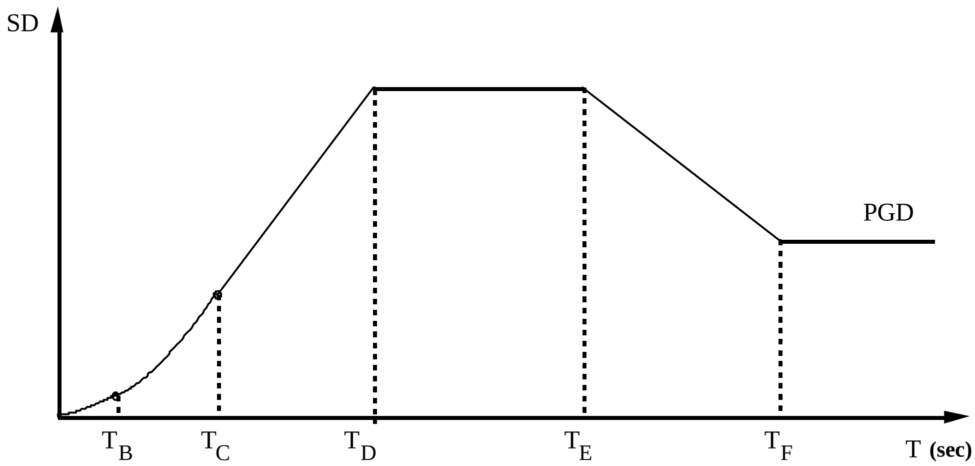


Figure 1. Control periods for compatible acceleration and displacement spectra.

CONSTRUCTION OF COMPATIBLE SPECTRA

Once the appropriate values of PGA, PGV and PGD have been read from the hazard maps, the acceleration spectrum can be constructed using the following simple expressions:

$$T_C = 5\left(\frac{PGV}{PGA}\right) \qquad T_B = \frac{T_C}{4} \qquad T_D = 8\left(\frac{PGD}{PGV}\right) \tag{6}$$

$$0 \le T \le T_B \qquad SA(T) = PGA\left[1 + \frac{T}{T_B}(2.5\eta - 1)\right] \tag{7}$$

$$T_B \le T \le T_C \qquad SA(T) = PGA(2.5\eta) \tag{8}$$

$$T_C \le T \le T_D \qquad SA(T) = PGA(2.5\eta)\left(\frac{T_C}{T}\right) \tag{9}$$

$$T_D \leq T \leq T_E \qquad SA(T) = PGA(2.5\eta)\left(\frac{T_C T_D}{T^2}\right) \qquad (10)$$

The value of η can be obtained from Eq.(2) and the displacement spectrum can be determined form Eq.(1). The data currently available does not permit the determination of the period T_E although it is likely to be at least 3-4 seconds and to increase further for larger magnitudes and softer sites. The vast majority of buildings would be more than adequately covered by the spectrum defined up to periods of 4 seconds (Bommer & Elnashai, 1999) and exceptional structures of longer fundamental period would in any case warrant special studies.

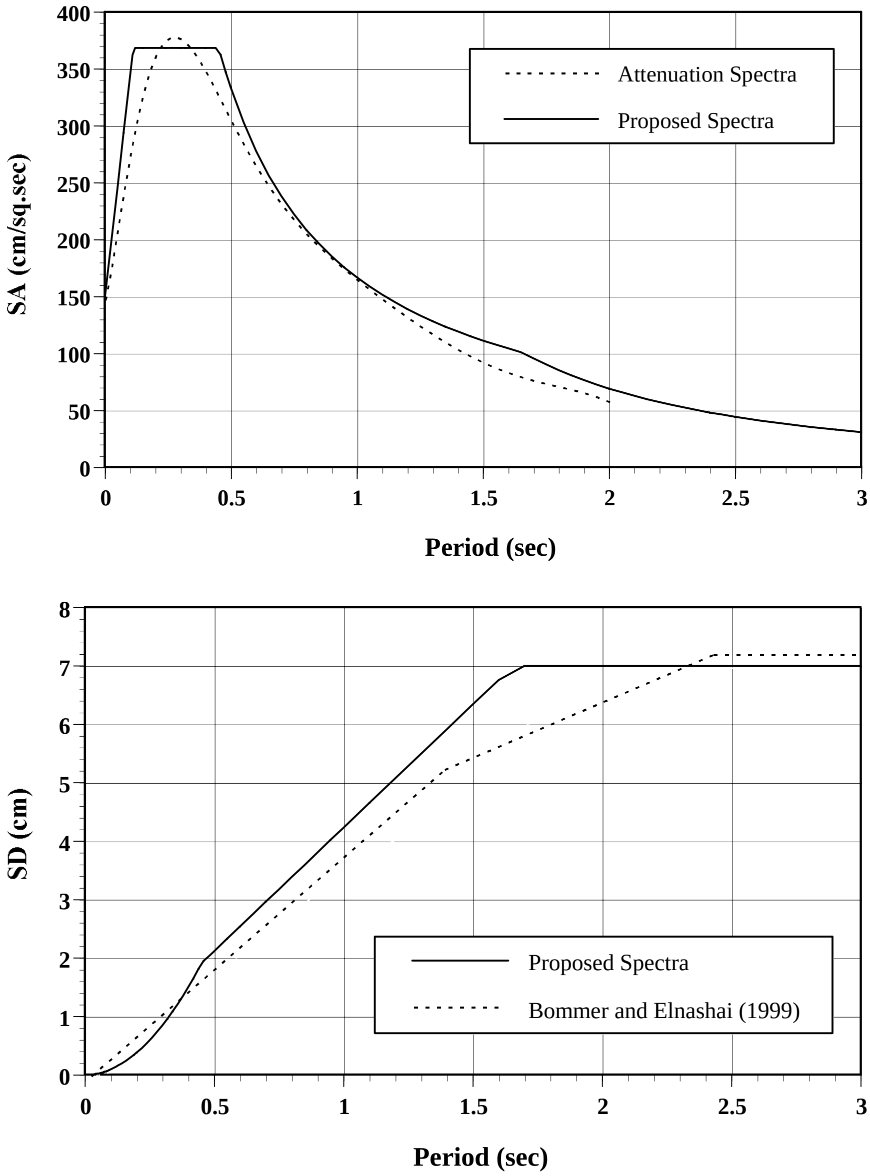


Figure 2. Acceleration and displacement spectra (ξ=5%) for rock site at 20 km from an M_s 7 earthquake.

Figures 2-3 show acceleration and displacement spectra for 5% damping for a number of scenarios in terms of magnitude, distance and site classification. The spectrum are constructed using the attenuation coefficients in Table 1 and are compared with the acceleration spectrum of Ambraseys *et al.* (1996), using the coefficients as smoothed by Tsangaris (1996), and the displacement spectrum of Bommer & Elnashai (1999).

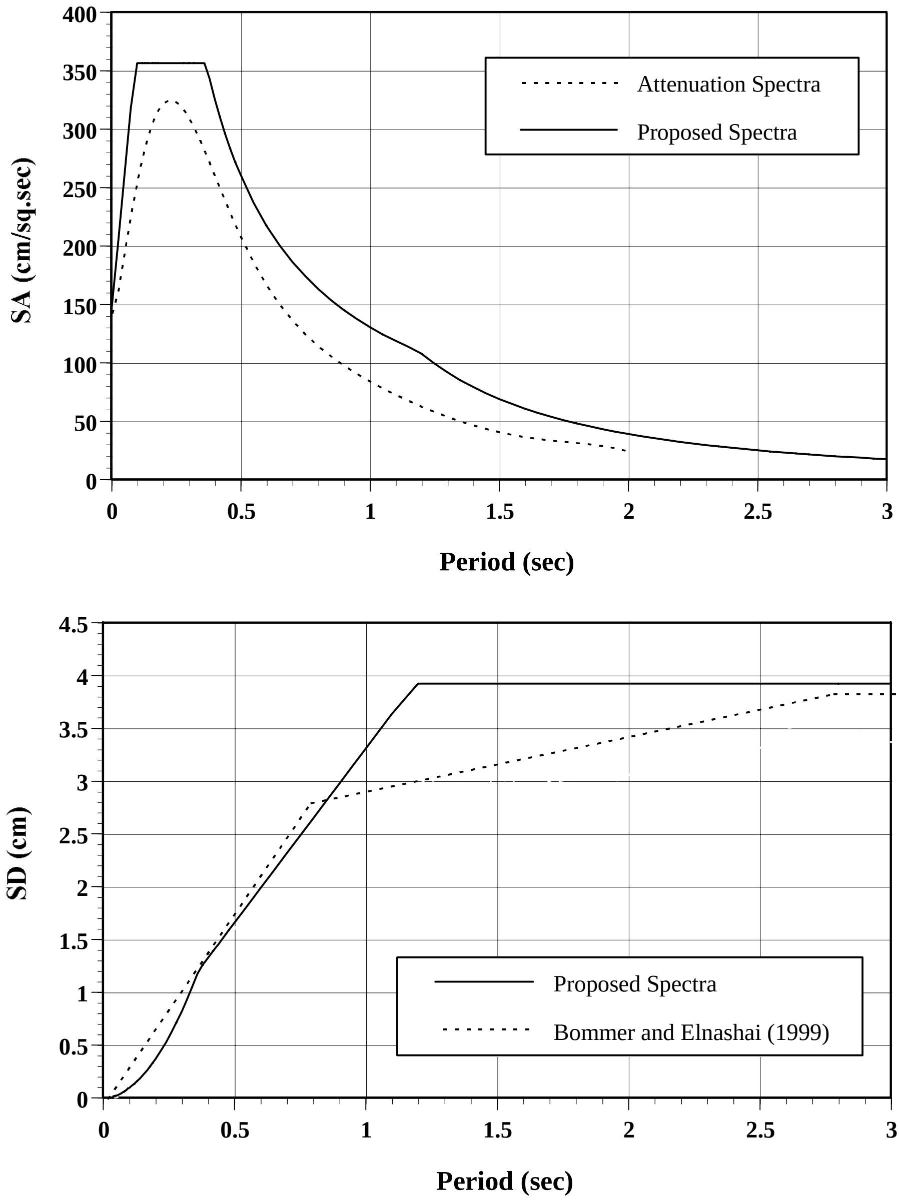


Figure 3. Acceleration and displacement spectra (ξ=5%) for a soft soil site at 10 km from an M_s 5.5 earthquake.

DISCUSSION AND CONCLUSION

A simple procedure has been presented which facilitates the incorporation of compatible acceleration and displacement response spectra into seismic design codes. The spectra can be constructed using values of PGA, PGV and PGD, for which hazard zonation maps can be easily developed. The resulting spectra provide an acceptable fit to both the acceleration and displacement response ordinates predicted by attenuation relationships derived using European strong-motion data. Since both spectra reflect the influence of magnitude on both amplitude and shape, this new approach also represents an improvement for mapping acceleration spectra for codes as well as simultaneously providing reliable displacement spectra. As more reliable strong-motion records are obtained from digital accelerographs it will be possible to continue to refine the attenuation relationships for PGV and particularly for PGD. High quality digital data from large earthquakes may also be able to provide insight into the selection of appropriate values for the control periods T_E and T_F.

The authors believe that future refinements to the attenuation relationships are unlikely to result in large changes and therefore it is proposed that the new approach could be adopted in seismic codes as they pass through the continuous process of revision and updating. The method provides a simple and elegant procedure to obtain elastic response spectra for both force-based and displacement-based design procedures.

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