PORTFOLIO THEORY FOR EARTHQUAKE INSURANCE RISK ASSESSMENT

Weimin DONG[1] And Felix S WONG[2]

SUMMARY

This paper presents an approach to quantifying portfolio risks that acknowledges the importance of correlation between losses at different locations (loss correlation, in shorthand). The approach is event-based: a group of events and their respective occurrence rates are identified to represent the potential overall risk in the region, and the loss of the portfolio for each event as given by engineering models is recorded. From these losses, the exceedance probability curve is developed to predict the probability that the portfolio loss will exceed a certain threshold. A key innovation of the method is the introduction of a general correlation function that embodies loss correlation contributions from geographic concentration, variability in building vulnerability, uncertainty in soil amplification, and choice of attenuation model. These effects are quantified using "diversification factors" which can be computed readily on the policy and portfolio levels. The variance of portfolio loss is then calculated based on this function. The method is not exact when compared with the complete simulation approach (which is currently viable only in theory), but enjoys an attractive balance of accuracy and computation efficiency that makes quantitative portfolio risk assessment possible. Compatibility and integration with contemporary financial risk assessment methodologies are discussed

INTRODUCTION

Insurance companies that issue catastrophe policies such as for earthquakes and hurricanes are concerned with their probable maximum loss (PML). Their portfolios consist of many individual policies, and the maximum loss is an aggregation of the losses from the individual policies. Reinsurance companies that issue catastrophe treaties to primary insurers are also concerned with their portfolio risk. In this case, the risk is an aggregation of the risks of the primary insurers' portfolios.

This paper discusses how portfolio losses can be determined from their component (policy or cedant insurer) losses. For a portfolio with assets at many different locations, the mean of the aggregated loss is simply the sum of the mean location losses. However, the computation of the standard deviation of the aggregated loss is more complicated because losses at any two locations may be correlated. Such correlation is known to have significant impact on the distribution of the aggregated loss. Less well known but equally important is the fact that the allocation of losses to the reinsurer, such as under an excess-loss treaty, depends not only on the mean of the aggregated loss but even more so on the loss distribution. Hence, location loss correlation plays an important part in quantifying portfolio risk.

Loss correlation can materialize in two ways. The losses at any two locations due to all relevant events (i.e., events deemed to have an effect on at least one of the locations of interest) may be correlated because the same event(s) may lead to damage at both locations. Such correlation exists even if the event occurrence rates and the corresponding loss estimates have no uncertainties. We refer to this type of correlation, **events correlation** (note the plural form for event). The other type refers to correlation of losses at two locations given an event, viz., how the uncertainty in the aggregated loss is affected by the uncertainties in the location losses due to a particular event and the tendency of those uncertainties to vary together. We refer to this type as **correlation given an**

[1] *Risk Management Solutions Inc, Menlo Park, CA. USA. Email:weimind@ca.wai.com*
[2] *Weidlinger Associates Inc, Los Altos, CA. USA. Email:felix@ca.wai.com*

event. Note that correlation given an event exists only if the losses are uncertain. As we shall show, both types contribute to the uncertainty in portfolio risk.

The methodology that addresses the former is called the Event-Loss Table (ELT) approach, while the method for the latter is called the global diversification factor approach. They will be described herein following the presentation of a simple example to highlight the importance of loss correlation in insurance decisions, in case that is not immediately obvious. We also indicate how the two sources of correlation interact with each other, and how the ELT and diversification factor methods can be combined to form a complete methodology for risk assessment that applies to not only insurance/reinsurance portfolios but also financial and market risks as well.

EXAMPLE EFFECT OF CORRELATION ON INSURANCE LOSS ALLOCATION

Consider a portfolio with 100 locations and the policy coverage is \$100,000 per location, for a total coverage of \$10 million (100 × \$100,000). The policy has a 5% deductible so that the amount of deductible is 5% of \$10 million or \$0.5 million as indicated in Fig.1b. The treaty terms as shown includes FAC for Facultatives, QS for Quota Shares and SS for Surplus Shares. Note that FAC1 and FAC2 have 20% participation, but different attachment points and limits. QS and SS have participation of 20% and 10%, respectively, and their attachments and limits as indicated in the figure.

Suppose further that the mean damage to all locations is the same, at 10% damage. Hence, the expected loss to the portfolio is \$1 million (10% of \$10 million is \$1 million) as indicated in Fig.1a. For comparison, the distribution of the portfolio loss is presented in Fig.1c and 1d for the totally independent and totally correlated cases, respectively, assuming a coefficient of variation of 1.6 for each location loss. It is clear from these figures that in the totally independent case, the portfolio loss distribution is concentrated (i.e., small standard deviation) and the distribution is centered on the deterministic value of \$1 million (compare Fig.1c with 1a). When the losses are totally correlated, the distribution is spread out over a wide range (i.e., large standard deviation; see Fig.1d).

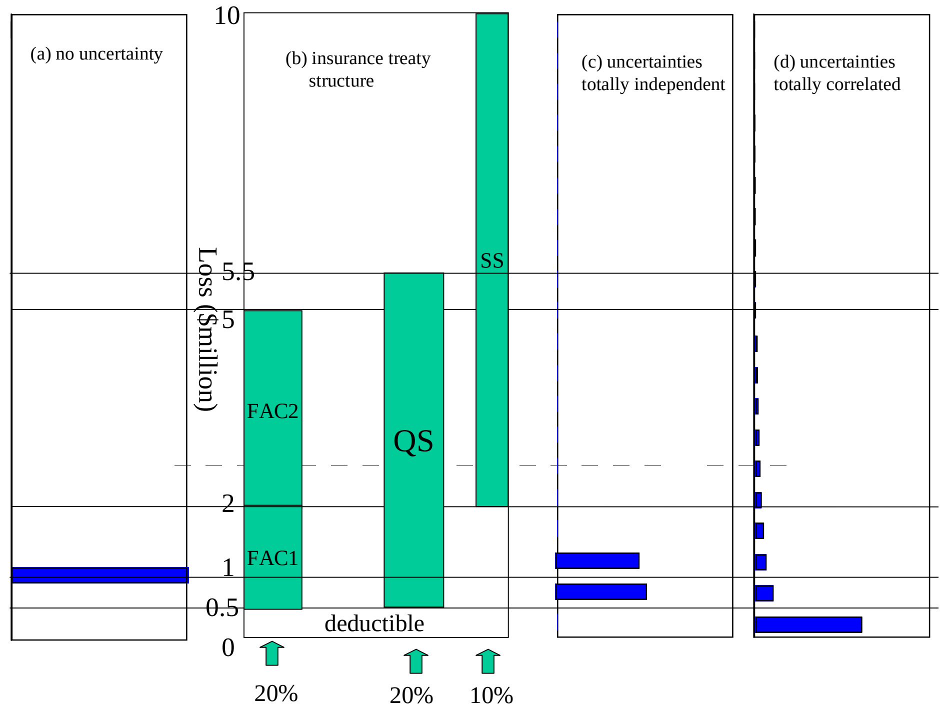

Figure 1. Insurance structure used in example.

These differences in distribution have significant impact on the allocation of insurer/reinsurer losses, as an overlay of Fig.1c (or Fig.1d) with Fig.1b readily indicate. Numerical results summarized in Table 1 serve as a reminder of the importance of including correlation in portfolio loss estimation, although a detailed discussion cannot be included due to space limitation. Note that Insured loss, insurer loss, and reinsurer loss are all effected due to correlation given event.

Table 1. Comparison of Losses (in $) Allocated to Treaties of the Policy.

Treaty	No uncertainty	Uncertainty, but losses between locations are independent	Uncertainty, but losses between locations are totally correlated
FAC1	100,000	100,000	79,446
FAC2	0	0	58,015
QS	100,000	100,000	141,164
SS	0	0	34,980
Reinsurer Loss	200,000	200,000	313,605
Insurer Loss	300,000	300,000	430,486
Insured Loss	500,000	500,000	255,909
Total Loss	1,000,000	1,000,000	1,000,000

THE EVENT LOSS TABLE APPROACH (EVENTS CORRELATION)

Suppose that the risk for a property at a particular location is in the form of a group of events (earthquakes from nearby faults that are judged to have substantial effects on the assets should they occur) with various occurrence rates. Given that an event has occurred, the loss sustained by the property is computed using standard techniques. The event losses for all relevant events are then compiled and collected in a table, called the Event Loss Table or ELT (see Table 2). Each row of the ELT corresponds to a catastrophe event in the group of credible scenarios, and is identified by a number, e.g., Event ID = *j*, with λ_j as the corresponding annual rate of occurrence. It is customary to arrange the events according to the losses L_j. Note that for the moment λ_j and L_j are assumed known without any uncertainty, i.e., they are point estimate values. Uncertainties on rate and loss will be brought in as an extension of the ELT.

Table 2. An Event Loss Table (ELT).

Event ID	Annual Rate	Loss
1	λ_1	L_1
2	λ_2	L_2
:	:	:
j	λ_j	L_j
:	:	:
J	λ_J	L_J

Suppose that each event is an independent Poisson process. For the property of interest, then the average annual loss *E(L)* and the standard deviation of the loss σ are, respectively:

$$E(L) = \sum_{j=1}^{J} \lambda_j L_j \qquad (1)$$

and

$$\sigma = \sqrt{\sum_{j=1}^{J} \lambda_j L_j^2} \qquad (2)$$

where the summation index *J* corresponds to the total number of *independent* events in the ELT, i.e., number of rows in the table.

Given the ELTs for two (properties at two) locations, the correlation between losses at the two locations can be readily established as follows. Let's denote the properties by A and B. The ELTs for A and B are rearranged in such a way that the events in the tables are in the same order. In particular, $L_{j,A}$ and $L_{j,B}$ are the respective losses from A and B given event *j* with the event rate λ_j . It is easy to see that for the combined loss, the average annual loss *E(L)* is simply,

$$\begin{aligned} E(L) &= \sum_{j=1}^{J} \lambda_j (L_{j,A} + L_{j,B}) \\ &= E_A(L) + E_B(L) \end{aligned} \qquad (3)$$

Furthermore, for property A, the standard deviation of loss according to Eq.2 is:

$$\sigma_A = \sqrt{\sum_j \lambda_j L_{j,A}{}^2} \tag{4}$$

whereas that for B, it is:

$$\sigma_B = \sqrt{\sum_j \lambda_j L_{j,B}{}^2} \tag{5}$$

On the other hand, working with the combined loss for both properties, the standard deviation of the combined loss is:

$$\sigma_{AB} = \sqrt{\sum_j \lambda_j (L_{j,A} + L_{j,B})^2} \tag{6}$$

By definition, then, the correlation coefficient, ρ, can be obtained from Eqs.4-6 as:

$$\rho = \frac{\sigma_{AB}{}^2 - \sigma_A{}^2 - \sigma_B{}^2}{2\sigma_A \sigma_B} \tag{7}$$

Note that the concept of ELT as shown in Table 2 is general, and applies readily to an insurance company's portfolio as well. In that case, entries in the loss column denote, instead of losses for a particular property, the portfolio losses for the events. Equation 7 then defines the correlation between portfolios. As illustration, consider the (insurance) industry's portfolio and portfolios from three companies as indicated in Table 3. Company A has a 5% market share and is totally correlated with the industry loss. The correlation coefficient, ρ_A, is then 1. Company B is partially correlated with the industry and, using Eq.6 above, we calculate the correlation coefficient, ρ_B, to be 0.727. Company C is outside of the USA so that any event that affects Company C does not affect the U.S. industry and vise versa. Hence, the correlation coefficient for Company C with U.S. industry, ρ_C, is 0.

Table 3. ELTs for an Industry Portfolio and Three Individual Company Portfolios.

Event	Industry Loss	Comp. A Loss	Comp. B Loss	Comp. C Loss	Annual Rate
1	150	7.5	6	0	0.001
2	100	5.0	5	0	0.002
3	100	5.0	0	0	0.002
4	80	4.0	3	0	0.003
5	60	3.0	0	0	0.003
6	50	2.5	5	0	0.004
7	40	2.0	0	0	0.004
8	30	1.5	4	0	0.005
9	20	1.0	0	0	0.005
10	10	0.5	1	0	0.006
11	0	0	0	15	0.003
12	0	0	0	35	0.005
13	0	0	0	65	0.004
14	0	0	0	32	0.006
15	0	0	0	16	0.004
16	0	0	0	24	0.002

EXTENDED EVENT LOSS TABLE (EVENTS CORRELATION)

When uncertainties associated with the rate of occurrence of an event and the loss estimate given an event are taken into account, the ELT approach can be enriched with the respective uncertainty information incorporated into an extended table, called extended ELT or EELT for short. An example is shown in Table 4, where the "Rate" and "Loss" columns now denote the mean rates and mean losses, respectively, and additional columns such as “CV of Rate” or “CV of Loss” denote the additional information on uncertainty. Even higher levels of uncertainty information can be included in the same fashion[1].

Table 4. An Extended Event Loss Table Showing Additional Uncertainty Information.

Event ID	Mean Rate	CV of Rate	Mean Loss	CV of Loss
1	$\overline{\lambda}_1$	CV_{λ_1}	$\overline{L}_1$	CV_{L_1}
2	$\overline{\lambda}_2$	CV_{λ_2}	$\overline{L}_2$	CV_{L_2}
:	:	:	:	:
j	$\overline{\lambda}_j$	CV_{λ_j}	$\overline{L}_j$	CV_{L_j}
:	:	:	:	:
J	$\overline{\lambda}_J$	CV_{λ_J}	$\overline{L}_J$	CV_{L_J}

The development given in the previous section can be readily applied to the EELT. In particular, it can be shown that with loss and rate uncertainties incorporated, the standard deviation of a portfolio loss can be approximated by[2]

$$\sigma = \sqrt{\sum_{j=1}^{J}\left\{\overline{\lambda}_j \overline{L}_j^2 (1+CV_{L_j}^2)\right\}} \quad (8)$$

which should be compared with Eq.2. We can use this formula to calculate the correlation between two portfolios as done with Eq.7 for the ELT.

Like the ELT, the EELT as shown in Table 4 is general, and applies to a single location or an insurance company’s portfolio as well. In the latter case, entries in the loss columns denote, instead of losses for a particular property, the portfolio losses. Whereas the mean portfolio loss, *L*, is simply the sum of the component losses, the quantification of the *CV* of the portfolio loss and, by extension, the loss distribution, is by no means trivial and constitutes the main subject of this paper. The reason is correlation, but this correlation arises from uncertainties in the estimate of the component losses. It exists even if we are considering only a single event, as delineated in the following. We call this correlation given an event.

WEIGHTING FACTOR APPROACH (CORRELATION GIVEN AN EVENT)

Now, we focus on a single event (e.g., any row in the EELT) and the loss estimates for the portfolio in question. Suppose the portfolio covers *n* locations (or cedant portfolios), and denote the location losses by random variables X_i, *i=1, n* and the portfolio loss by *Y*. *Y* is then a random variable given by

$$Y = \sum_i X_i, \qquad i = 1, n \quad (9)$$

The task is: Given the mean loss for each location, $\overline{X}_i$, and the standard deviation of loss for the location, σ_{X_i}, compute portfolio mean, $\overline{Y}$, and the portfolio standard deviation, σ_Y.

As is well known, the mean of the total loss for the portfolio, or portfolio mean, is:

$$\overline{Y} = \sum_i \overline{X}_i, \qquad i = 1, n \quad (10)$$

and the variance of the total loss, or portfolio variance, is:

$$Var[Y] = \sigma_Y^2 = \sum_{i=1}^{n} Var[X_i] + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \rho_{i,j}\sigma_{X_i}\sigma_{X_j} \quad (11)$$

and $\rho_{i,j}$ is the correlation coefficient between losses at locations *i* and *j*. Note that $\rho_{i,j}$ in Eq.11 is different from ρ in Eq.7; $\rho_{i,j}$ is the correlation between two losses given an event and ρ in Eq.7 is the overall correlation between two losses for all events.

While $\rho_{i,j}$ can be determined, in theory, by considering all possible pairs of locations according to Eq.11, such brute-force approach is impracticable. As an alternative, we suggest using a “global” approach in the form of a weighting factor *f* such that the standard deviation of the total loss can be approximated as follows:

[1] Uncertainties in occurrence rate and property losses are discussed in a companion paper in the same proceedings (see [Wong, Chen and Dong., 2000]).

[2] Space limitation excludes a presentation of the derivation of Eq.8, but the interested reader can consult [Dong, 1999].

$$\sigma_Y = f\sum_{i=1}^{n}\sigma_{X_i} + (1-f)\sqrt{\sum_{i=1}^{n}\sigma_{X_i}^2} \qquad (12)$$

The idea is to use, f, a judiciously chosen weight for the portfolio, to reflect the portfolio's *overall* correlation within a scale defined by total correlation ($f = 1$) at one end and total independence ($f = 0$) at the other. The weight f is then an encapsulation of the major factors affecting correlation of losses at dispersed locations. These factors are discussed below, and a general approach to quantifying them is outlined. Briefly, "diversification factors" are used to express the degree of diversification (concentration) in the policy/portfolio in these factors, in conformance to the fact that a well-diversified policy/portfolio has smaller loss correlation whereas a concentrated policy/portfolio has large loss correlation. The diversification factors, in turn, are related to the equivalent, composite weight f.

FACTORS INFLUENCING LOSS CORRELATION (DIVERSIFICATION)

Major factors of diversification (concentration) affecting policy/portfolio loss correlation include geographic location, vulnerability modeling, soil amplification and hazard attenuation modeling:

- **Geographic Concentration**: Observations in past earthquakes point out the existence of local pockets in which all structures suffered more (or less) severe damage as a group than in neighboring areas. These phenomena are caused by local conditions such as basin effect, and are not included in general attenuation models. From observations of past events, the swing due to these local effects can cause a maximum of one intensity level from the expected value. Assuming uniform distribution within the same pocket, the intensity swings for two locations in the pocket will be totally correlated. The effect of geographic concentration on portfolio loss depends on the portion of assets in such pockets.
- **Parameter Uncertainty in Vulnerability**: If the mean damage ratio for a building class is actually larger than the model, e.g., wood-frame buildings in the Northridge earthquake, then the damage of all such buildings will be underestimated. Hence, the correlation effect on portfolio loss is higher for portfolios consisting of single building types than for portfolios of many mixed types[3]. From ATC data, the average coefficient of variation for the mean damage ratio is 0.387, which can be used to calculate the correlation coefficient for portfolios consisting of buildings of the same class. If a portfolio is well diversified, say, uniformly distributed among five classes, then the coefficient of variation will be smaller, namely, $\frac{0.387}{\sqrt{5}} = 0.173$; there is much less correlation.
- **Uncertainty in Soil Amplification**: When the buildings in a portfolio are spread out over all types of soils, the correlation in loss will be small compared that for a portfolio with buildings on the same soil. For example, suppose soil amplification may lead to a swing in intensity of ±0.5, uniformly distributed. The swing when all buildings are seated on the same soil will be correlated. When the buildings are seated in four different soils, the four uniformly distributed swings will convolve into a bell-shaped distribution.
- **Attenuation Model Uncertainty**: If all buildings are located at the same distance from the rupture, there is a good chance that all ground motion estimates are off due to the use of one attenuation model or another, when compared with portfolios with buildings located at a wide range of distances from the rupture. In the latter case, some buildings at a given distance may be underestimated, while others at a different distance may be overestimated.

Let D_G, D_c, D_s and D_d represent the diversification factors for the above sources, respectively. Computer loss simulation is used to determine the relationship between these diversification factors and the overall weighting factor f, which is stated below without elaboration[4]:

$$f = 0.4076 - 0.02375 * D_c - 0.02775 * D_s - 0.038 * D_d - 0.19975 * D_G \qquad (13)$$

[3] It is assumed that parameter uncertainty in vulnerability of one building type is independent of that in another type, i.e., vulnerability of wood-frame buildings may be underestimated, but that of masonry buildings may be overestimated.

[4] Details are not presented due to space limitation but can be found in [Dong, 1999].

With f given by Eq.13, the standard deviation of the total loss shown in the EELT can be obtained from Eq.12, and the methodology is complete.

SUMMARY AND FINANCIAL APPLICATIONS

This paper has addressed correlation issues that are crucial to portfolio loss estimation. There is correlation between the variations of the mean losses of two locations (or two portfolios) for different events; there is correlation between uncertain losses of two locations given a particular event. Using an overall weight factor that summarizes the correlation contributions from the main physical elements, the aggregated portfolio loss for different participants (insurer or reinsurer) can be reasonably estimated. Ignoring correlation given the event will significantly affect the loss allocation among the insured, insurer and reinsurer. The proposed EELT and weight factor methodology can be used for portfolio loss assessment; it can be used to estimate the average annual loss and variation of the loss for the portfolio. Furthermore, the method can be used to quantify the covariance matrix of various regional losses for an insurance portfolio, and to support optimal capital allocation (e.g., using quadratic algorithms to minimize the overall variance [Markowitz, 1991]). Similarly, a reinsurance company can use the covariance matrix of various cedant portfolios to optimize its global exposure.

In modeling financial portfolio risk, J.P. Morgan developed an application called *RiskMetrics* to determine a quantity called *VaR* (Value at Risk, [Longerstaey and Spencer, 1996]). It uses historical data to obtain the covariance matrix for different sectors of the market, and from this information, derives the portfolio variance and the percentile loss for the risk. In particular, the *VaR* of a portfolio of n instruments at 95% probability is given by:

$$VaR = \sqrt{\vec{V} R \vec{V}^T} \tag{14}$$

where $\vec{V}$ is a vector of *VaR* estimates for the respective instruments,

$$\vec{V} = [w_1 \times 1.65\sigma_1,\ w_2 \times 1.65\sigma_2,\ \ldots,\ w_n \times 1.65\sigma_n] \tag{15}$$

w_i is the fraction of the investment allocated to the i^{th} instrument, and σ_i is the standard deviation of the value of the i^{th} instrument (1.65 σ_i corresponds to its 95% bounds in fluctuation). R is the correlation matrix,

$$R = \begin{bmatrix} 1 & \rho_{12} & . & . & \rho_{1n} \\ \rho_{21} & 1 & . & . & \rho_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \rho_{n1} & \rho_{n2} & . & . & 1 \end{bmatrix} \tag{16}$$

ρ_{ij} is the correlation between the values of the i^{th} and j^{th} instrument, and $\vec{V}^T$ is the transpose of $\vec{V}$. The *RiskMetrics* data bank provides all the instrument performance statistics that are required by Eqs.14-16, notably, the standard deviations σ_i and the correlations ρ_{ij} used in Eqs.15 and 16, respectively.

Properties subject to catastrophe damage such as due to earthquakes and hurricanes can be viewed as another form of asset or instrument that is at risk, and can be treated by *RiskMetrics* in exactly the same way as foreign exchanges or bonds. In particular, the overall risk of a collection of portfolios of properties is the aggregate of the constituent portfolio *VaR*s as computed by Eqs.14-16, when the statistics of property damage due to catastrophes and their correlation are known. Furthermore, real estate assets can be assembled with other financial assets to form a balanced portfolio within the *RiskMetrics* framework since losses due to catastrophes are, under most circumstances, fairly independent of losses due to financial turmoil or currency fluctuations. The methodology described in this paper for catastrophe portfolio risk is compatible with the RiskMetrics approach. The portfolio loss correlation coefficients computed using the EELT and weighting function can be incorporated into the covariance matrix in exactly the same way that risks of portfolios in foreign exchange and bonds are evaluated.

Table 5 below compares the requirements of financial risk with catastrophe risk assessment within the framework of *RiskMetrics*. The column under "catastrophe risk" also serves as a road map for reviewing the developments presented in this paper. The ELT is a convenient and succinct summary of the losses predicted by engineering models. Starting from the basic portfolio ELT, the effects of uncertainties are incorporated and summarized in an extended portfolio table, the EELT, and the portfolio EELTs are then combined per aggregation needs.

Table 5. Financial Risk and Catastrophe Risk Assessment.

	Financial Risk	Catastrophe Risk
Objective	Minimize risk, maximize return; diversification; solvency	
Database	Vast historical database	Very limited and dated • demographic changes • valuation changes • industry concentration changes • applicability concerns
Asset loss prediction	Financial models and historical data	Engineering models calibrated with limited data • propagate uncertainties within engineering models and approximation • aleatory and epistemic uncertainties
Correlation between asset losses	Historical data	Diversification factors at the policy and portfolio levels
Portfolio	Collection of assets	
Portfolio risk (theory)	Aggregate asset risks; use standard deviations and correlation coefficients derived from data	Aggregate asset risks; use standard deviations obtained from engineering model uncertainty analysis, and correlation coefficients derived from diversification factors
Aggregating portfolios	Collection of portfolios	
Portfolio risk (theory)	Aggregate portfolio risks; use standard deviations and correlation coefficients derived from data	Aggregate portfolio risks; use standard deviations of portfolios, and correlation coefficients derived from portfolio diversification factors

ACKNOWLEDGEMENT

The work presented herein is partially supported by NSF grant CMS97-14401, under a joint project between the Wharton School of the University of Pennsyvalnia and RMS, Inc.

REFERENCES

1. Dong, Weimin, *Modern Portfolio Theory with Application to Catastrophe Insurance*, RMS Special Publication, 1999.
2. Longerstaey, J., and Spencer, M., *RiskMetrics – Technical Document*, Morgan Guaranty Trust Company and Reuters Ltd., Fourth Edition, New York, December 1996.
3. Markowitz, H. M., *Portfolio Selection – Efficient Diversification Investment*, Blackwell Publishers, 1991.

Wong, F., Chen, H., Dong, W., and, " Uncertainty Modeling for Disaster Loss Estimation", Proceedings of 12WCEE, Auckland, New Zealand, January 2000.