3D-SEISMIC WAVES IN A DIPPING SURFACE LAYER

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SUMMARY

The method of generalized ray, originally developed for wave guides of parallel layers, was recently generalized to include the divergence effects of up to two layers overlying a penetrable half-space bedrock. The objective of the paper is to investigate the waves from an oblique single point force source, buried and, as well in the limits, acting on the surface and at the interface. Using the source ray of the vertical force, we determine the source rays of the horizontal force components by rotating the Cartesian coordinates and comparing the emittance functions. The phase functions, or arrival times, of a given wave mode for a fixed source - receiver configuration remain invariant. Successive total reflections at the free surface and refraction at the interface render the solution in the Laplace transform domain. A modified Cagniard-de Hoop technique is applied to corresponding pairs of generalized rays to obtain their inverse Laplace transforms. The total seismic response is obtained by summing responses due to all generalized rays arriving at a receiver in a prescribed observation time. The present semi-numerical solution as such serves also as a benchmark for the direct numerical analyses by Finite Difference or Finite Element Discretization.

Recently, the dynamic Green's functions of the linear elastic background of the visco-plastic soil were applied to model the waves emitted from "localized" plastic sources in the region of the plastic zone within the background. The latter, so called multiple field approach, is worked out, currently under the restrictions of plane strain or plain stress, at the expense of an additional convolution integral.

INTRODUCTION

The expansion into plane waves of the spherical waves emitted from a point source renders ray integrals which are ordered systematically according to their arrival times at a fixed receiver in a given source - receiver configuration, for a review see Pao and Gajewski [1]. For line sources of various kinds acting in a layered half space, the generalized ray integrals are numerically integrated using a modified Cagniard-de Hoop method, see e.g. [2-7]. A dipping surface layer overlying penetrable bedrock was considered at first in [5] as the wave guide of SH-waves emitted from a parallel line source. The transformation of the ray integrals for rotated coordinate systems was developed and consequently applied, see again [2-7]. Using Fourier spectra of the received signals and the Dasgupta-Sackman numerical version of the elastic-viscoelastic correspondence, phenomena like the formation of two Rayleigh waves propagating in the viscoelastic half space were explored in [8]. Axisymmetric waves due to point loads have been considered in [1] by means of the Laplace-Hankel transform and extended to the problem of an obliquely applied concentrated force. However, these solutions are only applicable to the parallel layer case. Since we focus on the extension of the plane problem of dipping layers to the three-dimensional wave systems produced by such an oblique single force, we stick to the Cartesian coordinates and consider the three non vanishing displacement potentials of the source rays, respectively and individually for the force components in the vertical and in the two horizontal directions of the infinite space. We also refer to [9] and [10] for preliminary work on the asymmetric point source. Since the source rays of a vertical single instantaneous force are readily available, we use the method of rotated coordinates to determine the non vanishing potentials of the waves emitted by the two horizontal force components. For the plane problem the line

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source of vertical forces was rotated into horizontal position, thus providing the solution to a horizontal line of forces, see e.g. [4] for the surface load. In the three-dimensional case a second coordinate rotation about the vertical axis is performed and, thus, provides the full set of nine non vanishing displacement potentials. Figure 1 illustrates the soil models when restricted to a single surface layer.

SOURCE RAYS FROM AN OBLIQUE POINT FORCE

The solution for the vertical point force with time signature *f(t)*, not necessarily of impulse type,

$$\boldsymbol{F}_z(t) = Q_z f(t)\delta(x)\delta(y)\delta(z-z_0)\boldsymbol{e}_z \tag{1.1}$$

in terms of the displacement potentials (according to the Helmholtz decomposition) renders the source rays in the infinite elastic space. The P-wave,

$$\begin{aligned} \overline{\varphi}^z(x,y,z,s) &= s^2 Q_z \bar{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_P^z \exp(s g_P)\, \mathrm{d}\xi \mathrm{d}\kappa \\ g_P &= i\xi x + i\kappa y - \eta\left|z-z_0\right|,\ \eta = \sqrt{c^{-2}+\xi^2+\kappa^2} \end{aligned} \tag{1.2}$$

and the S-wave,

$$\bar{\psi}_x^z(x,y,z,s) = s^2 Q_z \bar{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Sx}^z \exp(s g_S)\, \mathrm{d}\xi \mathrm{d}\kappa \tag{1.3}$$

$$\begin{aligned} \bar{\psi}_y^z(x,y,z,s) &= s^2 Q_z \bar{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Sy}^z \exp(s g_S)\, \mathrm{d}\xi \mathrm{d}\kappa \\ g_S &= i\xi x + i\kappa y - \zeta\left|z-z_0\right|,\ \zeta = \sqrt{C^{-2}+\xi^2+\kappa^2} \end{aligned} \tag{1.4}$$

$$\bar{\psi}_z^z(x,y,z,s) = 0 \tag{1.5}$$

The density of the carrier medium enters explicitly in addition to the two body (P- and S-) wave speeds, *c* and *C*, respectively,

$$\bar{F}(s) = \frac{\dot{f}(s)}{8\pi^2 s^2 \rho} \tag{1.6}$$

and the emittance functions become with the direction factor with respect to propagation in the positive vertical direction, respectively,

$$S_P^z = -\varepsilon_z,\quad S_{Sx}^z = -\frac{i\kappa}{\zeta},\quad S_{Sy}^z = \frac{i\xi}{\zeta}. \tag{1.7}$$

Equations (1.2)-(1.5) can be rewritten in terms of rotated coordinates, say at first rotated clockwise about the *y*-axis such that

$$x^r = z,\ z^r = -x \tag{1.8}$$

Hence, consequently,

$$Q_z^r = -Q_x,$$

the (negative) component of the horizontal force. The expansion into plane waves requires invariance of the phase function and preservation of the amplitudes. For sake of convenience, we put the point source into the origin, furthermore, the wave modes remain understood:

$$g = g^r = i\xi x + i\kappa y - (\eta,\zeta)\,|z| \;\; = i\xi^r x^r + i\kappa y - (\eta,\zeta)^r |z^r| \;, \quad S\,\mathrm{d}\xi\,\mathrm{d}\kappa = S^r\,\mathrm{d}\xi^r\,\mathrm{d}\kappa \tag{1.9}$$

From equations (1.2) and (1.4) we note, the rotated slowness are understood,

$$\left(\frac{\mathrm{d}\eta}{\mathrm{d}\xi} = \frac{\xi}{\eta}\right)^r, \; \left(\frac{\mathrm{d}\zeta}{\mathrm{d}\xi} = \frac{\xi}{\zeta}\right)^r$$

Hence, from the first of equation (1.9) follows upon substitution of equation (1.8), for each wave mode,

$$i\,\xi = (\eta^r,\, \zeta^r),\; -(\eta,\, \zeta) = i\,\xi^r \tag{1.10}$$

and, substituting dξ into the second equation (1.9), we get, equation (1.10) is used to eliminate the slowness in the unrotated directions, the source functions of the horizontal force excitation,

$$S_P^{z,\,r} = \varepsilon_{z,\,r}\,\frac{i\,\xi^r}{\eta^r} = S_P^x = \frac{i\,\xi}{\eta}\;, \quad S_{Sx}^{z,\,r} = -\frac{i\kappa}{\zeta^r} = S_{Sz}^x = -\frac{i\,\kappa}{\zeta}\;, \quad S_{Sy}^{z,\,r} = \varepsilon_{z,\,r} = S_{Sy}^x = -\varepsilon_z \tag{1.11}$$

The three non vanishing potentials are, the emittance functions in equation (1.11) are properly recalled,

$$\overline{\varphi^x}\,(x, y, z, s) = s^2 Q_x \bar{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_P^x \exp(s g_P)\,\mathrm{d}\xi\,\mathrm{d}\kappa \tag{1.12}$$

$$\bar{\psi}_x^x\,(x, y, z, s) = 0 \tag{1.13}$$

$$\bar{\psi}_y^x\,(x, y, z, s) = s^2 Q_x \bar{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Sy}^x \exp(s g_S)\,\mathrm{d}\xi\,\mathrm{d}\kappa \tag{1.14}$$

$$\bar{\psi}_z^x\,(x, y, z, s) = s^2 Q_x \bar{F}(s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{Sz}^x \exp(s g_S)\,\mathrm{d}\xi\,\mathrm{d}\kappa \tag{1.15}$$

Subsequently, we rotate the coordinates about the vertical axis, such that

$$x^{rr} = y\;,\;\; y^{rr} = -x \tag{1.16}$$

Substituting into the phase invariance, superscript *rr* is understood in the first of equation (1.9), analogously to the above, renders now the relations,

$$\kappa^{rr} = -\,\xi\;,\;\; \xi^{rr} = \kappa \tag{1.17}$$

Note the invariance of the sum of squares of these horizontal slowness.

The second of equation (1.9) becomes

$$S^x\,\mathrm{d}\xi\,\mathrm{d}\kappa = S^{rr}\,\mathrm{d}\xi^{rr}\,\mathrm{d}\kappa^{rr} \tag{1.18}$$

and thus yields the new set of emittance functions upon elimination of the unrotated slowness,

$$S_P^{x,\,rr} = S_P^y = -\frac{i\,\kappa}{\eta}\;, \quad S_{Sy}^{x,\,rr} = S_{Sx}^y = \varepsilon_z\;, \quad S_{Sz}^{x,\,rr} = S_{Sz}^y = -\frac{i\,\xi}{\zeta} \tag{1.19}$$

The displacement potentials for the lateral horizontal force with respect to the original coordinate system are consequently identified as

$$\overline{\varphi^y}\left(x, y, z, s\right) = s^2 Q_y \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_P^y \exp\left(s g_P\right) \mathrm{d}\xi\, \mathrm{d}\kappa \tag{1.20}$$

$$\bar{\psi}_x^y\left(x, y, z, s\right) = s^2 Q_y \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{Sx}^y \exp\left(s g_S\right) \mathrm{d}\xi\, \mathrm{d}\kappa \tag{1.21}$$

$$\bar{\psi}_y^y\left(x, y, z, s\right) = 0 \tag{1.22}$$

$$\bar{\psi}_z^y\left(x, y, z, s\right) = s^2 Q_y \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{Sz}^y \exp\left(s g_S\right) \mathrm{d}\xi\, \mathrm{d}\kappa \tag{1.23}$$

Since one and the same time function applies to all three force components, the resulting displacement potentials are given by simple summation of the individual source functions, when pre multiplied by the amplitudes of the force components,

$$\bar{\varphi}\left(x, y, z, s\right) = s^2 \bar{F}(s) \int_{\infty}^{\infty} \int_{\infty}^{\infty} S_P \exp\left(s g_P\right) \mathrm{d}\xi \mathrm{d}\kappa, \quad S_P = Q_x S_P^x + Q_y S_P^y + Q_z S_P^z = \frac{1}{\eta}\left(Q_x i\xi - Q_y i\kappa\right) - Q_z \varepsilon_z, \tag{1.24}$$

$$\bar{\psi}_x\left(x, y, z, s\right) = s^2 \bar{F}(s) \int_{-\infty}^{\infty} \int_{\infty}^{\infty} S_{Sx} \exp\left(s g_S\right) \mathrm{d}\xi \mathrm{d}\kappa, \quad S_{Sx} = Q_y S_{Sx}^y + Q_z S_{Sx}^z = Q_y \varepsilon_z - Q_z \frac{i\kappa}{\zeta}, \tag{1.25}$$

$$\bar{\psi}_y\left(x, y, z, s\right) = s^2 \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{Sy} \exp\left(s g_S\right) \mathrm{d}\xi \mathrm{d}\kappa, \quad S_{Sy} = Q_x S_{Sy}^x + Q_z S_{Sy}^z = -Q_x \varepsilon_z + Q_z \frac{i\xi}{\zeta}, \tag{1.26}$$

$$\bar{\psi}_z\left(x, y, z, s\right) = s^2 \bar{F}(s) \int_{\infty}^{\infty} \int_{\infty}^{\infty} S_{Sz} \exp\left(s g_S\right) \mathrm{d}\xi \mathrm{d}\kappa, \quad S_{Sz} = Q_x S_{Sz}^x + Q_y S_{Sz}^y = -\frac{1}{\zeta}\left(Q_x i\kappa + Q_y i\xi\right). \tag{1.27}$$

SOURCE RAYS FROM AN OBLIQUE SURFACE OR INTERFACE POINT FORCE

The emittance functions of the point source at the traction-free surface of a half space are derived by first receiving the reflected rays originally emitted from the buried source in the upward direction, and subsequently taking the limit of the source depth to zero in

$$\overline{\varphi_{Pp}} = s^2 \bar{F}(s) \int_{\infty}^{\infty} \int_{-\infty}^{\infty} S_P R^{PP} \exp\left(s g_{Pp}\right) \mathrm{d}\xi \mathrm{d}\kappa \tag{2.1}$$

$$\bar{\psi}_{Ps,j} = s^2 \bar{F}(s) \int_{\infty}^{\infty} \int_{\infty}^{\infty} S_P R^{PS,j} \exp\left(s g_{Ps}\right) \mathrm{d}\xi\, \mathrm{d}\kappa \tag{2.2}$$

$$\overline{\varphi_{S,kp}} = s^2 \bar{F}(s) \int_{\infty}^{\infty} \int_{\infty}^{\infty} S_{S,k} R^{S,kP} \exp\left(s g_{Sp}\right) \mathrm{d}\xi \mathrm{d}\kappa \tag{2.3}$$

$$\bar{\psi}_{S,ks,j} = s^2 \bar{F}(s) \int_{\infty}^{\infty} \int_{\infty}^{\infty} S_{S,k} R^{S,k\,S,j} \exp\left(s g_{Ss}\right) \mathrm{d}\xi \mathrm{d}\kappa \tag{2.4}$$

Note the direction factor in the emittance functions (1.24)-(1.27) is negative for the upward propagating rays. The numbers *k* of the incident S-wave and the numbers *j* of the reflected S-wave indicate the *x, y, z* -components, respectively.

The notation follows from the incident source rays given by equations (1.24)-(1.27) and the reflection coefficients of the potentials of three-dimensional plane waves at the traction free surface are understood, [11] and [12]. Since the wave mode is determined by the last ray segment, the potentials are given by proper summation: the direct rays emitted from an oblique force are added and denoted by the subscript zero, also taking into account the downward propagation, the direction factor is positive in equations (1.24)-(1.27), *p* or *s* indicate the mode of the downward pointing rays,

$$\overline{\varphi_p} = \overline{\varphi_0} + \lim_{z_0 \to 0} \left(\overline{\varphi_{P\,p}} + \sum_k \overline{\varphi_{S,k\,p}} \right), \quad S_p = S_{0p} + S_P R^{P\,P} + \sum_k S_{S,k} R^{S,k\,P} \tag{2.5}$$

$$\bar{\psi}_{s,j} = \bar{\psi}_{0s,j} + \lim_{z_0 \to 0} \left(\bar{\psi}_{P\,s,j} + \sum_k \bar{\psi}_{S,k\,s,j} \right), \quad S_{s,j} = S_{0s,j} + S_P R^{P\,S,j} + \sum_k S_{S,k} R^{S,k\,S,j} \tag{2.6}$$

The phase functions are simply given by the two wave modes as above and the absolute value of the vertical coordinate is simply substituted by $z > 0$.

When considering the reflections of the downward pointing rays emitted by the buried source at the "parallel" interface at the distance $z = h$ from the surface between two dissimilar half spaces, yields the potential ray integrals within the surface layer and, taking the limit of the source depth to layer thickness, thus the solution for the interface source. In this case, the resulting rays are propagating upward. The source rays in the adjacent medium are determined by substituting the three-dimensional transmission coefficients of plane waves at the plane interface, however, these rays are oriented downward and not further evaluated. In the first case, we have, analogously to equations (2.1)-(2.6), however with the three-dimensional reflection coefficients of plane waves taken at the interface, the direction factor is positive in equations (2.7)-(2.10),

$$\overline{\varphi_{pP}} = s^2 \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_P R_{PP} \exp\left(s g_{pP}\right) \mathrm{d}\xi \mathrm{d}\kappa, \tag{2.7}$$

$$\bar{\psi}_{pS,j} = s^2 \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_P R_{PS,j} \exp\left(s g_{pS}\right) \mathrm{d}\xi \mathrm{d}\kappa, \tag{2.8}$$

$$\overline{\varphi_{s,k\,P}} = s^2 \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{S,k} R_{S,k\,P} \exp\left(s g_{sP}\right) \mathrm{d}\xi \mathrm{d}\kappa, \tag{2.9}$$

$$\bar{\psi}_{s,k\,S,j} = s^2 \bar{F}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{S,k} R_{S,k\,S,j} \exp\left(s g_{sS}\right) \mathrm{d}\xi \mathrm{d}\kappa, \tag{2.10}$$

$$\overline{\varphi_P} = \overline{\varphi_{0P}} + \lim_{z_0 \to h} \left(\overline{\varphi_{pP}} + \sum_k \overline{\varphi_{s,kP}} \right), \quad S_P = S_{0P} + S_p R_{P\,P} + \sum_k S_{s,k} R_{S,\,kP}, \tag{2.11}$$

$$\bar{\psi}_{S,j} = \bar{\psi}_{0S,j} + \lim_{z_0 \to 0} \left(\bar{\psi}_{pS,j} + \sum_k \bar{\psi}_{s,k\,S,j} \right), \quad S_{S,j} = S_{0S,j} + S_p R_{P\,S,j} + \sum_k S_{s,k} R_{S,k\,S,j}. \tag{2.12}$$

The directly emitted rays carry the subscript zero. They are propagating upwards and the direction factor thus is negative in the first terms of the emittance functions in equations (2.11) and (2.12). The phase functions are set up according to the two wave modes and the absolute vertical segment becomes $(h - z)$. In all these cases we end up with a fully coupled problem.

TRANSFORMATIONS FOR DIPPING LAYERS

A wedge shaped source layer with a dipping angle α measured clockwise against the free surface is considered next. The source rays from the vertical point force, equations (1.2)-(1.5), are exemplary treated in the rotated (through α and primed) coordinate system. Wave slowness in the y-direction remains unaffected. Phase functions and amplitudes are invariant under coordinate rotation. Considering the horizontal epicentral distance from the wedge tip renders exemplary for the P-wave

$$\begin{gathered} g_P = i\xi x + i\kappa y - \eta\left|z - z_0\right| = g'_P = i\xi'\left(x' - x'_0\right) + i\kappa y - \eta'\left|z' - z'_0\right| \\ x' = x\cos\alpha + z\sin\alpha,\; z' = -x\cos\alpha + z\sin\alpha,\; x'_0 = x_0\cos\alpha,\; z'_0 = z_0\cos\alpha \end{gathered} \tag{3.1}$$

$$S_P^z \,\mathrm{d}\xi\, \mathrm{d}\kappa = S'^{\,z}_P \,\mathrm{d}\xi' \,\mathrm{d}\kappa \tag{3.2}$$

Hence, the travel time remains constant for a fixed source-receiver configuration, if and only if the slowness are related by

$$i\xi' = i\xi \cos\alpha - (\eta, \zeta) \sin\alpha, \quad (\eta, \zeta)' = i\xi \sin\alpha + (\eta, \zeta) \cos\alpha \tag{3.3}$$

In the rotated coordinates, the reflection of the source ray at the interface is done classically. The reflected P- and S-waves are subsequently referred to the unprimed coordinates for further consideration. The phase functions are for the incident P-ray, Snell's law applies,

$$g_{pP} = i\xi_1 x + i\kappa y - \eta\left(h - z_0\right) - \eta_1 \left(h - z\right), \ \xi_1' = \xi', \ h = x_0 \tan\alpha \tag{3.4}$$

$$g_{pS} = i\xi_1 x + i\kappa y - \eta\left(h - z_0\right) - \zeta_1 \left(h - z\right) \tag{3.5}$$

$$i\xi_1 = i\xi' \cos\alpha - (\eta', \zeta_1') \sin\alpha, \quad \eta_1 = \sqrt{c^{-2} + \xi_1^2 + \kappa^2}, \ \zeta_1 = \sqrt{C^{-2} + \xi_1^2 + \kappa^2} \tag{3.6}$$

PLASTIC SOURCES IN THE ELASTIC BACKGROUND SOIL

In [13], the generalized dynamic Maysel's formula was derived. It contains a time convolution between the plastic strain (or any strain imposed to the linear background) and the force Green's function and it holds even for finite bodies. In the incremental form, resulting from stepping the time, non homogeneous initial conditions are to be considered as well. In the time Laplace domain, note the slightly different notation, the particular solution of displacements resulting from these sources becomes

$$u_i^*\left(x_0; s\right) = \int_V \varepsilon^*_{\alpha\beta}\left(x; s\right) \tilde{\sigma}_{\alpha\beta(i)}\left(x, x_0; s\right) \mathrm{d}V(x) + \rho \int_V \left[u^*_{\alpha 0}\left(x\right) \tilde{u}_{\alpha(i)}\left(x, x_0; s\right) + v^*_{\alpha 0}\left(x\right) \tilde{u}_{\alpha(i)}\left(x, x_0; s\right)\right] \mathrm{d}V(x) \tag{4.1}$$

The dynamic Green's functions of the elastic background (infinite, semi infinite, or layered body), are given through their displacement potentials and, consequently expanded into plane waves, see the representation given above, the Dirac Delta function in time is applied in equations (1.1) and (1.6). In plane strain, the source rays from the line of oblique forces per unit of length are considered and the table of receiver functions is included in [14].

CONCLUSIONS

Implementations like receiver functions of displacements and stresses, surface receiver functions etc., see [1] and [14] for the line source and [12] for the twofold simultaneous numerical integration in the course of inversion of the Laplace transformed ray integrals, are reported elsewhere. In the course of developments in parallel computing and at increased speeds of modern workstations, the method of generalized ray has become competitive in computational seismology. The representation of the solution to the single oblique force in Cartesian coordinates, presented above, is suitable to consider the divergence effect of a dipping layer (including back scattering of the updip traveling source ray) and as well the phenomenon known as horizontal refraction, see also[15] for the none-penetrable wedge. The damage statistics of the city of Skopje was reported in [16] and the relation to the dipping angle of the interface illustrated. The solution serves also as the Green's function of the linear background of an elastic-viscoplastic soil, where in the plastic zones waves are emitted from so-called plastic sources, see [14] for the two-dimensional problem. For line sources with random time functions and correlation theory of generalized rays, see [17].

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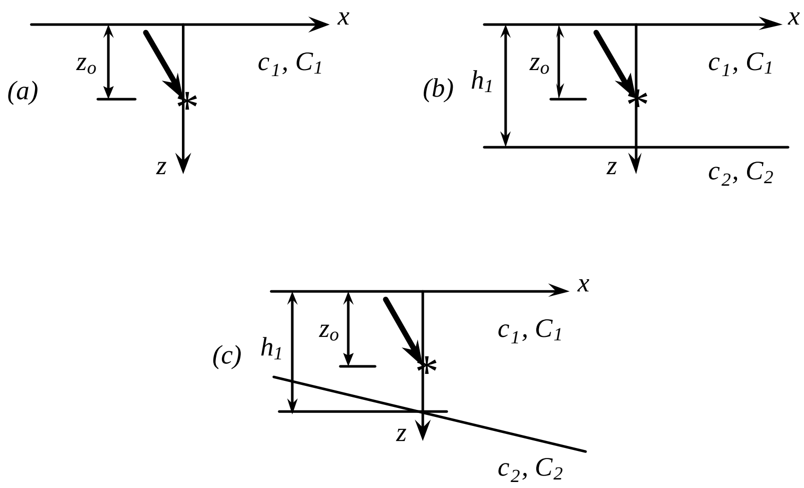

Figure 1. Three models of the soil; (a) a homogeneous half space (b) a horizontal layer overlying a dissimilar bedrock half space (infinite number of ray integrals) (c) a dipping layer overlying a dissimilar bedrock half space (finite number of ray integrals).