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STATISTICAL ANALYSIS OF MODAL PARAMETERS FOR RC BUILDINGS

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SUMMARY

For the seismic design of buildings, the modal parameters are intensively used by the engineers. The seismographs installed in 19 RC buildings in Taiwan offer the opportunity to evaluate the real modal parameters of existing buildings. A hybrid identification scheme used here includes an ARX model to identify the frequency transfer functions in the time domain and a nonlinear regression analysis to estimate the modal parameters in the frequency domain. The identified results of each building triggered by four events are compared to ensure the consistency and then averaged to increase the accuracy. All the averaged identified modal parameters from different buildings offer the database to derive empirical formulas, which relate the modal parameters to the height of building. The probability distributions of the predicted modal parameters are then tested. The results show that the modal periods conditional on the height have the lognormal distribution, and so do the effective participation factors, which are assumed to be the cubic polynomial functions of the normalized floor height. The modal damping ratios are also lognormal variates, but are independent of the height. The time of usage shows no definite influence in predicting the modal periods and damping ratios.

INTRODUCTION

The assumptions of linear elasticity and modal analysis are intensively used by the engineers in the seismic design of buildings, and the associated parameters are usually estimated from theoretical models based on ideal conditions. The real parameters, however, could be quite different from the theoretical ones due to the uncertainty on the material properties, construction, and usage. Hence, using the system identification technique to find the dynamic parameters of structures under use has became more and more important [Beck and Jennings, 1980; Mcverry, 1980; Masri et al., 1982; Li and Mau, 1991; Peng and Iwan, 1992; Loh and Chung, 1993].

In Taiwan the seismographs installed in RC buildings offer the opportunity to evaluate the real modal parameters of existing buildings. A hybrid scheme used here to identify the modal parameters by using the seismic records includes an ARX model [Ljung, 1987] to identify the frequency transfer functions and then a nonlinear regression analysis on those frequency transfer functions to estimate the modal parameters. The identified results for the same building triggered by several events are compared to ensure the consistency and then averaged to increase the accuracy.

One of the benefits in studying the identified modal parameters comes from the need of empirical formulas, which relate the modal parameters to the height or other factors of buildings. Such formulas are useful in the seismic design codes [e.g. *UBC*, 1997]. Several researches have been done to derive the empirical formulas and then to compare them with those presented in the codes [Chen et al., 1992; Goel and Chopra 1997].

Furthermore, the probability distributions of the modal parameters predicted by the empirical formulas are tested. Such probability distributions could be helpful in determining the capacity curves of RC buildings when the damage criteria are specified and then in constituting the fragility curves when the ground excitations are assessed by the seismic hazard analysis.

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STRUCTURAL ARRAY MONITORING SYSTEMS

Several tens of structural array monitoring systems have been installed in the buildings and bridges around Taiwan. Each system contains a central recording system supplied by two personal computers for real-time processing and others. The central recording system controls up to thirty channels of accelerometers. A typical accelerograph includes three channels at the free field and the others at the first basement, ground level, roof level, and other middle floors in a building. In the measured floor, there are usually one channel in the longitudinal direction and two or three channels in the transverse direction.

The installation of structural array monitoring systems began in 1992. Nineteen RC and SRC buildings were triggered by more than four events since operation and the distribution of their heights is shown in Fig. 1. Those buildings are beam-column moment-resistance frame systems with a few shear walls surrounding the elevators. They were originally selected to install the arrays on purpose to cover different heights and site conditions. It is clearly seen in Fig. 1 that the heights vary almost linearly. Such a distribution offers the excellent database to perform the regression analysis of the identified modal parameters on the heights.

For each building, the records of four events are selected to identify the modal parameters. In additional to the larger ground vibration level, the wider separation of the occurring time is also the major concern in selecting those events. Those events occurred from 1993 to 1996. The peak values of the acceleration records at the ground and the floors range from several gals to a few hundred gals. Obviously, those events are not strong shaking. However, they could just be in the scale to ensure the linearly elastic responses.

SYSTEM IDENTIFICATION

A two-step procedure is used here to identify the modal parameters of the 19 buildings. The first step is the ARX model, which is a non-parametric identification method to estimate the frequency transfer functions of the output responses to the input excitation in the time domain. The second one takes the nonlinear regression analysis on the theoretical frequency transfer functions to fit the identified ones in order to estimate the modal parameters. In fact, the former is a method of linear regression analysis in which the noises are considered, and the latter is a parametric identification method in the frequency domain. These two schemes belong to the least-square methods, but are different in the target models or functions to fit in the data in either the time or frequency domain.

Both the acceleration records at the free field and those at the basement would be chosen as the input excitation to the buildings. After comparing the results identified from the ARX model, it is found that using the records at the basement as the input excitation gives more consistent frequency transfer functions among events. Usually the free field is located several tens of meters away from the building, and its records are easily disturbed by the traffic nearby.

In each horizontal direction, a single-input (record at the basement) and single-output (record at the upper floors) relation is identified in the ARX model. Since there are more than one channel in the transverse direction, the records are combined into one, by the method of interpolation (for two channels) or regression analysis (for three channels), at the rigid center of each floor [Huang and Hong, 1996]. Hence, one frequency transfer function is identified in each direction for each upper measured floor after using the ARX model.

Theoretically, the frequency transfer function of the *k*-th floor acceleration excited by the ground acceleration can be represented as follows:

$$H_k(\omega) = \sum_{j=1}^{n} P_{kj} \frac{\omega_j^2 + 2i\zeta_j\omega_j\omega}{\omega_j^2 - \omega^2 + 2i\zeta_j\omega_j\omega} \tag{1}$$

where ω is the frequency in rps, n the total number of modes, P_{kj} the effective participation factor at the *k*-th floor due to the *j*-th mode, ω_j the *j*-th modal frequency, and ζ_j the *j*-th modal damping ratio. After the frequency transfer functions at the measured floors are all identified in the ARX model, a nonlinear regression analysis based on Eq. (1) is performed for the combination of all the identified frequency transfer functions to estimate the modal parameters. However, only the parameters in the first few modes are concerned as usual because not only they are dominant in evaluating the responses but also the identified results are more reliable.

In each building, there are four sets of identified parameters due to the four events, and they are averaged to be the final results.

For example, the identified and averaged vibration periods of the first three modes in the longitudinal direction are shown in Fig. 2. In general, the higher the mode is, the wider the identified modal periods disperse. It is also true for the identified modal damping ratios and effective participation factors. In each mode, the identified periods are more consistent among events than the damping ratios and effective participation factors. It is also seen in Fig. 2 that the modal period enlarges as the height of building increases.

STATISTICAL ANALYSIS

The modal parameters identified in the proceeding section offers the database for simple statistical analysis and even for regression analysis on the height. The common empirical relation to predict the fundamental vibration period *T* conditional on the height h_n is [*UBC*, 1997]

$$T = a h_n^b \tag{2}$$

where *a* and *b* are the coefficients. After performing the linear regression analysis to $\ln T$ on $\ln h_n$ respectively in the longitudinal, transverse, and both two horizontal directions, the unbiased estimates of *a*, *b*, the conditional standard deviation $\sigma_{\ln T|h}$, and the standard deviation $\sigma_{\ln T}$ are all listed in Table 1, where *T* is in unit of sec and h_n in meter. In any mode and direction, it is noted that $\sigma_{\ln T}$ is always significantly greater than $\sigma_{\ln T|h}$, which implies the validity of the height on predicting the modal periods. It is not true, however, for the modal damping ratios. In other words, the modal damping ratios are almost independent of the height. Therefore, only the results of the statistical analysis on the logarithm of the identified modal damping ratios are listed in Table 2.

On the basis of the K-S test [Ang and Tang, 1972], the assumptions of lognormal distributions for the modal periods conditional on the height can be accepted at the 5% significant level, and so do the modal damping ratios themselves. In view of the standard deviation listed in Tables 1 and 2, the parameters in the higher modes disperse increasingly. The central values of the modal damping ratios also enlarge at the higher modes.

In order to predict the effective participation factors up to the third mode, a cubic polynomial function of the normalized height is proposed in the following:

$$P_{kj} = c_{1j}\left(\frac{h_k}{h_n}\right) + c_{2j}\left(\frac{h_k}{h_n}\right)^2 + c_{3j}\left(\frac{h_k}{h_n}\right)^3 \tag{3}$$

where h_k is the height of the *k*-th floor to the ground, and c_{1j}, c_{2j}, and c_{3j} are the coefficients of the polynomial. Since there is a constraint on $P_{kj} = 0$ when $h_k = 0$, the non-constant conditional standard deviation $\sigma_{P_{kj}|h_k}$ is represented as

$$\sigma_{P_{kj}|h_k} = \sigma\left\{1 - \exp\left[-\lambda\left(\frac{h_k}{h_n}\right)\right]\right\} \tag{4}$$

where λ is a given constant, and σ is the coefficient under estimation. After performing the nonlinear regression analysis to the identified effective participation factors, the results are given in Table 3. The heights of the nodes, h_{node1} and h_{node2}, which are obtained by using Eq. (3) are also listed in Table 3 for reference in selecting the floors to install the accelerometers. The assumptions of lognormal distributions for the effective participation factors with the mean in Eq. (3) and the standard deviation in Eq. (4) can be accepted at the 5% significant level when the K-S test is performed.

For comparing the modal parameters to the regression results, the identified modal periods are drawn in Fig. 3 together with the regression lines based on Eq. (2) and $\sigma_{\ln T|h}$. It is noted again in Fig. 3 that the vibration periods in the first three modes enlarge when the height of building increases. The identified modal damping ratios and the statistical results are shown in Fig. 4. As stated before, the central values and the dispersion of the modal damping ratios increase at the higher modes. The identified effective participation factors and the regression results are shown in Fig. 5. In general, the regression results of the effective participation factors in the first mode are most reliable.

The 19 RC buildings were constructed from a few months to twenty-five years before collecting the records. It would be possible to check if the modal parameters change with the time. Therefore, the ratios of the identified fundamental vibration periods to the predicted ones in Eq. (2) are shown in Fig. 6. Since those ratios fluctuate as the time after use increases, no obvious correlation is observed. The same phenomenon is also found in the modal damping ratios, which is shown in Fig. 7. Hence, the uncertainty due to the time after use in predicting the modal parameters could be implicitly covered by the corresponding (conditional) standard deviation.

CONCLUSIONS

On the basis of the regression and statistical results using the identified modal parameters of 19 RC buildings, the modal periods strongly depend on the height and the effective participation factors could be represent by the cubic polynomial functions of the normalized height. The modal damping ratios, however, are independent of the height. The modal parameters are all lognormal variates with the mean (or median) and (conditional) standard deviation presented in this study. The higher the mode is, the larger the central value and the dispersion of the modal damping ratio are. In predicting the modal parameters, the uncertainty due to the time of building usage could be implicitly contained in the corresponding (conditional) standard deviation.

ACKNOWLEDGMENTS

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Table 1: Regression analysis of identified modal periods on height

	$(T_1)_L$	$(T_2)_L$	$(T_3)_L$	$(T_1)_T$	$(T_2)_T$	$(T_3)_T$	T_1	T_2	T_3
a	0.037	0.022	0.011	0.019	0.023	0.007	0.027	0.023	0.009
b	0.741	0.547	0.574	0.947	0.556	0.734	0.844	0.550	0.655
$\sigma_{\ln T\|h}$	0.255	0.194	0.221	0.259	0.290	0.271	0.258	0.245	0.243
$\sigma_{\ln T}$	0.478	0.356	0.330	0.572	0.413	0.414	0.522	0.382	0.370

Table 2: Statistical analysis of identified modal damping ratios

	$(\zeta_1)_L$	$(\zeta_2)_L$	$(\zeta_3)_L$	$(\zeta_1)_T$	$(\zeta_2)_T$	$(\zeta_3)_T$	ζ_1	ζ_2	ζ_3
Median	0.031	0.056	0.141	0.037	0.051	0.103	0.034	0.053	0.120
Mean	0.037	0.078	0.297	0.046	0.071	0.180	0.041	0.074	0.228
$\sigma_{\ln\zeta}$	0.603	0.815	1.222	0.654	0.830	1.056	0.629	0.813	1.133

Table 3: Regression analysis of identified effective participation factors

	$(P_{k1})_L$	$(P_{k2})_L$	$(P_{k3})_L$	$(P_{k1})_T$	$(P_{k2})_T$	$(P_{k3})_T$
c_{1j}	0.559	2.853	2.432	1.300	2.372	2.523
c_{2j}	2.351	-5.240	-7.881	-0.249	-3.614	-8.010
c_{3j}	-1.569	1.936	5.646	0.150	0.827	5.620
σ	0.115	0.162	0.182	0.360	0.198	0.170
λ	5	5	5	5	5	5
h_{node1}		$0.76h_n$	$0.46h_n$		$0.80h_n$	$0.47h_n$
h_{node2}			$0.94h_n$			$0.96h_n$

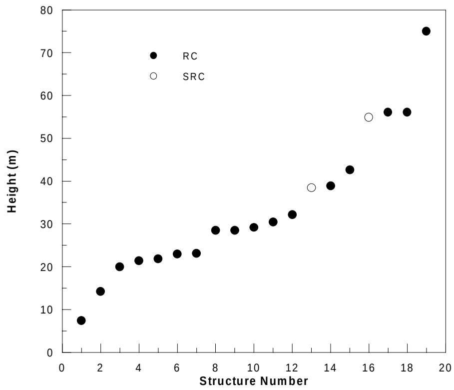

Figure 1: Distribution of building heights

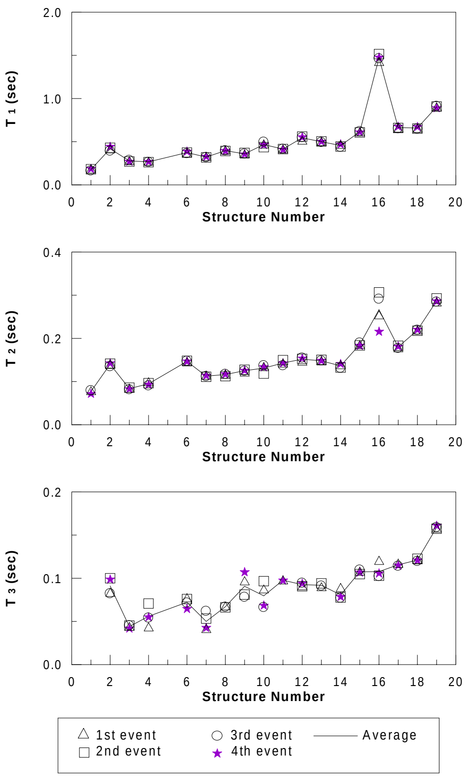

Figure 2: Identified and averaged modal periods in the longitudinal direction

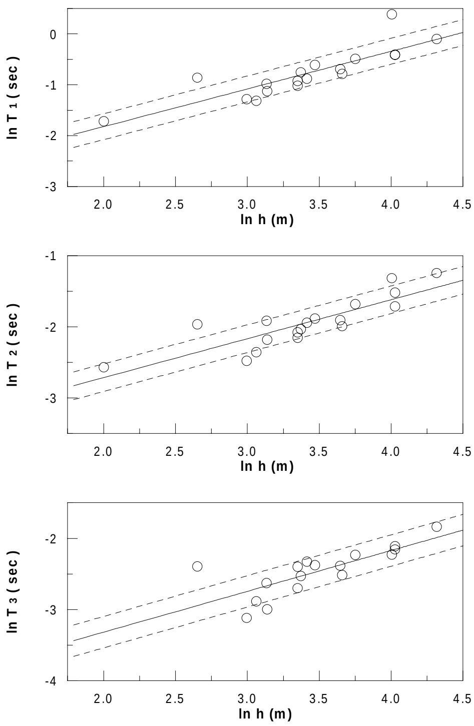

Figure 3: Identified modal periods and regression lines in the longitudinal direction
(———— mean, ----------- mean ± one standard deviation)

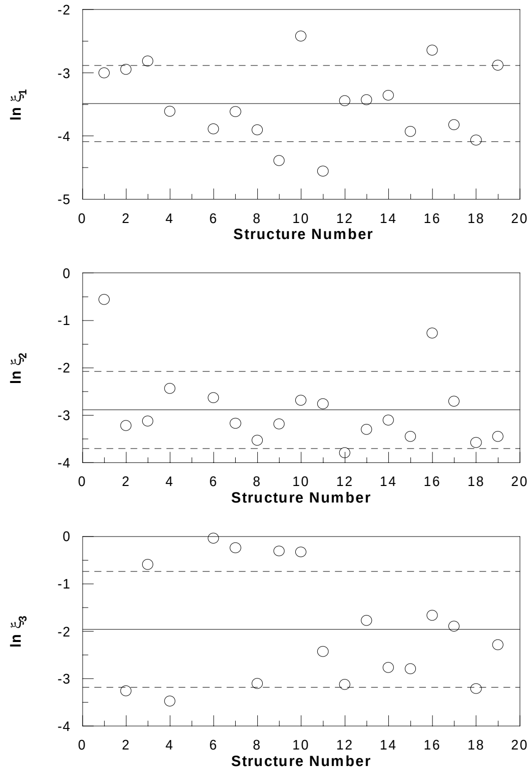

Figure 4: Identified modal damping ratios and statistical results in the longitudinal direction
(———— mean, ----------- mean ± one standard deviation)

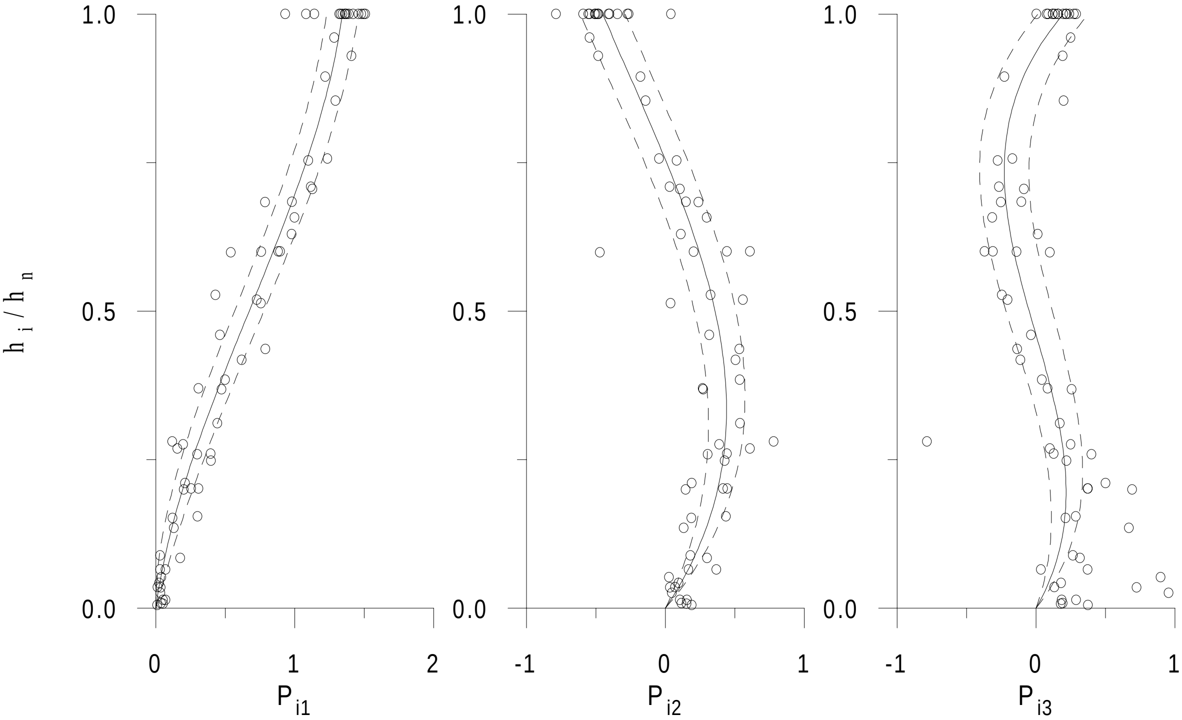

Figure 5: Identified effective participation factors and regression curves in the longitudinal direction (———— mean, ------------ mean ± one standard deviation)

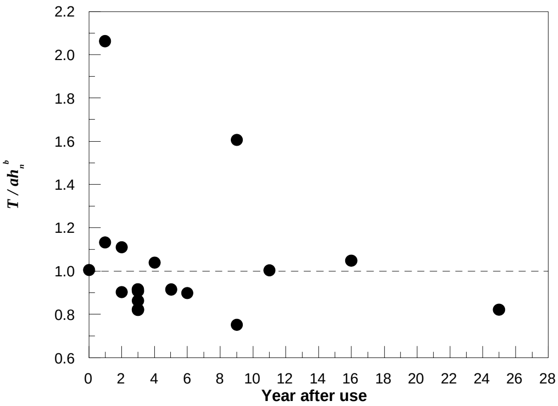

Figure 6: Ratios of identified fundamental periods to predicted ones in the longitudinal direction

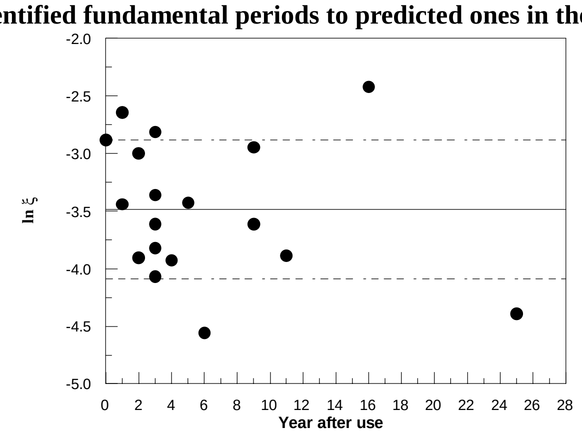

Figure 7: Identified damping ratios of the first mode in the longitudinal direction (———— mean, ------------ mean ± one standard deviation)

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