



## LOW-FREQUENCY BEHAVIOR OF COHERENCY FOR STRONG GROUND MOTIONS IN MEXICO CITY AND JAPAN

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### SUMMARY

The behavior of earthquake ground motion at low frequencies is studied and the adequacy of different models in representing the low frequency coherency variation is discussed. Acceleration records from four events recorded by seismic arrays in the valley of Mexico and at the Chiba experimental station in Japan are analyzed. Reliable estimates of coherency at very low frequencies can be obtained due to the length of the records. Estimated coherency functions are compared with several models proposed by various investigators.

### INTRODUCTION

The coherency spectrum  $\chi(f)$  is a key component in the characterization of spatially varying earthquake ground motion (SVEGM) and describes the incoherence between accelerations recorded at different spatial locations. Some functional forms have been proposed for the coherency function  $|\chi(f)|$  [see e.g., Harichandran and Vanmarcke 1986, Hao 1989, Abrahamson 1993], but studies on how these different forms affect the computed responses have been few. A recent study on the response of an earth dam to SVEGM has shown that the variation of coherency at very low frequencies can have a substantial impact on the safety assessment of the dam [Chen and Harichandran 1998]. Since the low-frequency behavior of coherency is significantly different amongst some of the commonly used models, it is important to resolve contradictions and provide guidance to engineers.

Very long duration accelerograms are necessary to accurately estimate coherency at very low frequencies using conventional spectral estimation techniques. Sufficient resolution can be obtained without compromising stability for such records. This paper discusses the low-frequency behavior of coherency for four strong earthquake ground motions recorded in the valley of Mexico and at the Chiba experimental station in Japan. The length of the records permit reliable coherency values to be estimated at frequencies as low as 0.2. The ability of different models to fit the estimated coherency functions is examined.

### ESTIMATION OF COHRENCY SPECTRA

The acceleration records from the valley of Mexico and the Chiba experimental station are quite long and allow high resolution coherency spectra to be estimated while keeping the variance of the estimates at acceptable levels. Records in the radial and transverse direction from four events recorded at these sites were analyzed. The events and their characteristics are listed in Table 1.

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**Table 1. Characteristics of Seismic Events**

Event	M-1	M-2	C-1	C-2
Date	25/04/89	09/10/95	12/02/86	24/06/86
Magnitude	6.9	8.0	6.1	6.5
Epicerter distance (km)	250	515.	125	105
Focal depth (km)	19	20	44	73
Azimuth	6°	84°	45°	148°
Peak acceleration (cm/s <sup>2</sup> )	12.2	2.2	15.4	54.0
Record Duration (s)	140	140	75	190

Coherency estimates were found to be unreliable for frequencies lower than 0.2 Hz because of the very low power in the acceleration records for events M-1 and M-2. Thus, coherency spectra were estimated using a Hamming spectral window with a bandwidth  $b = 0.2$  Hz for frequencies  $f \geq 0.2$  Hz. For events M-1 and M-2, 34 and 58 pairs of acceleration records were analyzed, respectively, and station separations varied from 150 to 1500 m. For the Chiba data, 210 pairs of acceleration records were analyzed for events C-1 and C-2 in the radial and transverse directions, and station separations varied from 5 to 320 m. As expected, in both cases the absolute values of ground motion coherency decreased with increasing frequency and station separation for both the radial and transverse components.

Based on pairwise coherency estimates, the inverse hyperbolic tangent of the coherency function was assembled for specific separation distances  $\nu$  according to

$$Y(\nu, f) = \tanh^{-1}(|\gamma(\nu, f)|) = \frac{\sum_{i=1}^n \beta_i(f) \kappa_i(\nu)}{\sum_{i=1}^n \kappa_i(\nu)} \quad (1)$$

where  $n$  is the number of pairs of records,  $\beta_i(f) = \tanh^{-1}(|\hat{\gamma}_i(f)|)$  are transformed coherency estimates for the  $i$ th station pair,  $\kappa_i(\nu) = \exp[-((\nu - \nu_i)/\Delta\nu)^2/2]$  is a smoothing function and  $\Delta\nu$  is a smoothing parameter. Since no significant differences were found in the pairwise coherency functions for the radial and transverse components, the smoothing was performed using results for both components.

Fig. 1 shows the assembled curves for events M-1 and M-2 using  $\Delta\nu = 33$  m. Due to filtering of the seismic waves by the soft soil in the valley of Mexico, the ground motions have significant power only in the frequency range  $0.2 < f < 0.8$  Hz. The results show higher coherency values for event M-1 than for event M-2, indicating that coherency is possibly dependent on the event characteristics, i.e., magnitude, epicentral distance, peak ground acceleration, etc. At frequencies around 0.2 Hz, the coherency is significantly less than 1.0 for station separations greater than 1000 m and 800 m for events M-1 and M-2, respectively. However, estimates for  $d < 1000$  m are less reliable since more data is available for station separations around 1200 m. Note that  $\nu$  and  $d$  are both used to represent station separation.

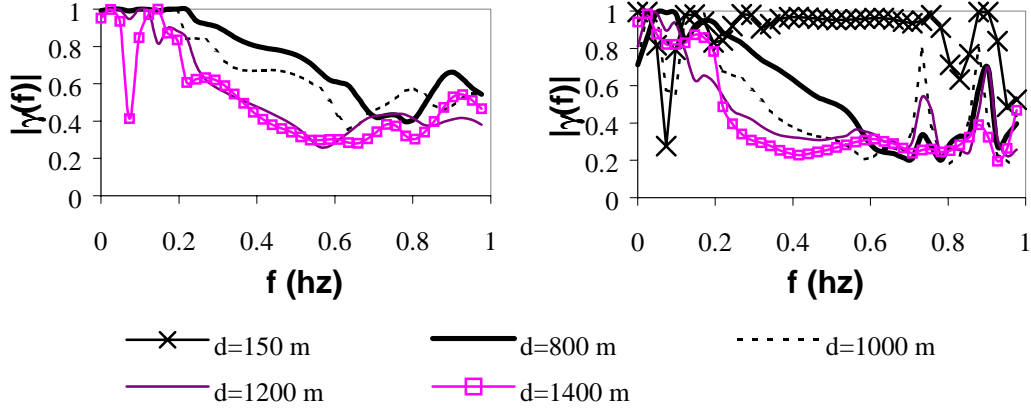


Fig. 1: Assembled  $|\gamma(f)|$  for M-1 and M-2

The variance of  $\beta_i = \tanh^{-1}(|\hat{\gamma}_i(f)|)$  is  $\sigma^2 = 1/(2bT)$ , where  $b$  = bandwidth of the spectral window and  $T$  = length of the records [Jenkins and Watts 1969]. The variance of  $Y(\nu, f)$  can be calculated from

$$Var[Y(\nu, f)] = \frac{\sum_{i=1}^n Var[\beta_i(f)] \kappa_i^2 + \sum_{i=1}^n \sum_{j=1, j \neq i}^n \kappa_i \kappa_j Cov[\beta_i \beta_j]}{\left( \sum_{i=1}^n \kappa_i \right)^2} \quad (2)$$

The 95% confidence intervals for the assembled coherency functions can be obtained using (2). Since the variance of  $Y(\nu, f)$  is maximum when the estimates  $\beta_i$  and  $\beta_j$  are fully correlated, it is conservatively assumed that  $Cov(\beta_i, \beta_j) = Var(\beta_i)$ . Figs. 2 and 3 show the 95% confidence intervals for events M-1 and M-2.

Due to the small station separation distances for the Chiba array, the estimated coherency values are close to 1.0 for frequencies around 0.2 Hz. These results cannot be extrapolated to greater distances and do not give further insight into the behavior of coherency at very low frequencies. Estimates were also obtained with a wider spectral bandwidth  $b = 1$  Hz to better visualize the coherency function. The assembling was performed according to (1) for station separations  $\nu = 30, 100, 200$  and  $300$  m, and  $\Delta\nu = 16.6$  m, and the results are shown in Fig. 4. There is no significant difference in the assembled coherency for events C-1 and C-2, which may be due to the similarity between the characteristics of both events (see Table 1).

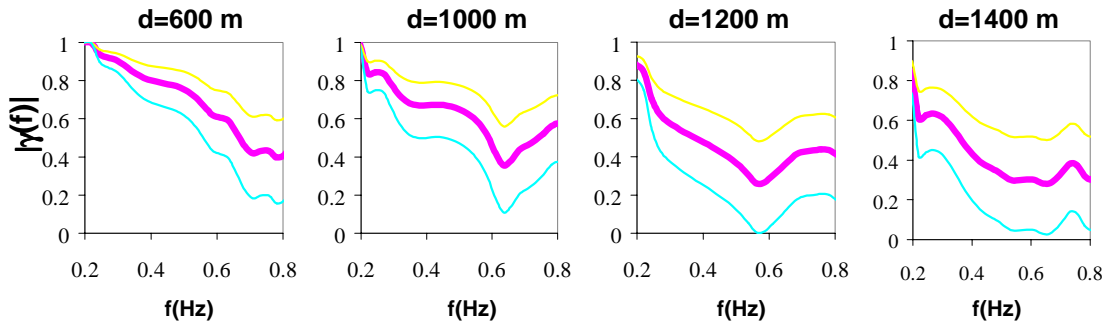
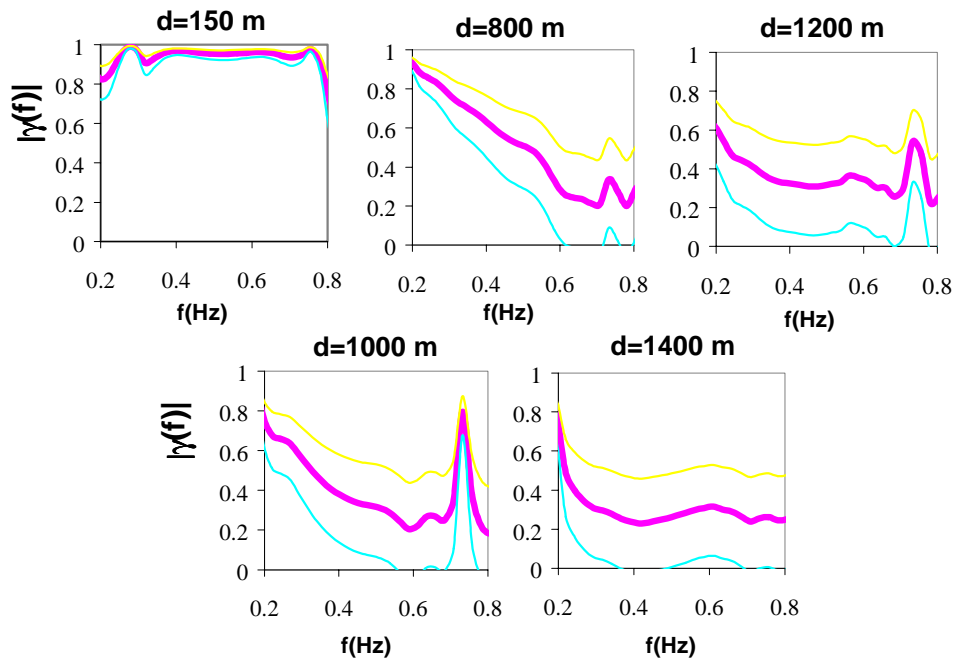
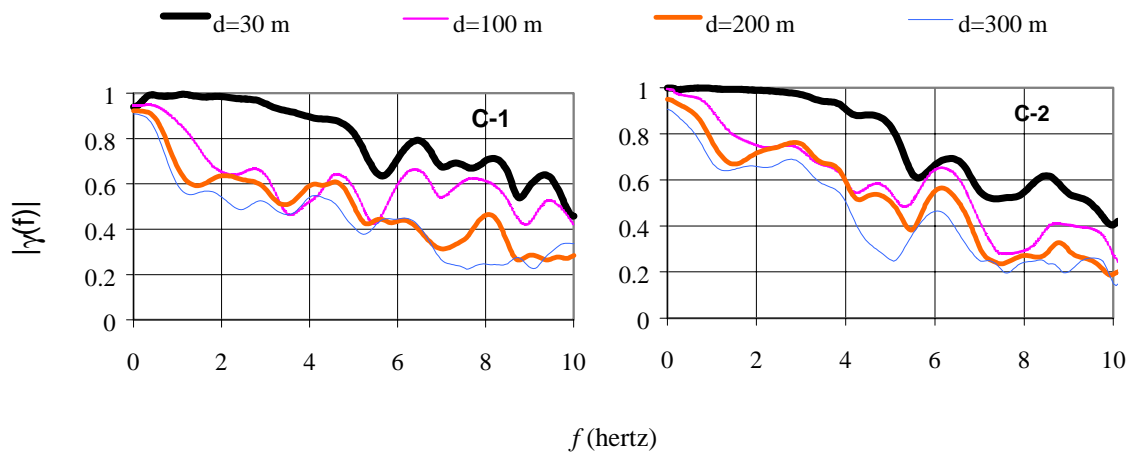


Fig. 2. 95% Confidence Intervals for Event M-1

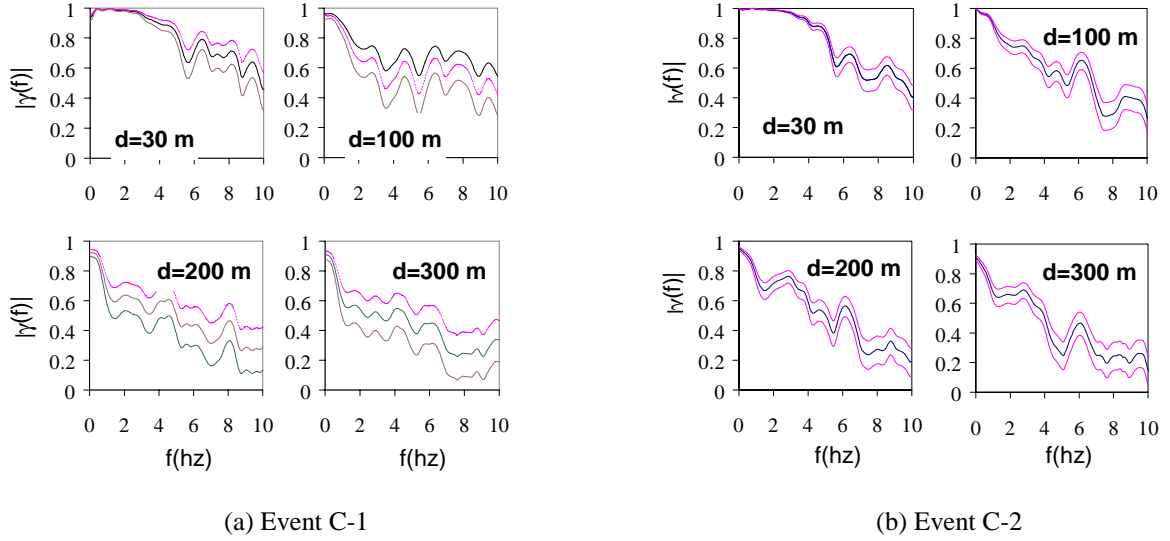


**Fig. 3. 95% Confidence Interval for Event M-2**



**Fig. 4. Assembled  $|\gamma(f)|$  for Events C-1 and C-2**

Fig. 5 shows the 95% confidence intervals for events C-1 and C-2. Notice that the intervals for C-1 are broader than those for C-2 because the record lengths are shorter for the former event. However, in both cases the confidence intervals are reasonably narrow because many station pairs were available for the analysis.



**Fig. 5. 95% Confidence Intervals for Events C-1 and C-2**

### COHERENCY MODELS

Several coherency models have been proposed by different investigators based on theoretical and empirical studies. Abrahamson [1993] proposed an empirical model based on the analysis of recordings from various dense arrays. The model involves a frequency and distance dependent plane wave factor and assumes that coherency does not depend on the local site or event characteristics. Hindy and Novak [1980] proposed the following functional form

$$|\gamma(v, \omega)| = \exp\left\{- (\alpha v \omega)^\beta\right\} \quad (3)$$

where  $\alpha$  and  $\beta$  are model parameters. The Luco and Wong [1986] model can be considered a particular case of (3) when  $\beta = 2$ ,  $\alpha = \eta/V_s$ , and  $V_s$  is the shear wave velocity. Based on the statistical analysis of strong ground motion data from the SMART-1 dense array, Harichandran and Vanmarcke [1986] advanced the following model

$$|\gamma_H(v, f)| = A \exp\left(-\frac{2v}{\alpha\theta(f)}(1-A+\alpha A)\right) + (1-A) \exp\left(-\frac{2v}{\theta(f)}(1-A+\alpha A)\right) \quad (4)$$

$$\theta(f) = k \left(1 + \left(\frac{f}{f_o}\right)^b\right)^{-1/2}$$

where  $\theta(f)$  is the frequency-dependent spatial scale of fluctuation [Vanmarcke 1983].

The Luco and Wong, Novak, and Harichandran models were fitted to the coherency functions estimated for the four events studied. Abrahamson's model was not fitted due to its complex form, and because it is supposed to be event independent. Models were fitted to the data using a nonlinear least squares algorithm. Because the variance of the inverse hyperbolic tangent of the coherency estimator is constant (i.e., the transformed data is homoscedastic), all fitting was done in the hyperbolic tangent space. All coherency functions were estimated using a Hamming window with a bandwidth  $b = 0.2$  Hz. The frequency and separation ranges used for the fitting

were  $0.2 < f < 0.8$  Hz and  $0 < v < 1300$  m for the Mexican events, and  $0.2 < f < 8$  Hz and  $0 < v < 320$  m for the Japanese events, respectively. The estimated parameters applicable to these frequency and separation intervals, are listed in Tables 2, 3 and 4.

Fig. 6 shows a comparison between the models and the assembled coherency functions for event M-1. The Hindy and Novak, and Harichandran models are flexible and fit the data well. The Luco and Wong model is less flexible. While it fits well for separations less than 600 m, it falls below the estimated coherency values at greater separations. Abrahamson's model significantly overestimates the coherency for Mexico City and does not seem appropriate for it.

Fig. 7 shows results for event C-1. Again, the Hindy and Novak, and Harichandran models fit the data well. The models by Luco and Wong, and Abrahamson fit well for very short distances ( $d = 30$  m) but at greater separations they differ significantly from the estimated coherency functions.

**Table 2. Coherency Parameters for the Luco and Wong Model**

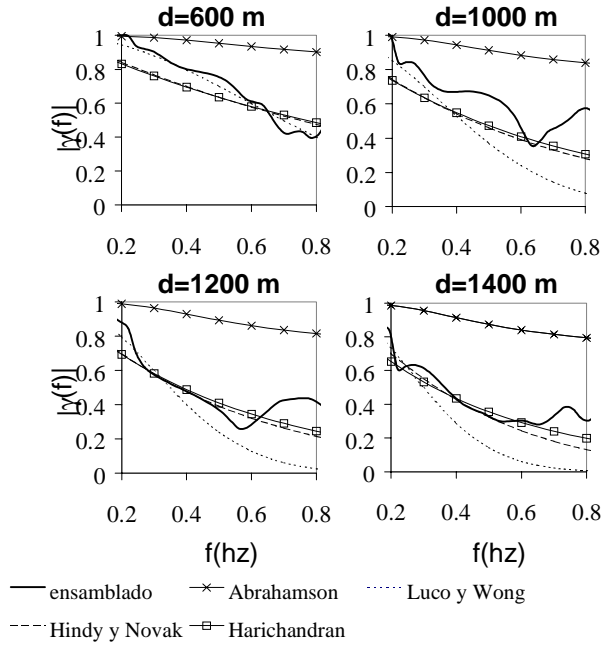
	M-1	M-2	C-1	C-2
$\eta/V_s$	$3.17 \times 10^{-4}$	$5.55 \times 10^{-4}$	$5.38 \times 10^{-4}$	$5.51 \times 10^{-4}$

**Table 3. Coherency Parameters for the Novak Model**

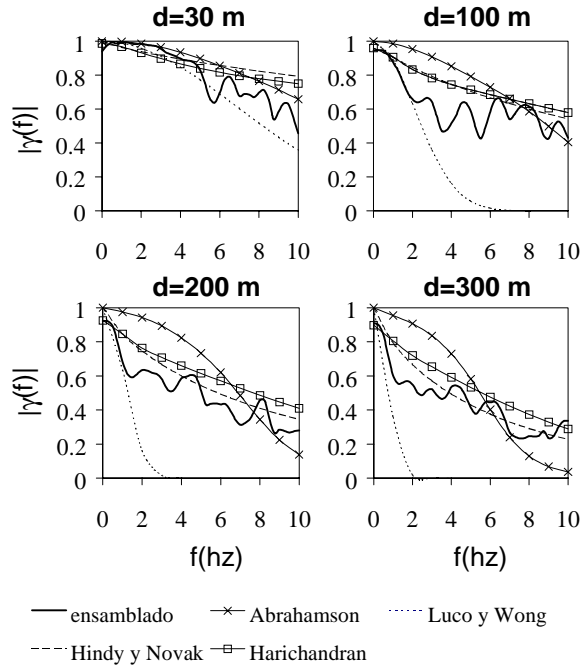
	M-1	M-2	C-1	C-2
$\alpha$	$2.50 \times 10^{-4}$	$4.75 \times 10^{-4}$	$8.61 \times 10^{-5}$	$8.43 \times 10^{-5}$
$\beta$	1.05	1.59	0.81	0.80

**Table 4. Coherency Parameters for the Harichandran Model**

	M-1	M-2	C-1	C-2
$A$	1.418	0.429	0.178	0.182
$\alpha$	0.339	0.105	$3.652 \times 10^{-2}$	$3.854 \times 10^{-2}$
$k$	3618.8	2867.8	21589.3	23970.0
$f_o$ (Hz)	0.0104	3.095	0.494	0.444
$b$	2.093	4.571	2.534	2.518



**Fig. 6. Fitted Coherency Functions for Event M-1**



**Fig. 7: Fitted Coherency Functions for Event C-1**

## CONCLUSIONS

Coherency functions are estimated for four seismic events recorded in the valley of Mexico and at the Chiba experimental station in Japan. No significant differences were found in the coherency functions for the radial and transverse components of ground motion. For frequencies as low as 0.2 Hz, the absolute value of coherency can be considerably less than 1.0 for separation distances greater than 1200 m. The results also display variation of coherency from event to event and site to site, indicating a possible dependence on event characteristics and local site conditions. The coherency models proposed by Hindy and Novak, and by Harichandran, are flexible and fit the coherency functions estimated from the data. In general, the Luco and Wong model and that proposed by Abrahamson provide poorer fits over the frequency and separation ranges of interest.

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