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A SIMPLE DISPLACEMENT COMPATIBILITY-BASED SEISMIC DESIGN STRATEGY FOR REINFORCED CONCRETE BUILDINGS

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SUMMARY

Recent studies provided an opportunity to review some of the principles, which have been used in the formulation of internationally accepted code recommendations relevant to the seismic design of buildings also subjected to torsional phenomena. In the context of specific design criteria this review led to definitions of strength, element and system yield displacements and hence relevant stiffness. A liberal strategy in the assignment to lateral force resisting elements of strength, very different from that based on traditionally defined stiffness, is postulated. This necessitates an examination of displacement compatibility criteria applicable to ductile mixed structures subjected to uniform translations. Applications of the findings are illustrated. The relevance of displacement compatibility for the ultimate limit state is extended to address also systems in which, due to static and dynamic torsional phenomena, element displacements, and hence displacement ductility demands, may be different. Only design and behaviour oriented simple concepts, combined with static and dynamic equilibrium criteria, are used to encourage designers to impart desirable properties to structures, which may be subjected to significant earthquake - induced displacements.

INTRODUCTION

In countries where severe earthquakes are expected, with few exceptions, intentional ductile structural response is the basis of the adopted design strategy. Hence, the main features, which are to be addressed here, are those relevant to displacements at the perceived ultimate limit state. Most current code provisions are based on linear elastic structural response and embody rules by which element strengths should be increased to compensate for adverse effects of torsional phenomena. In contrast, this brief study reviews some features considered to be important when attempting to quantify critical element displacements at the ultimate limit state of an inelastic system.

Existing design procedures are based on the magnitude of the stiffness eccentricity, adjusted to provide increased strength to elements, which are subsequently expected to respond in the inelastic domain. Codes do not address explicitly issues affecting ductile torsional response. Strength allocation to elements is generally based on the stiffness of elastically responding elements. Therefore, existing codified techniques manifest contradictions in the prediction of elastic and ductile structural behaviour of building systems subjected to translational and torsional motions. Although, code recommended eccentricity amplifications were found in numerous analyses to lead to satisfactory inelastic behaviour, the nature and understanding of the structure's behaviour remains shrouded.

DESIGN CRITERIA

The primary purpose of this study is to address means by which performance criteria of systems, conforming to the ultimate limit state, may be more rationally executed. These criteria are:

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- Earthquake-induced deformations, including system twist, should limit the expected displacement ductility demand on any element to its stipulated ductility capacity, $\mu_{\Delta imax}$.
- Magnitudes of ultimate interstorey displacements, to be expected at locations remote from the centre of twist, should not exceed those considered acceptable for the building, typically 2-2.5% of the storey height.

The purpose of considering torsion in ductile systems should be, therefore, to account for twist-induced displacements on certain elements, additional to those associated with uniform translations of the system, rather than to provide specific torsional resistance.

DEFINITIONS OF YIELD DISPLACEMENT, STRENGTH, STIFFNESS AND DUCTILITY

Traditional Definitions

In terms of the theory of elasticity, the flexural stiffness of, for example, prismatic elements, such as shown in Fig. 1(a), is readily expressed in terms of the flexural rigidity of the cross section, EI, where E is the modules of elasticity of the material used and I is the second moment of the cross sectional area. For the evaluation of elastic response, the translational and torsional stiffness of the system are then used. The procedure is embodied in relevant seismic codes[1]. Accordingly lateral design forces are assigned to elements in proportion of element stiffness and displacements. This procedure is traditionally used[2] to determine the required nominal strength of elements, even though element response is no longer expected to be elastic. The advantage in this design procedure is simplicity and the fact that equilibrium criteria are satisfied. The procedure assumes that the intended strength of each element in Fig. 1(a) is associated with the same displacement, Δ, shown in Fig. 1(b). Traditional usage then mistakenly defines this as the element as well as the system yield displacement.

These familiar relationships are illustrated with the use of the example structure shown in Fig. 1(a). Four rectangular cantilever walls with identical widths and heights are assumed to be interconnected at each floor level, while resisting a distributed lateral force, V_E. The same principles also apply when ductile frames provide the necessary lateral strength. The lengths of the walls, l_{wi}, (1) to (4) are such that the flexural rigidities of the sections have relative values of 1, 2, 4 and 8. The corresponding linear response of the elements and the system, associated with identical displacements of Δ, are shown in Fig. 1(b).

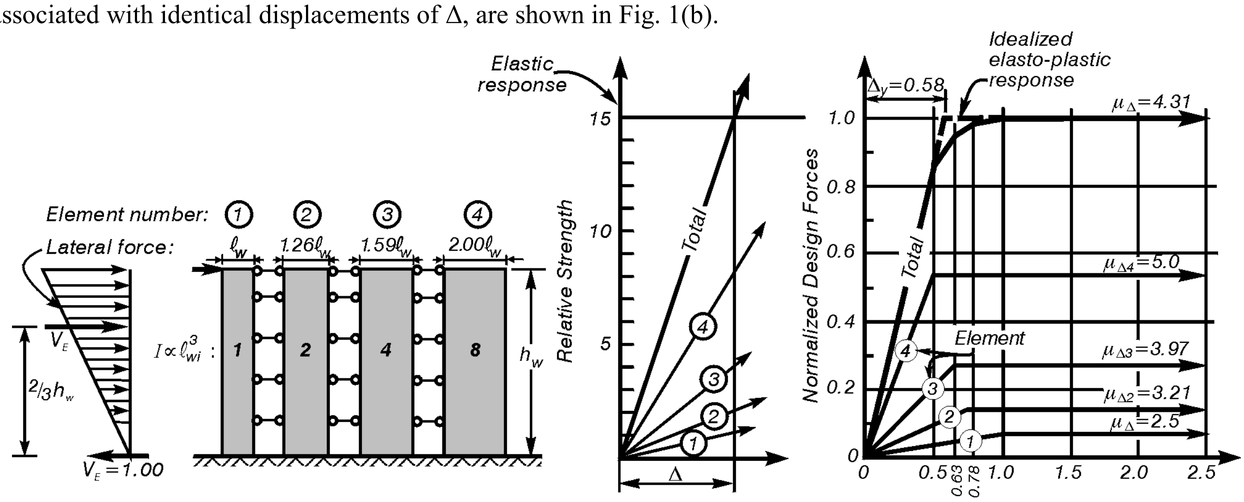

Figure 1: Simulation of Wall Element Force-Displacement Responses

Element Yield Displacement

The estimation of the yield displacement of elements, Δ_{yi}, for example those of reinforced concrete cantilever walls shown in Fig. 1(a), is best based on the yield curvature at the critical base section. The curvature at first yield, φ_y, is defined by the yield strain of the steel, ε_y, at the extreme fibre and the relevant neutral axis depth, kl_w. After some repeated loading, not exceeding this elastic limit, the moment-curvature response is very close to linear. With additional imposed curvature, nonlinearity becomes evident. Eventually the nominal flexural strength of the section is attained. The reference yield curvature, φ_y, at this level of resistance, subsequently simply referred to as yield curvature, may then be obtained by linear extrapolation[3]. Extensive analyses[4] have shown that the yield curvature is relatively insensitive to the quantity of reinforcement used and the intensity of the axial compression load, as long as this does not exceed $0.1f\underline{c}A_g$, where A_g is the gross concrete area of the section and $f\underline{c}$ is the compression strength of the concrete. Therefore, for design purposes very satisfactory approximations may be made[4] for rectangular wall sections when their yield curvature is estimated by

$$\varphi_{yi} = 2\varepsilon_y/l_{wi} \tag{1}$$

Consequently the yield displacement of an element i, such as a cantilever wall, subjected to lateral forces with a given pattern, may be satisfactorily estimated as:

$$\Delta_{yi} = C\phi_{yi} h_{wi}^2 = 2C\text{\_}_{y}\ h_{wi}^2 / l_{wi} \tag{2}$$

where h_{wi} is the height of the element and C is a constant corresponding with the vertical distribution of the magnitudes of lateral forces. In a structural system values of the yield strain of the steel and wall heights are generally constant. It is thus seen that in such common cases the yield displacement of a prismatic element is inversely proportional to its length, ie,

$$\Delta_{yi}\ \mu\ 1/l_{wi} \tag{3}$$

The flexural rigidity of the section, EI, widely used in analytical studies of ductile seismic response, is not involved in the definition of yield displacement. It should be noted that, contrary to traditional usage, the yield displacement of an element is independent of its strength. The relative yield displacements, based on eq. (3), of the four elements of the example structure with values 0.5 £ Δ_{yi} £ 1.0, are shown along the horizontal axis of Fig. 1(c). For more refined estimates of $\Delta_{yi,}$ the extent of cracking, shear and anchorage deformations may be included.

Element and System Strength

The strength of elements used here is that corresponding with ultimate limit state criteria for material strengths and strains. This may be the nominal strength, V_{ni}, or the probable strength of the element. Hence the translatory strength of a system, such as shown in Fig. 1(a), is $V_E = \Sigma V_{ni}$.

Element and System Stiffness

The application of bilinear modelling of ductile behaviour, eminently suited for the assessment of seismic performance of building structures, is examined . Details presented are relevant to the model structure shown in Fig. 1(a). It is assumed first that strength to elements has been assigned, as in traditional design practice, in proportion to the flexural rigidity of the prismatic wall elements, ie, V_{ni} μ l_{wi}^3. Hence, the strength of element (1), in terms of the unit base shear, V_E, is 1/15, and that of element (4) is 8/15, as implied in Fig. 1(b). The different yield displacements of the elements, normalised in terms of the length of element (1), are distinctly shown in Fig. 1(c). It is evident that in accordance with the bilinear modelling of element behaviour, stiffness should be defined by:

$$k_i = V_{ni}/\Delta_{yi} \tag{4}$$

stating that, contrary to traditional usage, stiffness is proportional strength!

Designers who, for the purpose of defining element stiffness, prefer to use the terms of flexural rigidity, the following substitution may be made: $E_cI_e = M_n/\varphi_{y,}$ where I_e is the equivalent second moment of area of the element section and E_c is the modules of elasticity of the concrete and M_n is the nominal flexural strength of the base section.

It is thus evident that with the use of bilinear modeling, the strength of an element must be known before its stiffness can be evaluated. The translatory stiffness of the system is then Σk_i.

Element and System Displacement Ductility

The displacement ductility applicable to element i can then be readily defined by the familiar expression, $\mu_{\Delta i} = \Delta_{ui}/\Delta_{yi,}$ where Δ_{ui} is the ultimate displacement imposed on the element.

The superposition of the bilinear response of elements, shown in Fig. 1(c), leads to the corresponding force-displacement response of the system, as shown by the heavy curved line. The nonlinear transition from the elastic to fully plastic behaviour of the system, can again be adequately simulated by a bilinear elasto-plastic relationship. This allows the reference yield displacement of the system to be conveniently defined as:

$$\Delta_y = \Sigma V_{ni}/\Sigma k_I \qquad (5)$$

Because of the different yield displacements of the four elements, under increasing displacements imposed on the system, simultaneous yielding of all elements is not possible. As Fig. 1(c) shows, some elements will commence yielding before the reference yield displacement of the system has been attained, while others will yield under larger displacements. Equation (5) allows also the system displacement ductility, $\mu_\Delta = \Delta_u/\Delta_y$, to be conveniently defined. The displacement of the system at the relevant limit state, with reference to its centre of mass, CM, is Δ_u.

It is thus seen that if the first seismic design criterion, listed in Section 2, is to be satisfied, the displacement ductility demand, μ_Δ, imposed on the system under uniform translation, may need to be restricted in order to ensure that the displacement ductility capacity of the critical element i, $\mu_{\Delta imax}$, ie, that with the smallest yield displacement, will not be exceeded. The displacement ductility capacity of an element will depend on the quality of detailing, which will ensure adequate curvature ductility to be developed in plastic hinges[5].

Figure 1(c) illustrates a situation[3] where the system displacement is to be limited to the maximum acceptable displacement for element (4). With $\Delta_{y4} = 0.5$ and $\mu_{\Delta 4max} = 5$, $\Delta_u = 5 \times 0.5 = 2.5$ displacement units. Hence the system displacement ductility demand must be limited to $\mu_\Delta = 2.5/0.58 = 4.31$. The value of Δ_y was evaluated with Eq. 5. As Fig. 1(c) shows, all other elements will be subjected to smaller displacement ductility demands.

The Assignment of Strength to Elements

It is shown in Fig. 1(c), as stated earlier, that, irrespective of their strength, elements subjected to increasing identical displacements will commence yielding in a predetermined sequence. It may, therefore, be construed, that the strength to elements of such a system could be assigned arbitrarily. Limitations on this arbitrariness stem from rational engineering practice in providing the chosen strength and the consideration of the effects of this choice on the seismic response of the system.

It will be postulated in Section 4 that the critical quantity affecting the torsional response of ductile structural systems is the strength eccentricity, ie, the distance between the centre of mass, CM, and the centre of strength, CV, defined by eq. (6). The significant advantage, which an arbitrary but astute choice of element strengths offers to the designer, is the deterministic location of CV. As a subsequent example will show, the designer is now in a position to choose a strength eccentricity that is perceived to minimize the detrimental effects of torsional phenomena. With the traditional practice, embodied in most building codes[1], such a choice was not possible. To illustrate this simple concept, the structure shown in Fig. 1(a), slightly modified, as shown in Fig. 2(a), will be examined. The four wall elements are interconnected by an infinitely rigid floor diaphragm. Hence, when this floor is subjected to translation without twist, the response of the four elements will be as presented in Fig. 1(c).

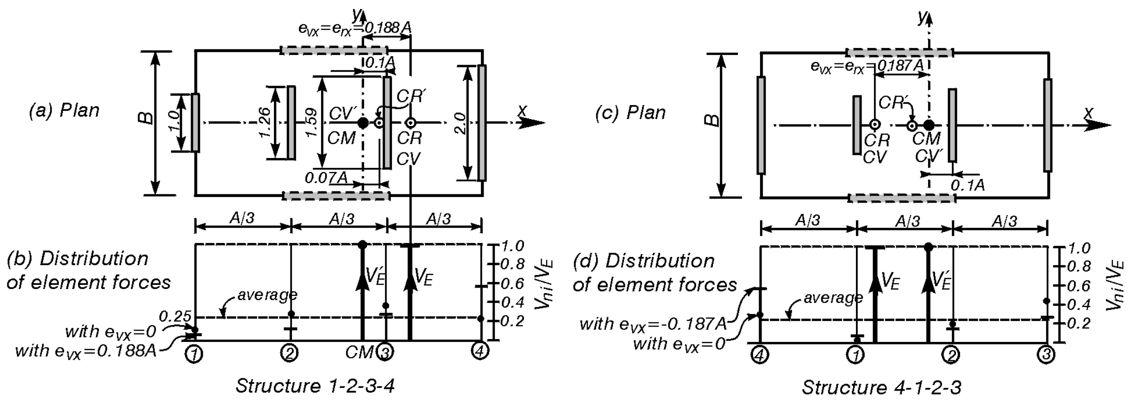

Figure 2: Strength and Stiffness Relationships in Multi-element Ductile Translatory Systems

When strength to the elements in this 1-2-3-4 type structure is assigned in accordance with traditional practice, as in Fig. 1(c), the centre of the strength, CV, is located to the right of the centre of mass, CM. This location, ie, the strength eccentricity, is found from:

$$e_{vx} = x_i V_{ni} / \Sigma V_{ni} \tag{6}$$

where x_i is the distance of element i, with a nominal strength V_{ni}, from CM. In this case it is found that e_{vx} = 0.188A. Because element strengths were made proportional to the traditionally defined element stiffness ie, EI, CV coincides, as Fig. 2(a) shows, with the centre of rigidity, CR, of the system. Element strength so established are shown to scale, in terms of the unit base shear, by the small horizontal bars in Fig. 2(b).

Using simple experience-based engineering judgement, designers may distribute strengths to elements, without changing the total base shear capacity of the structure, V_E, in such a way that the strength eccentricity is eliminated. For the system presented in Fig. 2(a), for convenience named the 1-2-3-4 structure, element strengths may be readily redistributed in such a way that the new location of the base shear, $V\underline{E} = V_E$, will coincide with CM. Thereby $e_{vx} = 0$. The relative magnitudes of element strengths so found are shown by dots in Fig. 2(b).

Because element stiffness is proportional to element strength (eq. (4)), the centre of rigidity, CR¢, associated with $e_{vx} = 0$, as shown in Fig. 2(a), will be at a different location. It should be noted that whenever the designer changes the originally assumed nominal strength of an element, its stiffness will also need to be proportionally changed. A set of element strengths will result in a specific stiffness eccentricity, a quantity assumed in the existing design procedures [1] to be constant once element geometry is finalized.

Another example is considered in Fig. 2(c). The same walls, as in Fig. 1(a), are used, except the positions have been changed, with the largest wall being placed at the left edge. The ensuing strength eccentricity in this 4-1-2-3 structure is now -0.187A. Strength redistribution, different from that used for the 1-2-3-4 structure, will also eliminate the strength eccentricity, and hence significant torsional phenomena. The location of CR¢, associated with the latter distribution of strengths is seen in Fig. 2(c), while element strengths, corresponding with the two locations of CV, are plotted in Fig. 2(d).

When e_{vx} = 0.188A, the reference yield displacement of the system is from eq. (5) is $\Delta_y = 1.00/1.727 = 0.58$, as shown in Fig. 1(c). The restriction on the system displacement ductility demand, to ensure that under uniform translation the capacity of element (4), $\mu_{\Delta 4max} = 5.0$, is not exceeded, was described in Section 3.3 and is illustrated in Fig. 1(c).

Very significant redistribution of element strengths, relative to those assumed in Fig. 1(c), has relatively small effects on expected overall translatory response. Element strengths, resulting in zero strength eccentricity, as shown in Figs. 2(a) and (b), and corresponding stiffness values indicate that $\Delta_y = 0.66$. Therefore, the system displacement ductility demand should be restricted to $\mu_\Delta = \Delta_{u4}/\Delta_y = 5 \times 0.5/0.66 = 3.79$.

TORSIONAL PHENOMENA

The Traditional Approach to Earthquake-induced Torsion in Buildings

Design approaches considering torsional effects during seismic excitations of structural systems, as incorporated in building codes (1,6) are very similar. They are based on linear elastic behaviour under the application of lateral forces. Because the principles and applications involved are well established, a detailed description is not provided here. Highlights of the relevant features, particularly those fundamentally different from those proposed here, are:

The torque to be resisted within the elastic domain of response results from the stiffness eccentricity, e_r, of the storey shear, ie, the distance between the centre of mass, CM, and the centre of rigidity; CR, of the system. Element stiffness are based on section geometry only, ie, relevant EI values. Strengths are assigned proportionally to this stiffness and element displacement.

For design purposes this eccentricity, $e_{r,}$ is modified to allow for factors which may adversely affect the strength of elements situated at various positions within the plan of the building.

The aim of the design is to provide strength to elements, additional to that resulting from satisfying criteria of equilibrium and compatibility of elastic deformations. Because elements at different locations are affected by different specified values of design stiffness eccentricities, storey shear capacities so derived, will increase as the stiffness eccentricity increases. Thereby a corresponding reduction in the system displacement ductility demand can be expected.

Existing codified procedures do not address the amplification of element displacements as a consequence of torsional effects on inelastic, ie, ductile structural response. Neither are features associated with the bilinear modelling of element response, as discussed in Section 3, necessary to quantify inelastic displacements, considered in these codes. Strength eccentricity is not considered.

The principal aim of recent studies (3) was to explore the behaviour of fully or partially plastic mechanism, as affected by system twist, and to find approaches which would possibly enable element displacements at the ultimate limit state to be directly addressed. To this end structural models, ie, mechanisms, relevant to torsionally unrestrained and restrained systems were introduced. A brief examination of these is presented in the next sections.

Torsionally Unrestrained Ductile Systems

The simplest forms of torsionally unrestrained systems with respect to a base shear, V_{Ey}, are assembled in Fig. 3. For the sake of simplicity rectangular wall elements, properties of which were presented in Section 3, are used again.

With the appropriate evaluation of the storey yield displacements, the same principles apply also to ductile frames[7]. An important property of these ductile systems is considered to be the location of the centre of strength or centre of resistance, CV, of the system, defined by eq. (6).

For the chosen examples in Fig. 3, element strengths have been deliberately assigned so that CV coincides with CM, and hence the strength eccentricity, e_{vx}, is zero. This implies that when, under the action of the base shear, V_{Ey}, all elements develop their nominal strength, no torque is generated. Because transverse elements, shown in Fig. 3, providing base shear resistance, V_{Ex}, in the x direction, are located in single plane, no restraint against twist, should this occur, is available. It is for this reason that these systems have been designated [3] as torsionally unrestrained.

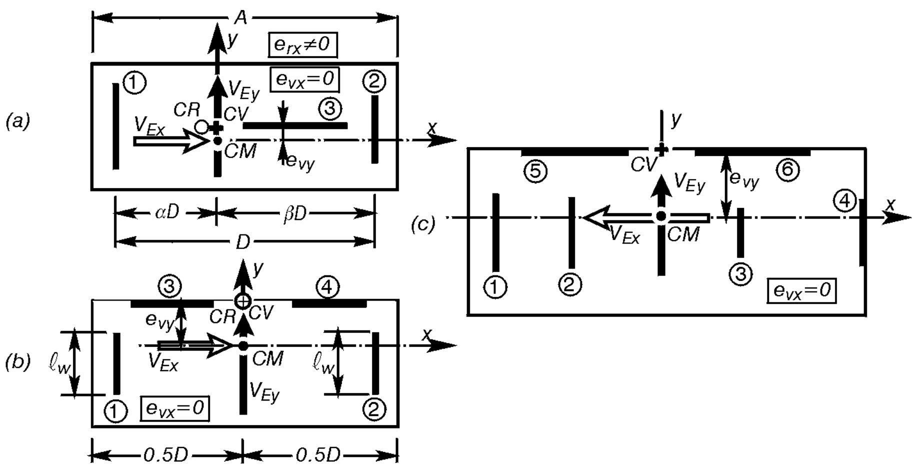

Figure 3: Examples of Torsionally Unrestrained Systems

Strength eccentricity is eliminated in the system shown in Fig. 3(a) when element nominal strengths, V_{ni}, satisfy static equilibrium requirements. It may be shown, however, that there will be a stiffness eccentricity. Therefore, in the elastic domain of response a twist of the system will occur. This in turn will engage the rotary inertia of the distributed mass. As a corollary it will be found that when strengths to elements (1) and (2) in Fig. 3(a) have been assigned so that there is no stiffness eccentricity (e_{rx} = 0), there will be a strength eccentricity (e_{vx} ¹ 0).

The system in Fig. 3(b) shows exceptional perfect symmetry with respect to the y axes. Elements (1) and (2) are assumed to have nominal strengths of $0.5V_{Ey}$. With identical wall lengths, $l_{w,}$ element stiffness are, therefore, also identical. Hence $e_{rx} = e_{vx} = 0$. Element deformations in both the elastic and inelastic domains of response will be identical and twist should not occur. Fig. 3(c) shows, in terms of V_{Ey}, a multi-element system, traditionally defined as being statically indeterminate. However, when ductile response is addressed, as explained in Section 3.5, strength allocation may be arbitrary. Of the infinite possibilities the designer may chose one attractive solution, whereby, for example, strength eccentricity is eliminated. Figure 2 illustrated examples how this could be achieved. In the absence of strength eccentricity, no torque need to or can be sustained by the structure in Fig. 3(c) when, with all elements yielding, the ultimate limit state is being approached. However, because e_{rx} ¹ 0, in the elastic domain system twist will be inevitable.

In real structures the conditions depicted in Fig. 3 will not be possible to be achieved because the probable strengths of the elements will be inevitably different from those intended. Consequently some strength eccentricity will exist. The full strength of all elements can thus be engaged only if a torque, corresponding to e_{vx}, can be introduced. Under static conditions this situation cannot arise. However, with the twist-induced rotary inertia of the distributed mass during dynamic response, the development of the full system strength, associated with ductile response of all elements, will generally occur.

Torsionally Restrained Ductile Systems

Whenever elements, transverse to translatory ductile ones, are present and have

sufficient strength to resist a torque resulting from strength eccentricity, system twist will be restrained. The example structure shown in Fig. 2(a), with transverse elements present, shown with dashed outlines, is used to illustrate relevant features of behaviour. With a known static torque the forces induced in the transverse elements can be compared with their strength. Due to the rotary inertia of the mass, torsion-induced forces in these elements are likely to increase. The acceptable twist is controlled by the displacement capacity of the translatory elements. For the 1-2-3-4 structure Fig. 4(a) shows diaphragm displacement patterns associated with uniform and non-uniform translation with the optimum angle of twist, $\theta_t = 1.95/A$. The latter is controlled by the displacement ductility capacities of elements (3) and (4). The corresponding idealized force displacement responses of all four elements are presented in Fig. 4(b). Element responses associated with strength allocations corresponding to $e_{vx} = 0$ are shown in Fig. 4(c).

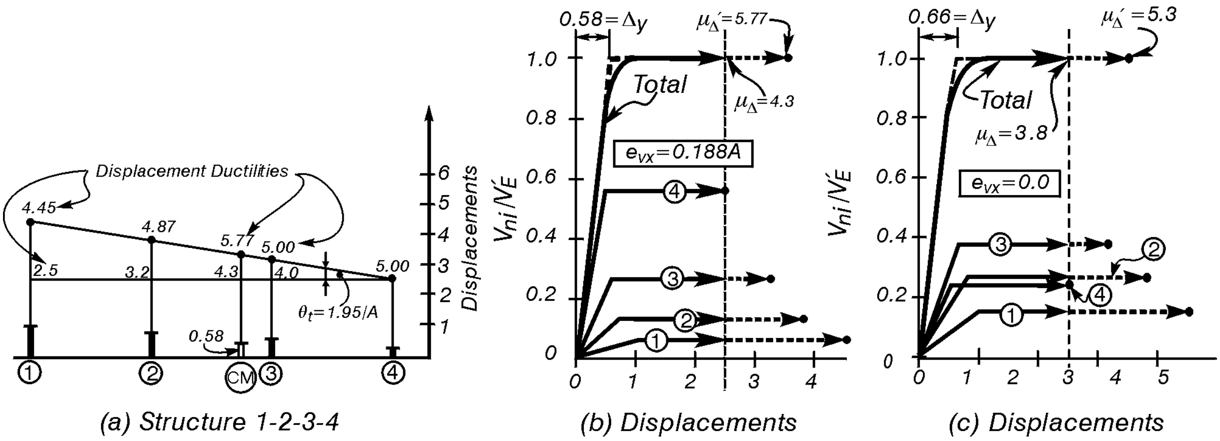

Figure 4 Limiting Displacement Profiles for a Torsionally Restrained Example Structure

This review suggests that, even with a significant strength eccentricity of the order of 0.1A, this ductile example structure should be tolerant with respect to torsional demands, also when dynamic effects are taken into consideration. An astute designer may use the available freedom in the choice of strength allocation to elements, to arrive at a solution that results in a moderate strength eccentricity. This then can be expected to result in more even utilisation of the displacement ductility capacities of all elements.

CONCLUSIONS

The primary aim in the seismic design of structural systems should be to address displacements corresponding to the stipulated performance criteria. Limitations relevant to the ultimate limit state are dictated by displacement ductility capacities that can be provided for elements, and the realistic evaluation of the absolute magnitudes of the corresponding displacements.

Reliable estimates of displacement ductilities can be made only if the estimation of the reference yield displacements of the system and its constituent elements is realistic. Current widely used design practice generally evaluates erroneously as well as grossly underestimates the reference yield displacements of reinforced concrete elements.

Studies, utilizing the exposure of some prevalent myths and fallacies in seismic design [8], drew attention to the fact that the yield displacement of an element is rather insensitive to the strength provided. For seismic design purposes, with few exceptions, the yield displacement of an element should be considered as being a geometric property, independent of element strength. This enables a simple bilinear modelling of the elasto-plastic monotonic behaviour of elements.

Contrary to traditional definitions, element stiffness, defined as the ratio of nominal strength to reference yield displacement, is strength-dependent. When features of ductile seismic response are addressed, the flexural rigidity, EI, of reinforced concrete elements, widely used for the purpose of estimating their stiffness should, therefore, be considered inappropriate, unless it is radically adjusted,

The utilisation of the above conclusions allows the the designer to assign strength to elements in an arbitrary manner dictated only by the desire, by means of the appropriate choice of strength eccentricity, to achieve optimal seismic performance.

It is suggested that in the formulation of a design strategy, catering also for the torsional response of ductile system, attention be focused on expected displacements rather than on torsional resistance. To this end the most important relevant parameter is claimed to be the strength rather than the traditionally used stiffness eccentricity.

These principles allow the system displacement ductility capacity, necessary to ensure that at the ultimate limit state displacement demands will not exceed corresponding capacities of critical elements, to be realistically and readily determined.

It is argued that the rotary inertia of the distributed mass of a system, mobilized by system twist, will beneficially affect element displacement ductility demands in torsionally unrestrained systems. The opposite is considered applicable to torsionally restrained systems. The validity of these postulates is being studied further.

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