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STRUCTURAL IDENTIFICATION AND POTENTIAL SYSTEMS

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SUMMARY

Structures behaviour is estimated by means of mathematical models that may be calibrated by dynamic identification procedures. The selection of right models is basic in the dynamic identification in order to obtain the minimum distance between the structural system dynamics and the mathematical model. At the same time the choice of right models is basic in order to obtain simple solving equations that can give results with high degree of reliability. The reaching of the minimum distance between the model and the real system by means of simple solving equations is a primary goal if one wants to obtain an effective identification procedure. Generally it is very difficult to reach this goal since the using of complex models needs in order to reproduce the complexity of structural systems and unfortunately, often, not reliable complex algorithms derive from complex models. So a right medium must be searched. In this paper is analysed the using of the so called potential models in structural identification. This models, by means a statistical approach, allow to formulate an identification procedure for systems under stochastic loads. This procedure allows one to obtain good results in terms of distance from the real system and consist of solving some linear algebraic equations. The purpose of the paper is to show that the same approach is not effective when different models are used. Furthermore through the paper is shown that, by means of potential models, the estimation of input probabilistic features can be performed in addition to the estimation of structural mechanical features. Even in this case the using of different models fails. Once more is underlined the effectiveness of potential models in identification.

INTRODUCTION

The interest in the general topics of identification has increased greatly during the past three decades [Eykhoff, 1974;Kozin and Natke, 1986; Ghanem and Shinozuka, 1995]. It follows from this fact: it is possible to have many kinds of mathematical models but it is very difficult to associate the right model to a structural system. Identification is a tool to solve this problem.

Generally identification needs: a) to fix a class of parametric models, that is depending on a set of parameters; b) to determine the model belonging to the fixed class which, better of each other, simulates the behaviour of the structure. Every model belonging to the fixed class is obtained when a set of values for the parameters are assigned. So determining the best model of the fixed class means to estimate the best set of parameters in such a way the model reproduces the structural system dynamics. Generally the knowledge of the time output history and the time input history needs for dynamic identification purpose. The choice of a right class of mathematical models is basic in the identification in order to obtain the minimum distance between the structure dynamics and the mathematical model. For example, supposing that the structure to be identified has a nonlinear behaviour and supposing to choose a class of linear parametric models for its identification, the estimation of the best set of parameters could converge to the best linear approximation of the real dynamic behaviour. No better result is possible.

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Many techniques are effective for the identification of the stiffness features [Yun and Shinozuka, 1980; Hoshiya and Saito, 1984; Koh and See, 1994; Ghanem and Shinozuka, 1995] but often these techniques fail for the identification of the dissipation characteristics. It happens in special way when the system have a non linear behaviour or is very complex in the sense that its modelling needs a multi degree of freedom approach. It can be said that more complex is the system and the model used for reproducing its behaviour as more unreliable is to identify the dissipation parameters. For this reason simple models are often preferred to complex models in spite of their degree of approximation. It must be underlined that even the use of simple models fails when any data are available about the input [Kozin and Natke, 1986].

System identification can be performed by means of time domain approaches or frequency domain ones. In [Cavaleri and Di Paola, 1999] it has been shown that good results can be obtained by using a time domain approach even in the case of unknown input. It means that, referring to dissipation features, an important improvement has been made in identification of systems. The results obtained in [Cavaleri and Di Paola, 1999] derive from the use of a special class of models, that is the so called restricted potential models (RPM) whose characteristics can be found extensively in [Cavaleri and Di Paola, 1998a]. It means that the using of models different from RPM does not guarantee the obtaining of algorithms with the same effectiveness as it will be shown through the paper. It must be stated that in literature the words "potential systems" are more generally used with the same meaning of "potential models".

RESTRICTED POTENTIAL MODELS

Let the equation of motion be

$$\ddot{\boldsymbol{X}} + \pi\left(\sum_{j=1}^{s} j a_j H(\boldsymbol{X},\dot{\boldsymbol{X}})^{j-1}\right)\boldsymbol{K}\dot{\boldsymbol{X}} + \boldsymbol{r}(\boldsymbol{X},\boldsymbol{b}) = \boldsymbol{W} \tag{1}$$

where $\boldsymbol{X}$ is a displacements n-vector, the dot upper $\boldsymbol{X}$ means derivative with respect of the time, $\boldsymbol{r(X,b)}$ is a vector of non linear functions of $\boldsymbol{X}$ and $\boldsymbol{b}$ with the meaning of elastic restoring forces and $\boldsymbol{b}$ is a vector of the time invariant stiffness parameters. $\boldsymbol{W}$ is a vector of gaussian white noises whose power spectral density matrix (PSD matrix) is $\boldsymbol{K}$, that is

$$E\left[\boldsymbol{W}(t_1)\boldsymbol{W}^T(t_2)\right] = \boldsymbol{K}\delta(t_1 - t_2) \tag{2}$$

$E[\cdot]$ being the average operator and δ the unitary impulse (Dirac's δ). Furthermore in eq.(1) a_j are the time invariant dissipation parameters and $H(\boldsymbol{X},\dot{\boldsymbol{X}})$ is the total energy of the system, that is

$$H(\boldsymbol{X},\dot{\boldsymbol{X}}) = \frac{1}{2}\sum_{i=1}^{N}\dot{X}_i^2 + \int_0^{\boldsymbol{X}} \boldsymbol{r}(\boldsymbol{x})^{\mathrm{T}}\,\mathrm{d}\boldsymbol{x} \tag{3}$$

Eq. (1) belongs to the class of potential models [Cai and Lin, 1996]. In the steady state the probability density function associated to $\boldsymbol{X}$ and $\dot{\boldsymbol{X}}$ has the form

$$p_{\boldsymbol{X},\dot{\boldsymbol{X}}}(\boldsymbol{x},\dot{\boldsymbol{x}}) = c\,exp\left(-\sum_{j=1}^{s} a_j h^j(\boldsymbol{x},\dot{\boldsymbol{x}})\right) \tag{4}$$

where $h(\boldsymbol{x},\dot{\boldsymbol{x}})$ is the domain of the stochastic process $H(\boldsymbol{X},\dot{\boldsymbol{X}})$, while $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$ are the domains of $\boldsymbol{X}$ and $\dot{\boldsymbol{X}}$ respectively. In eq.(4) c is a normalisation factor. Eq.(1) can be rewritten in the $It\hat{o}$ form as follows

$$d\boldsymbol{Z}_1 = \boldsymbol{Z}_2 dt \tag{5a}$$

$$d\boldsymbol{Z}_2 = -\pi\left(\sum_{j=1}^{s} j a_j H^{j-1}\right)\boldsymbol{K}\boldsymbol{Z}_2 dt - \boldsymbol{r}(\boldsymbol{Z}_1,\boldsymbol{b})dt + d\boldsymbol{B} \tag{5b}$$

where $\boldsymbol{Z}_1 = \boldsymbol{X}$ and $\boldsymbol{Z}_2 = \dot{\boldsymbol{X}}$ are the n-vectors of the state variables and $\boldsymbol{B}$ is the n-vector of the Wiener processes whose formal derivative is the n-vector of white noises $\boldsymbol{W}$.

The model (1) has been much investigated, as in [Cavaleri and Di Paola, 1998a; Cavaleri et al., 1999a], in order to know its ability of approximating the behaviour of many kind of non linear systems.

SYSTEM IDENTIFICATION ALGORITHM

By means of RPM can be formulated an identification procedure that can be summarised as follows. First the stiffness parameters b_i (entries of the vector $\boldsymbol{b}$) are estimated by means of the following equation

$$E\left[\ddot{\boldsymbol{X}} X_i^{2k-1}\right] + E\left[\boldsymbol{r}(\boldsymbol{X},\boldsymbol{b}) X_i^{2k-1}\right] = 0; \qquad k = 1,2,......,g\,; \quad i = 1,2,....,n \tag{6}$$

where the vector $\boldsymbol{r}(\boldsymbol{X},\boldsymbol{b})$ must be better defined for its resolution with respect of $\boldsymbol{b}$. Eq. (6) is obtained by multiplying the first an the second member of eq.(5b) for X_i^{2k-1}, dividing for dt, applying the average operator and taking into account the non anticipating property of the process d$\boldsymbol{B}$ [Jazwinsky, 1970] with respect of each function of $\boldsymbol{X}$ and $\dot{\boldsymbol{X}}$.

The dissipation parameters can be obtained by means of the equation

$$-\sum_{j=1}^{s} j a_j E[H^{m+j-2} \dot{\boldsymbol{X}}^T \dot{\boldsymbol{X}}] + E[H^{m-1}] + (m-1)E[H^{m-2} \dot{\boldsymbol{X}}^{\mathrm{T}} \dot{\boldsymbol{X}}] = 0; \qquad m = 1,2,...,\mathrm{s} \tag{7}$$

Eq.(7) is obtained by applying the $It\hat{o}$ rule [Jazwinsky, 1970] to the powers of the system energy H, applying the average operator and taking into account the non anticipating property of the process d$\boldsymbol{B}$.

The stationary condition has been supposed in the obtaining eq.(6) and eq.(7).

In the practical applications, once a response measurement of the system to be identified is performed, the averages contained in eq.(6) and eq.(7) can be evaluated and the parameters a_j and b_i can be estimated by solving eq.(6) and eq.(7). Generally $\boldsymbol{r}(\boldsymbol{X},\boldsymbol{b})$ is a linear function of the entries b_i of $\boldsymbol{b}$ so eq.(6) is linear with respect of the unknown b_i . On the other hand eq.(7) is linear with respect of the unknown a_j. The linearity of eq.(6) and eq.(7) with respect of the unknowns makes simple their resolution.

An extended treatment of the steps followed in the obtaining eq.(6) and eq.(7) is given in [Cavaleri and Di Paola, 1999].

The technique used for obtaining eq.(6) and eq.(7) can be performed referring to models different from RPM but it leads to different equations that in many cases are not effectiveness to solve a problem of structural identification as it will be shown in the next section.

THE EFFECTIVENESS OF RPM IN SYSTEM IDENTIFICATION

Eq.(6) shows that the stiffness identification can be performed without taking into account the dissipation parameters. As a matter of fact any information about the dissipation parameter appears into eq.(6). In this way any estimation error of the dissipation parameters (that generally produce the largest errors) can influences the estimation of the stiffness parameters. For this reason eq.(6) gives an estimation very reliable of the stiffness parameters with errors less than 1% as it shown in [Cavaleri, 1998]. If one considers a model different from RPM then, generally, a dependence between the stiffness parameters and the dissipation parameters is obtained with a loss of reliability of the algorithm.

For sake of clarity let the single degree of freedom model be

$$\ddot{X} + a_1 \dot{X} + a_2 \dot{X}\left|\dot{X}\right| + b_1 X + b_2 X^3 = W \tag{8}$$

where a_1 and a_2 are the dissipation parameters while b_1 and b_2 are the stiffness parameters. The using of the same technique performed for obtaining eq.(6) leads to

$$E\left[\ddot{X}\, X^{2k-1}\right] + a_2 E\left[\dot{X}\left|\dot{X}\right| X^{2k-1}\right] + b_1 E\left[X\, X^{2k-1}\right] + b_2 E\left[X^3 X^{2k-1}\right] = 0; \qquad k = 1,2,3 \tag{9}$$

where now the estimation of the stiffness parameters b_1 and b_2 depends on the estimation of the dissipation parameter a_2. It means that every error in the evaluation of a_2 produces an error in the evaluation of the stiffness parameters.

As a consequence of the independence of the stiffness parameters estimation from the dissipation parameters estimation, another fundamental remark can be derived. If one observes eq. (6) it is easy to recognise that it can be written, without modifying its contents, as

$$E\left[\ddot{X}_i \boldsymbol{X}^{2k-1}\right] + E\left[r_i(X, \boldsymbol{b}) \boldsymbol{X}^{2k-1}\right] = 0; \qquad k = 1,2,......,g\,; \qquad i = 1,2,....,n \tag{10}$$

$\boldsymbol{X}^{2k-1}$ being the n-vector whose entries are X_i^{2k-1}. Eq.(10) shows that an estimation can be made of stiffness parameters that appare in the *i-th* scalar equation of motion without involving any other parameter. As a matter of fact eq. (10), once i is fixed, contains only informations on the stiffness parameters of the i-th motion equation. It means that an error in the estimation of the stiffness parameters contained in the *i-th* of eq. (6) does not influences the estimation of any other stiffness parameter. This property derives from the analytical form of the potential models.

Once the stiffness parameters are estimated, the energy H can be evaluated and eq. (7) can be used to estimate the dissipation parameters. As it is possible to recognise, the evaluation of the dissipation parameters can be obtained without any information about the input intensity differently from the using of models which do not belongs to the class of RPM. That is to say, if one applies the technique used in deriving eq.(7) with a model different from RPM, it is not generally possible to obtain a set of algebraic equations effective for the evaluation of the dissipation parameters. For example, if one refers to the model (8) then the following equation can be obtained

$$-a_1 E\left[H^{i-1}\dot{X}^2\right] - a_2 E\left[H^{i-1}\dot{X}^2\left|\dot{X}\right|\right] + \frac{s}{2} E[H^{i-1}] + \frac{s}{2}(i-1)E[H^{i-2}\dot{X}^2] = 0\,; \quad i = 1,..... \tag{11}$$

where s is the intensity of the white noise W ($E[dB^2]=s\,dt$). Once the index i has been fixed eq. (11) constitutes an homogeneous equation with respect to the unknowns a_1, a_2 and s. So it cannot be solved before the knowledge of s.

In many usual cases eq. (7) can be transformed in

$$-\sum_{j=1}^{s} j a_j \alpha E[H^{m+j-1}] + E[H^{m-1}] + (m-1)\alpha E[H^{m-1}] = 0; \qquad m = 1,......,s+1 \tag{12}$$

α being a constant having the form [Cavaleri et al., 1999b]

$$\alpha = \frac{E[H^{m-2}\dot{\boldsymbol{X}}^T\dot{\boldsymbol{X}}]}{E[H^{m-1}]}, \quad \forall\, m \tag{13}$$

By using eq.(12) α and a_j (j =1,…, s) can be estimated after the evaluation of the statistic moments of the H powers. Eq.(12) is more effective than eq.(11) (supposing that it can be used for system identification) because the statistic moments of the kind $E[H^n]$ can be evaluated more exactly than the statistic moments of

$E[H^{n-1}\dot{X}^2]$ and $E[H^{n-1}\dot{X}^2|\dot{X}|]$. As a matter of fact H is a slowly varying process while X and $\dot{X}$ are rapidly varying processes.

In conclusion it must be underlined that eq.(6) and eq.(7) allows one to define completely the features of the system without any dependence from the input intensity. Furthermore if one wants to know the probability distribution of the state variables no more calculation must be performed because the probability function is already known exactly (see eq.(4)) if the stiffness parameters b_j and dissipation parameters a_j are known. On the other hand if eq.(8) is used as model, a Monte Carlo simulation have to be performed in order to know the statistics of the response, once the stiffness and dissipation parameters are estimated.

INPUT IDENTIFICATION ALGORITHM

In many practical cases can be useful to identify the input probability features, that is to estimate the entries of the PSD matrix ***K***. At this point no complete treatment has been performed in literature about the input identification by using RPM. In [Cavaleri, 1998] a first method has been proposed to estimate the input intensity of single degree of freedom system by means of RPM while in [Cavaleri and Di Paola, 1999] the trace of the PSD matrix ***K*** is evaluated. In this section will be shown as it is possible to evaluate the various entries of the matrix ***K*** as a development of the methods presented in [Cavaleri, 1998; Cavaleri and Di Paola,1999]. Even in this case the effectiveness of the algorithm is connected to the special form of the model used as it will be shown in the forward.

Theoretical formulation

Referring to the eq.(5b), the i-th entries can be written as

$$d\,Z_{2i}(t_1)=\left(-\pi\sum_{j=1}^{s} ja_j H^{j-1}\sum_{j=1}^{n}K_{ij}Z_{2j}-r_i(\boldsymbol{Z}_1)\right)dt_1+d\,B_i(t_1) \tag{14}$$

By multiplying the first and the second member of eq.(14) for $Z_{2k}(t_2)$, with $t_2 \le t_1$, dividing for dt_1, averaging and taking into account the non anticipating property of $d\boldsymbol{B}$, one obtains

$$\frac{\partial E[Z_{2i}(t_1)Z_{2k}(t_2)]}{\partial t_1}=\frac{\partial}{\partial \tau}R_{Z_{2i}Z_{2k}}(\tau)=-\pi E\left[\left(\sum_{j=1}^{s} ja_j H^{l-1}\right)\sum_{j=1}^{n}\left(K_{ij}Z_{2j}Z_{2k}\right)\right]- \\ -E[Z_{2k}(t_2)r_i(\boldsymbol{Z}_1,\boldsymbol{b})]; \qquad \tau>0 \tag{15}$$

$R_{Z_{2i}Z_{2k}}(\tau)$ being the correlation function of the state variables Z_{2i} and Z_{2k}. When $t_1 \to t_2$ eq.(15) becomes

$$\left[\frac{\partial}{\partial \tau}R_{Z_{2i}Z_{2k}}(\tau)\right]_{\tau\to 0^+}=-\pi E\left[\left(\sum_{j=1}^{s} ja_j H^{l-1}\right)\sum_{j=1}^{n}\left(K_{ij}Z_{2j}Z_{2k}\right)\right] \tag{16}$$

obtained by using a fundamental property of potential models [Cavaleri, 1998], that is

$$E[Z_{2k}(t_2)r_i(\boldsymbol{Z}_1,\boldsymbol{b})]=0 \tag{17}$$

Eq. (16) can be written in a more compact form

$$\boldsymbol{R}_i=\boldsymbol{k}_i^T\boldsymbol{V} \tag{18}$$

being $\boldsymbol{R}_i$ the vector containing the terms $\left[\frac{\partial}{\partial\tau}R_{Z_{2i}Z_{2k}}(\tau)\right]_{\tau\to 0^+}$, $\boldsymbol{k}_i$ the i-th column of the matrix $\boldsymbol{K}$ and $\boldsymbol{V}$ the matrix having the form

$$\boldsymbol{V} = -\pi\begin{bmatrix} E[\lambda Z_{2_1}Z_{2_1}] & E[\lambda Z_{2_1}Z_{2_2}] & .... & E[\lambda Z_{2_1}Z_{2_n}] \\ E[\lambda Z_2 Z_2] & E[\lambda Z_{2_2}Z_{2_2}] & .... & E[\lambda Z_{2_2}Z_{2_n}] \\ ...... & .... & .... & .... \\ E[\lambda Z_{2_n}Z_{2_1}] & E[\lambda Z_{2_n}Z_{2_2}] & .... & E[\lambda Z_{2_n}Z_{2_n}] \end{bmatrix} \tag{19}$$

λ having the following expression

$$\lambda = \left(a_1 + 2a_2 H + ... + na_n H^{n-1}\right) \tag{20}$$

In conclusion in order to estimate the input parameters the quantities $\left[\frac{\partial}{\partial\tau}\sum_{j=1}^{N}R_{Z_{2i}Z_{2k}}(\tau)\right]_{\tau\to 0^+}$ and $E\left[H^m Z_{2_i}Z_{2_k}\right]$ must be evaluated from the measured response of the system. For practical applications the first member of eq.(18) can be evaluated taking into account the following analytical steps

$$\begin{aligned}\left[\frac{\partial}{\partial\tau}R_{Z_{2i}Z_{2k}}(\tau)\right]_{\tau\to 0^+} &= \lim_{\tau\to 0^+}\frac{E\left[Z_{2_i}(t)Z_{2_k}(t+\tau) - Z_{2_i}(t)Z_{2_k}(t)\right]}{\tau} = \\ &= \lim_{\tau\to 0^+}\frac{E\left[Z_{2_i}(t)Z_{2_k}(t+\tau)\right] - E\left[Z_{2_i}(t)Z_{2_k}(t)\right]}{\tau} = \\ &= E\left[Z_{2_i}(t)\lim_{\tau\to 0^+}\frac{Z_{2_k}(t+\tau) - Z_{2_k}(t)}{\tau}\right] = E\left[Z_{2_i}(t)\dot{Z}^+_{2_k}(t)\right]\end{aligned} \tag{21}$$

where the signum (+) means that the values $\dot{Z}^+_{2_k}$ is shifted of dt with respect of $\dot{Z}_{2_i}$.

The property expressed by eq. (17) is fundamental in the proposed algorithm. That property can be simply obtained [Cavaleri, 1998] because of the knowledge of the state variables probability density function and cannot be derived directly from other kind of models.

In the following section the presented input identification algorithm will be applied to verify its effectiveness.

Numerical application

Let us suppose a system of two degree of freedom has the following govern equation

$$\ddot{\boldsymbol{X}} + \pi a_1\boldsymbol{K}\dot{\boldsymbol{X}} + \boldsymbol{Q}\boldsymbol{X} = \boldsymbol{W} \tag{22}$$

where the dissipation parameter have the value $a_1 = 10^{-2}$ while the entries of the stiffness matrix $\boldsymbol{Q}$ are

$$\boldsymbol{Q} = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1 \end{bmatrix} \tag{23}$$

Let us suppose that the Power Spectral Density matrix $\boldsymbol{K}$ has been fixed as

$$\boldsymbol{K} = \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} \tag{24}$$

For the input identification purpose the following class of models has been chosen

$$\ddot{\boldsymbol{X}} + \pi a_1 \overline{\boldsymbol{K}} \dot{\boldsymbol{X}} + \boldsymbol{Q}\boldsymbol{X} = \boldsymbol{W} \tag{25}$$

where $\overline{\boldsymbol{K}}$ have to be estimated. So in this case the physic system is governed by an RPM and the model used for identification purpose is an RPM. This choice is necessary to verify the effectiveness of the algorithm. As a matter of fact the algorithm will be effective if the identification procedure will give

$$\overline{\boldsymbol{K}} = \boldsymbol{K} \tag{26}$$

Once a response stationary time history has been generated its statistics can be calculated and the input identification algorithm can be applied to test its degree of convergence. In the examined case eq.(21) can be applied for the evaluation of the entries of the $\boldsymbol{R}_i$ vector. So from eq.(18) two blocks of independent equations can be obtained

$$\begin{bmatrix} k_{11} \\ k_{12} \end{bmatrix} = -\frac{1}{\pi a_1} \begin{bmatrix} E[\dot{X}_1 \dot{X}_1] & E[\dot{X}_1 \dot{X}_2] \\ E[\dot{X}_2 \dot{X}_1] & E[\dot{X}_2 \dot{X}_2] \end{bmatrix}^{-1} \begin{bmatrix} E[\dot{X}_1 \ddot{X}_1^+] \\ E[\dot{X}_1 \ddot{X}_2^+] \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} k_{12} \\ k_{22} \end{bmatrix} = -\frac{1}{\pi a_1} \begin{bmatrix} E[\dot{X}_1 \dot{X}_1] & E[\dot{X}_1 \dot{X}_2] \\ E[\dot{X}_2 \dot{X}_1] & E[\dot{X}_2 \dot{X}_2] \end{bmatrix}^{-1} \begin{bmatrix} E[\dot{X}_2 \ddot{X}_1^+] \\ E[\dot{X}_2 \ddot{X}_2^+] \end{bmatrix} \tag{28}$$

In the figs. [1-4] the values obtained for the input parameters and the comparison with the expected values derived by equation (24) are showed. The comparison are made for different time length of the observed system response. Obviously an increasing of the response observation produces an increasing of the effectiveness of the algorithm because of the possibility to evaluate the entries of eq.(27) and eq.(28) with more precision. As it can be seen an observation time greater than 500 sec needs for the algorithm convergence.

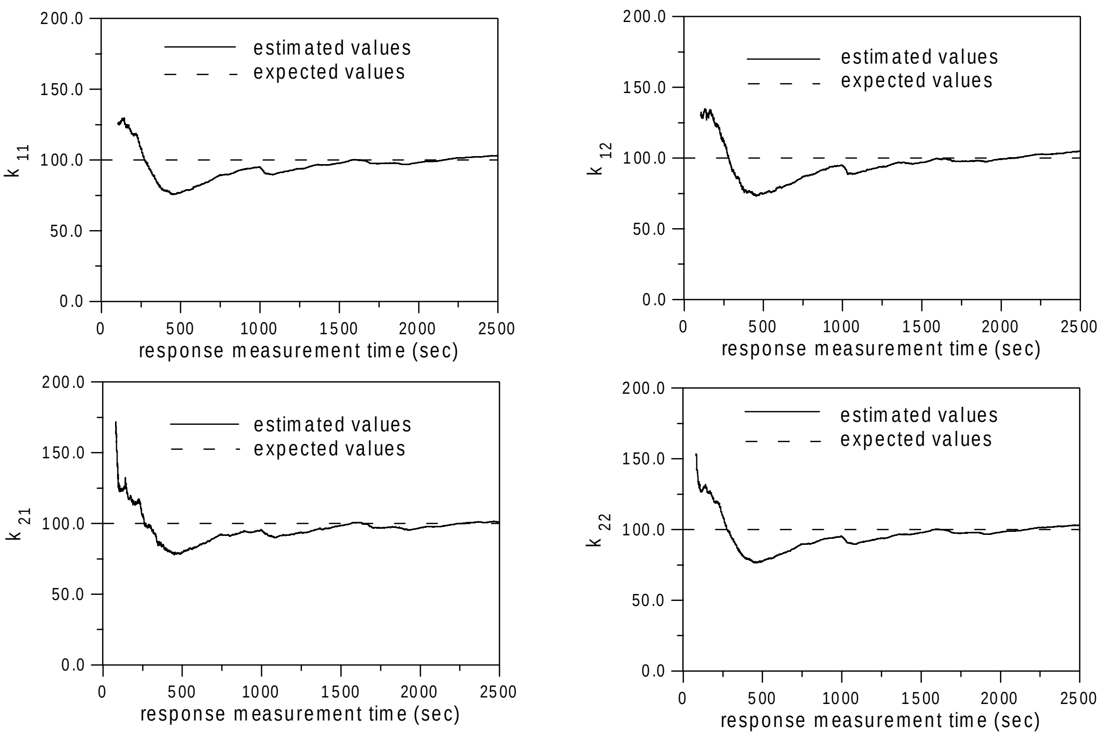

Figure 1: Estimation of the PSD matrix entries for different response measurement times

The possibility to estimate the parameters that define the input by means of limited observation time of the response is fundamental for an identification procedure since the input intensity varies much rapidly during a day producing a non stationary behaviour while for a correct application of the procedure the stationary conditions need. The same results can be obtained for different mechanical features of the system and different probabilistic features of the input.

CONCLUSIONS

The effectiveness of restricted potential models in identification of systems under stochastic loads has been discussed. For this purpose the moment statstic approch followed with potential models has been compared with the same approch with different kind of models. The comparison has evidenced that the algorithm obtained by using RPM is constituted by linear algebraic equations that can give results with high degree of reliability while by using different models in some cases, as the analysed one, cannot be obtained solving equation of the problem. The effectiveness of RPM has been even shown in the identification of the input probabilistic features. As a matter of fact it has been shown that the entries of Power Spectral Density matrix, that defines the features of the input, can be estimated following the same approach used in system identification parameters. Even in this case the results in input identification by RPM are obtained because of their special analitycal form: once more the effectiveness of RPM in identification is underlined.

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