12WCEE
2000

0030

EFFECT OF HYSTERETIC MODELS ON THE INELASTIC DESIGN SPECTRA

Young-Hun OH[1], Sang Whan HAN[2] And Li-Hyung LEE[3]

SUMMARY

The design response spectrum has been widely used in seismic design to estimate force and deformation demands of structures imposed by earthquake ground motion. Inelastic Design Response Spectra to specify design yielding strength in seismic codes are obtained by reducing the ordinates of Elastic Design Response Spectrum by strength reduction factor (R). However, Inelastic Response Spectra (IRS) depend not only on the characteristics of the expected ground motion at the site, but also on the nonlinear characteristics of the structural system. IRS for the selected ground motions should be computed taking into account the different hysteretic characteristics according to the expected behavior of the type of structural system and materials to be used. The objective of this study is to investigate the effect of hysteretic models on IRS. For this purpose, IRS are obtained to attain a given target ductility ratio using nonlinear dynamic analyses of SDOF system for earthquakes. Forty ground motions recorded at stiff soil site are used. The considered hysteretic models are elasto perfectly plastic, bilinear, strength degradation, stiffness degradation, and pinching models. Results indicate that IRS are strongly dependent on hysteretic model as well as structural period and target ductility. Statistical studies are also performed to investigate the effect of the parameters of each hysteretic model on IRS. Based on the results of this parametric study the functional form of IRS (=LERS/R) is established.

1. INTRODUCTION

The design response spectrum has been widely used in seismic design to determine the yield strength and deformation of the system necessary to limit the ductility demand imposed by the earthquake ground motion (EQGM). Current seismic design code has been developed based on the assumption that the structures designed according to the code will perform inelastically during severe EQGM. For this reason, the seismic design force calculated by the current design procedure is much lower than the force which makes structures responding elastically during the design level EQGM. Thus, structures designed according to this provision are likely subjected to significant inelastic deformations whose corresponding forces and deformations cannot be predicted with the use of linear elastic models. Smoothed inelastic design response spectra (SIDRS) for design yield level force (Vy) specified in codes are obtained by reducing smoothed linear elastic design response spectra (SLEDRS) using a strength reduction factor (R). However, many researchers have found the weaknesses of the R factor used in current seismic design codes. Specially it is very questionable that a single value of R factor is assigned for a given structural system irrespective of the height of the structure. Recent studies on the response of instrumented structures during recent earthquake, as well as on experimental investigation on the response of scaled down models of buildings, have concluded that there is a need for improved SIDRS (Bertero et al. 1991). The number of statistical studies of response spectra that have considered inelastic structural behavior is much smaller than those on linear elastic response spectra (LERS) and, in general, have only considered small number of earthquake ground motions and have not taken into account the effect of hysteretic models.

Inelastic response spectra (IRS) depend not only on the characteristics of the expected ground motion at the site, but also on the nonlinear characteristics of the structural system. Therefore, IRS for the selected ground motions

---

[1] *Visiting Scholar, Ph.D., Dept. of Civil Eng., University of Illinois at Urbana-Champaign, U.S.A*

[2] *Professor, Ph.D., Department of Architectural Eng., Hanyang University, Seoul, Korea*

[3] *Professor, Ph.D., Department of Architectural Eng., Hanyang University, Seoul, Korea*

should be computed taking into account the different hysteretic characteristics according to the expected behavior of the type of structural system and materials to be used.

The objective of this study is to investigate the effect of hysteretic models on IRS. For this purpose, inelastic strength demand are obtained to attain a given target ductility ratio using inelastic dynamic analyses of SDOF system for earthquakes. Forty ground motions are used, which were recorded at stiff soil site. The considered hysteretic models are elasto perfectly plastic, bilinear, strength degradation, stiffness degradation, and pinching models. Statistical studies are also performed to investigate the effect of the parameters of each hysteretic model on IRS. Based on the results of this parametric study the functional form of IRS (=LERS/R) is established.

2. REVIEW OF PREVIOUS STUDIES

Several studies have been conducted over the years with the purpose of improving the knowledge of design response spectra. In general, these studies have been improved in time as a result of a rapid increase in the number of recorded earthquake ground motions. Here, a brief summary of most relevant statistical studies on response spectra is presented.

The first attempt to study the characteristics of an ensemble of LERS of recorded ground motions was made by Housner (1959), who computed the average LERS of eight ground motions recorded during four earthquakes. Response spectra of inelastic systems were first studied by Veletsos (1969) who presented IRS to pulse-type excitations and two recorded ground motions. Newmark and Hall (1973) studied elastic and inelastic response spectra of a 5% damped SDOF system subjected to three recorded ground motions and pulse-type excitations. Based on statistical studies, they proposed the method to construct the inelastic response spectra from the elastic response spectra.

Riddell and Newmark (1979) performed the statistical studies for evaluating IRS using 10 different earthquake ground motions recorded at the rock and alluvium soil condition. They considered three different hysteretic models such as elasto perfectly plastic, bilinear and stiffness degradation models. According to their studies elasto prefectly plastic model gives conservative IRS. Riddell, et al. (1989) presented average IRS of four sets of earthquake records. Most of the ground motions included in this study were recorded on South America. Emphasis is given to reduction factors to construct SIDRS from LERS; However, no information is given on the dispersion of the recommended reduction factors. Nassar and Krawinkler (1991) evaluated average IRS of bilinear and stiffness degrading systems subjected to 15 ground motions recorded on firm soil sites in the western United States. They proposed functional form of R factor with respect to ductility, natural period and second slope of bilinear model. More recently, Miranda (1993) performed similar studies with that of Nassar and Krawinkler (1991). He used more earthquake records and considers the effect of different soil conditions on IRS.

3. HYSTERETIC MODELS AND EARTHQUAKE RECORDS USED IN THIS STUDY

Many previous investigations on IRS have been evaluated using either elasto perfectly plastic or bilinear models because of simplicity. In this study five different hysteretic models are considered which are (1) elasto perfectly plastic, (2) bilinear, (3) strength degradation, (4) stiffness degradation, and (5) pinching models. These models are shown in Figure 1. Among these models the elasto perfectly plastic (EPP) model is used as a basis model in this study. Thus, the effect of other hysteretic models on IRS is compared with those of EPP model. The characteristic parameters of each hysteretic model are shown in Table 1. The characteristic parameters of each hysteretic model are described by Kunnath et al (1990) in detail.

For the statistical study, 40 earthquake ground motions are used which are obtained from the Earthquake Strong Motion CD-ROM by National Geographical Data Center (1996) and U.S. Geological Survey digital data series, DDS-7, CD-ROM (1992). We selected the earthquake ground motions under the following conditions such as i) free field ground motion, ii) horizontal accelerograms, iii) station located stiff soil site (S1), and iv) wide range of earthquake ground motion records in terms of magnitude and epicentral distance. The software BAP (1992) is used for correcting the earthquake records. Also the software SMCAT (1989) by National Geographical Data Center is used for classifying the earthquake records according to soil type. The soil condition is classified into four types S_1, S_2, S_3, and S_4 according to Uniform Building Code 1988.

In this study the ground motion records at soil type 1 (S_1) are only considered. According to UBC 1988 soil type 1 (S_1) is classified as “A soil profile with either (a) a rock-like material characterized by a shear-wave velocity greater than 2,500 feet per second or by other suitable means of classification, or (b) Stiff or dense soil condition where the soil depth is less than 200 feet”. The inventory of selected earthquake records is shown in the study by Lee et. al..

4. METHOD OF ANALYSIS

The response of a damped SDOF system subjected to earthquake ground motions is given by

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + F(t) = -m\,\ddot{u}_g(t) \qquad (1)$$

where m, c, and F(t) = mass, damping coefficient, and restoring force of the system, respectively ; u(t) = relative displacement; ug(t) = ground displacement; and the overdot represents its derivative with respect to time.

Equation (1) can be rewitten by normalization as follows:

$$\ddot{u}(t) + 2\,\omega\,\xi\,\dot{u}(t) + \omega^2\,\frac{F(t)}{F_y} = -\frac{\omega^2}{\eta}\,\frac{\ddot{u}_g(t)}{\max|\ddot{u}_g|} \qquad (2)$$

where μ = displacement ductility ratio, defined as the maximum absolute value of the relative displacement divided by the yield displacement; F_y = system’s yield strength; and ω, ξ, and η = natural circular frequency, damping ratio, and nondimensional strength of the system, respectively. The latter three quantities are defined as

$$\omega = \sqrt{\frac{k}{m}} \qquad (3)$$

$$\xi = \frac{c}{2m\omega} \qquad (4)$$

$$\eta = \frac{F_y}{m \times \max|\ddot{u}_g|} \qquad (5)$$

where k = initial stiffness of the system.

A constant displacement ductility IRS is a plot of the yield strength of an SDOF system (with period T) required to limit the displacement to specified displacement ductility ratio, μ_t. This type of spectra is also referred to as strength demand spectra (Krawinkler and Nassar 1990). In this study, constant displacement ductility IRS were computed by iteration on the system`s nondimensional strength η until the ductility was, within a certain tolerance, the same as the specified targrt ductility ratio. The tolerance was chosen such that η was considered satisfactory if the computed ductility was within 2% of the target ductility ratio.

The following target ductility ratios were selected for this investigation: 1 (elastic behavior), 2, 3, 4, 5, 6, and 8. For each earthquake record and each target ductility ratio the IRS were computed for a set of 40 periods. In order to study the effect of hysteretic model on IRS, this study also considered SDOF systems that have a elasto perfectly plastic, bilinear, strength degradation , stiffness degradation, and pinching hysteretic behavior with a damping ratio of 5%. In this study, the damping is assumed to be of viscous type with a fixed damping coefficient. Therefore, further study is needed to examine other damping characteristics such as instantaneous stiffness proportional damping.

5. PRESENTATION OF RESULTS

For each hysteretic model and each period, normalized strength demands were averaged. The resulting mean strength demand spectra are shown in Figure 3 and compared with LEDRS of NEHRP provision. The spectra are plotted for displacement ductility ratios of 1-6 (from top to bottom). As shown in Figure 3, the shape of elastic spectra differ significantly from that of inelastic spectra. The larger the ductility demand, the larger this difference is. Furthermore, hysteretic model, that is strength degradation model, affects inelastic strength demands which are resulted in higher ordinates than elasto perfectly plastic model with no degradation. This can be seen more clearly in Figure 4, which compares the effects of key parameter of hysteretic characteristics on inelastic strength demand for target ductility ratio of 3. It is clearly shows that strength reduction factor that relate linear to inelastic spectra is dependent on hysteretic model.

Strength reduction factor, R_μ is defined as the ratio of the elastic strength demand $F_y(\mu = 1)$ to the inelastic yield strength demand $F_y(\mu_t)$ for a given target ductility ratio (μ_t), which is represented by the following equation.

$$R_\mu = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_t)} \qquad (6)$$

The relationship between $F_y(\mu = 1)$ and $F_y(\mu = \mu_t)$ is shown in Figure 2. In order to establish the functional form of strength reduction factor, $R(T, \mu)$ for each hysteretic model, statistical studies are carried out. Readers can be find a more detail description of the statistical studies in companion paper. Here only the most important data and results which have been used for the validation of the proposed procedure, are given. The results of above statistical studies have generally confirmed the main conclusions drawn by several other researchers. The R_μ factor is, in the medium- and long-period region, only slightly dependent on the period, T, and is roughly equal to the target ductility ratio μ. In the short-period region, however, the R_μ factor depends strongly on both T and μ. Furthermore, the considerable effect of hysteretic behavior can be observed in the whole period region.

Current seismic loading for building structures is based on the reduction of smoothed linear elastic design spectra (SLEDS) through empirical and period-independent reduction factors. As previously discussed, the difference between the shape of LERS and IRS increases with increase in ductility. Moreover, the ordinate of IRS for the same displacement ductility is different depending on hysteretic model used in this study. Thus, the error in using period-independent reduction factors to estimate IRS from LERS may also increases.

In figure 5, the approximate inelastic strength spectra calculated from strength reduction factor with each hysteretic model are compared with the corresponding mean inelastic strength spectra (actual spectra). It is confirmed that such approximation for inelastic strength demand is well estimated for actual demand. Consequently, inelastic strength demand can be easily determined from elastic strength demand, $F_y(\mu = 1)$ devided by strength reduction factor.

6. CONCLUSION

Results from a comprehensive statistical study of elastic and inelastic strength demands of SDOF systems when subjected to 40 ground motions recorded on stiff soil conditions has been presented. Based on these results, following conclusions are made.

1) Current seismic loading for building structures is based on the reduction of smoothed linear elastic design spectra (SLEDS) through empirical and period-independent reduction factors. The shape of inelastic strength response spectra differ significantly from the shape of elastic strength response spectra. This difference between the shape of LERS and IRS increases with increase in ductility.

2) Moreover, the ordinate of IRS for the same displacement ductility is different depending on hysteretic model used in this study. Thus, direct scaling by using a single strength reduction factor of elastic spectra to obtain inelastic strength demands is neither rational nor conservative.

3) The approximate inelastic strength spectra calculated from strength reduction factor with each hysteretic model are well estimated for actual demand. Consequently, inelastic strength demand can be easily determined from elastic strength demand, $F_y(\mu = 1)$ devided by strength reduction factor.

ACKNOWLEDGEMENT

This research was supported by Engineering Research Center for Advanced Structure Research Station (STRESS) of KOSEF at Hanyang University.

REFERENCES

ATC (1995), *A Critical Review of Current Approaches to Earthquake-Resistant Design*, Applied Technology Council Report ATC-34, Redwood City, California.

ATC (1995), *Structural Response Modification Factors*, Applied Technology Council Report ATC-19, Redwood City, California.

BAP (1992), *Basic Strong-Motion Accelerogram Processing Software Version 1.0*, Open File Report 92-296A, US Geological Survey.

Kunnath, S.K., Reinhorn, A.M., and Park, Y.J. (1990), “Analytical Modeling of Inelastic Seismic Response of R/C Structures”, Journal of Structural Engineering, Vol.116, No.4, ASCE.

Miranda E. (1993), “Site-dependent Strength Reduction Factors”, Journal of the Structural Engineering, ASCE, Vol.119, No.12, pp.3503-3519.

Nassar, A. A.and Krawinkler, H. (1991), *Seismic Demands for SDOF and MDOF Systems*, Report No. 95, The John A. Blume Earthquake Engineering Center, Stanford University, CA.

Newmark, N.M. and Hall, W.J. (1973), "Procedures and Criteria for Earthquake Resistant Design", Building Research Series No.46, National Bureau of Standards, U.S. Dept. of Commerce, Washington, pp.209-236.

NGDC (1989), *Earthquake Strong Motion CD-ROM Collection*, Product No. 1145-A27-001, National Geographical Data Center, Boulder, Colorado.

Riddell, R. and Newmark, N. M. (1979*), Statistical Analysis of the Response of Nonlinear Systems Subjected to Earthquakes*, Structural Research Series No. 468, Department of Civil Engineering, University of Illinois, Urbana.

Seekins, L.C., Brady, A.G., Carpenter, C. and Brown, N. (1992), *Digitized Strong- Motion Accelerograms of North and Central American Earthquake 1933-1986*, Digital Data Series DDS-7, US Geological Survey.

SMCAT (1989), *An Earthquake Strong Motion Data Catalog for Personal Computers*, National Geographical Data Center, Boulder, Colorado.

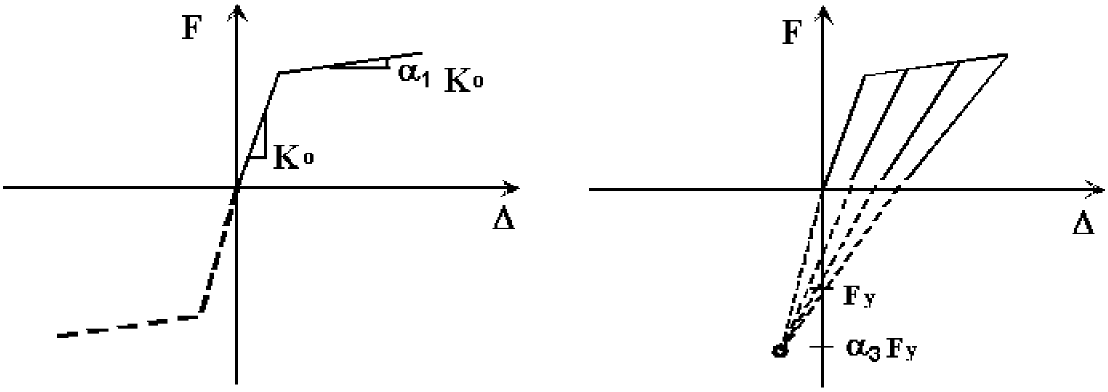
Bertero, V. V., Anderson, J. C., Krawinkler, H., and Miranda, E., (1991), *Design guidelines for ductility and drift limits*, Report No. EERC/UCB-91/15, Earthquake Engineering Research Center, Univ. of California, Berkely, Calif.

Housner, G. W. (1959). “Behavior of structures during earthquakes”, Journal of Engineering Mechanics, ASCE, 106(1), 69-86.

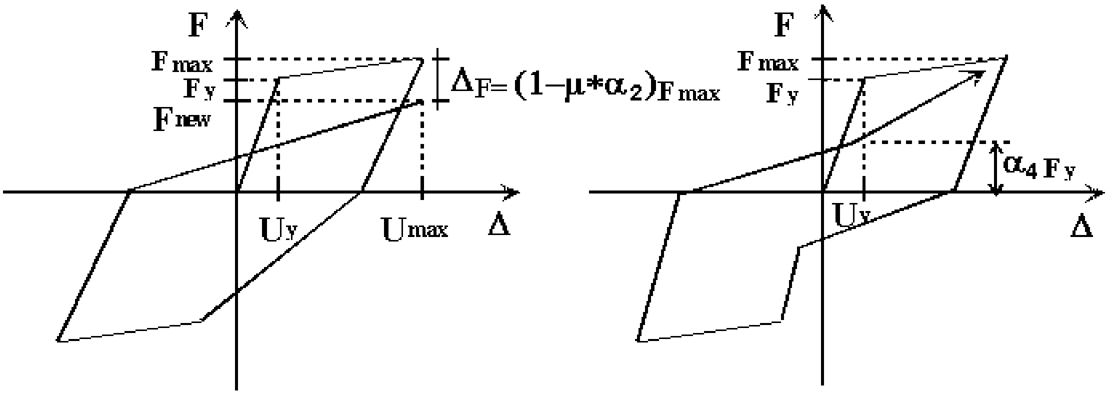
Li-Hyung Lee, Sang Whan Han, Young-Hun Oh, (1999), “Determination of Ductility Factor Considering Different Hysteretic Models”, Earthquake Engineering and Structural Dynamics, (in press).

Table 1. Key Parameters of Hysteretic Models

Hysteretic Model	Parameters	Effect
Elasto perfectly Plastic Model	K_0	Initial Stiffness
	U_y	Yield Displacement
Bilinear Model	K_0	Initial Stiffness
	U_y	Yield Displacement
	α_1	Second Slope (0, 2, 5, 7, 10, 15 %)
Strength Degradation Model	K_0	Initial Stiffness
	U_y	Yield Displacement
	α_2	Strength Degradation (0, 3, 6, 9, 12%)
Stiffness Degradation Model	K_0	Initial Stiffness
	U_y	Yield Displacement
	α_3	Stiffness Degradation(15, 4, 2, 1, 0.5, 0)
Pinching Model	K_0	Initial Stiffness
	U_y	Yield Displacement
	α_4	Pinching(100, 40, 30, 20, 10, 5%)



(a) Bilinear Model (c) Stiffness Degradation Model



(b) Strength Degradation Model (d) Pinching Model

Figure 1. Hysteretic Models Used in this Study

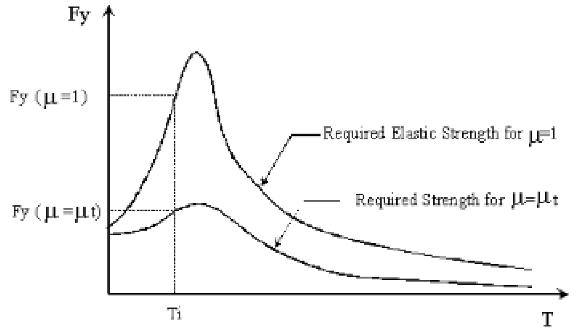
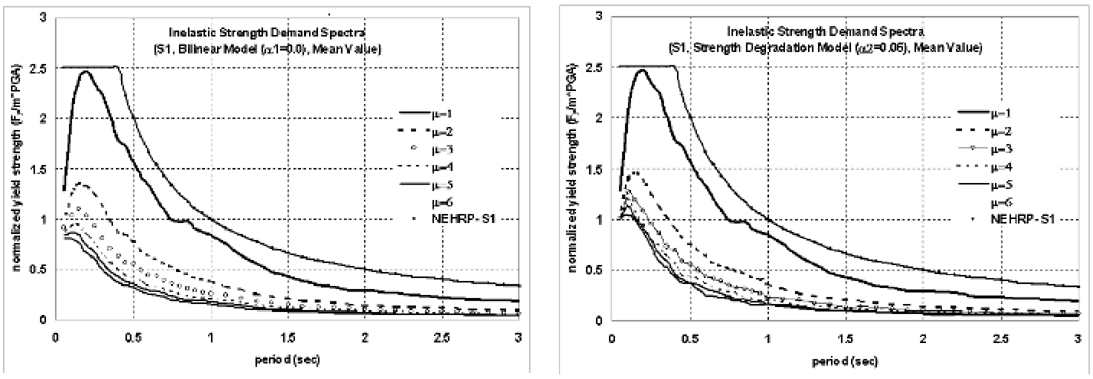
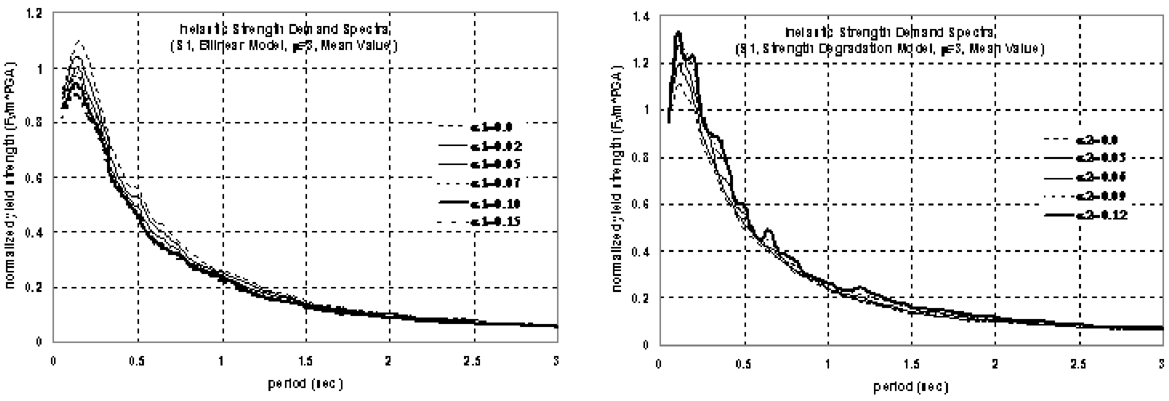


Figure 2. Yield Strength for a Given Target Ductility Ratio vs. Structural Period



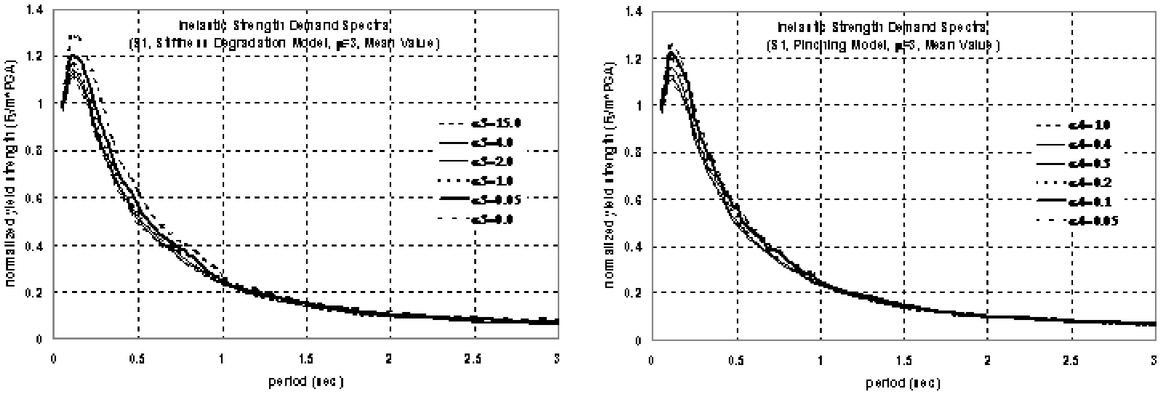
(a) Elasto Perfectly Plastic Model for $\alpha_1 = 0.0$ (b) Strength Degradation Model for $\alpha_2 = 0.06$

Figure 3. Inelastic Strength Demand Spectra with target ductility ratio



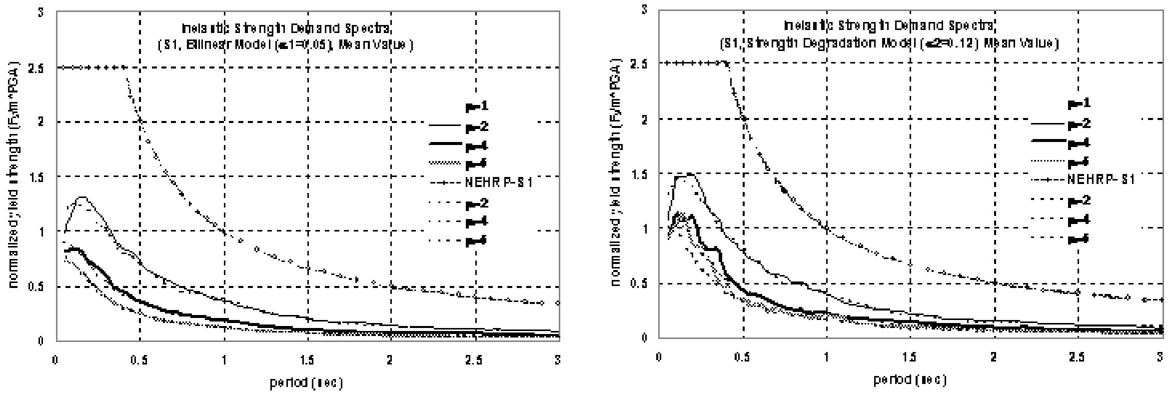
(a) Bilinear Model (b) Strength Degradation Model

Figure 4-1. Comparion of Inelastic Strength Demand Spectra for μ =3

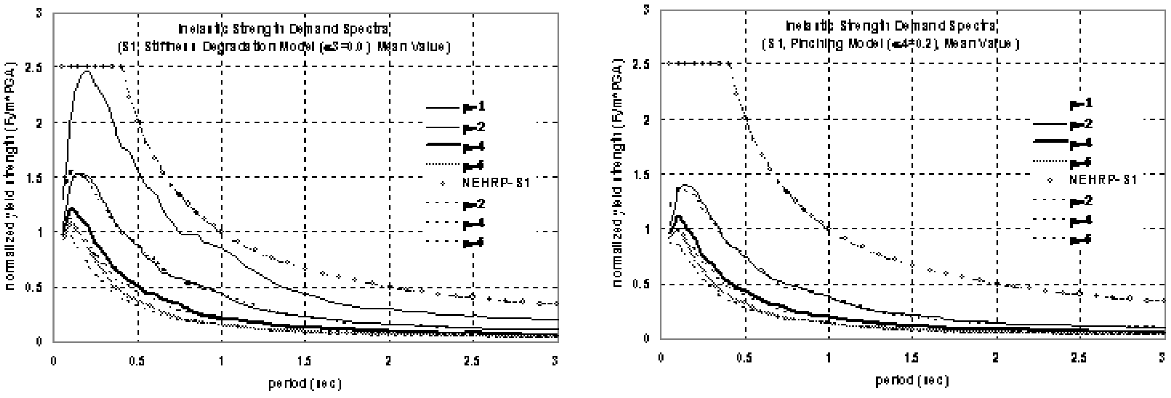


(c) Stiffness Degradation Model **(d) Pinching Model**

Figure 4-2. Comparion of Inelastic Strength Demand Spectra for μ =3



(a) Bilinear Model ($\alpha_1 = 0.05$) **(b) Strength Degradation Model ($\alpha_2 = 0.12$)**



(c) Stiffness Degradation Model ($\alpha_3 = 0.0$) **(d) Pinching Model ($\alpha_4 = 0.2$)**

Figure 6. Prediction of Inelastic Strength Demand Spectra for each Hysteretic Model