0006

ZOOMING PARADOX AND DAMPING MODELS IN DYNAMIC ANALYSIS

Alexander G TYAPIN[1]

SUMMARY

Zooming procedure» is the sequence of two dynamic analyses often used for large systems. The first analysis is performed using the model of the System and external excitation, the second one is made using the model of the Subsystem and the excitation derived from the response of the first model at the borders of the Subsystem inside the System. Usually the model of the Subsystem is more detailed, that is why the procedure is called “zooming”.

Usually the response spectra on the second step are unexpectedly high in the high frequency interval. For a long time this was considered to be the result of complicated model of Subsystem (thus more correct). Special investigation however has proved that even for the same model of Subsystem on the first and the second steps there appeared considerable differences between the results at the same points of Subsystem. This was called «zooming paradox».

The nature of «zooming paradox» is investigated using a very simple example. The recommendations are given to avoid errors in dynamic analysis through the «zooming procedure

INTRODUCTION

“Zooming paradox” is one of the problems hardly recognized by engineers, carrying out dynamic analysis, that is why some definitions are necessary. Let us define “zooming procedure” as the sequence of two analyses. The first one is performed using the model of the System and external excitation, the second one is made using the model of the Subsystem and the excitation derived from the response of the first model at the borders of the Subsystem inside the System.

Typical case of such procedure is the two-step dynamic analysis of the soil-structure system under seismic loading. The model of the System includes the soil part, rigid basement plate and the model of the structure (comparatively simple, e.g., a beam one). Dynamic response is obtained at the basement. The Subsystem is just the structure, excited by the movement of the basement. This movement is obtained from the first analysis and used in the second one as the kinematic excitation.

Usually the model of the Subsystem is more detailed, that is why the procedure is called “zooming”. Otherwise it makes no sense to carry out the second step. Sometimes special elements are used for the second step (e.g., for modeling axisymmetrical containment shells. The results of the second step, of course, are considered to be more precise and reliable.

“Zooming procedure” is based on the fundamental theorems of mechanics and on the common sense, that is why no doubts usually arise. The situation, however, is not so simple. Very often the results obtained during the first and the second steps in the same points of the structure are significantly different. For a long time the author explained the differences by the differences in modeling the structure in the first and in the second steps. But one day the author used the same models for Substructure in the second step and the part of the Structure in the first

[1] *Atomenergeoproject, Moscow, Russia.*

step and still got the difference. That was called “zooming paradox” and stimulated the author to make special investigations of the case.

OUTCOMES

It turned out that the same effect may be shown using a very simple model.

Let us consider vertical vibration of the mass M, placed at the top of the vertical massless rod with E - elastic modulus, L - length, F - cross-section and e - damping coefficient. The rod is placed on the rigid foundation, moving vertically. Harmonic vibrations of the foundation are set by amplitude xo and frequency ω. In the frequency domain the relative displacement y1 of the mass M is determined by the well-known SDOF equation:

$$-\omega 2^*y1+ i^*\omega^*2e^*\omega o^*y1 + \omega o2^*y1 = \omega 2^*xo \qquad (1)$$

where

$$\omega o2= E^*F / (L^*M) \qquad (2)$$

From (1)

$$y1=xo^*f1(L,\omega),\ \ f1(L,\omega)=1/(\omega o2/\omega 2- 1 + 2^*i^*e^*\omega o/\omega) \qquad (3)$$

Absolute displacement of the mass M is then

$$x1 = xo + y1 = xo^*[1 + f1(L,\omega)] \qquad (4)$$

Inside the massless rod the displacements are linearly varying, thus the absolute displacement of the internal point set at the distance r*L (r<1) from the mass M is

$$x2 = xo+ (1-r)^*y1 = xo^*[1 + (1-r)^*f1(L,\omega)] \qquad (5)$$

Let us implement “zooming” procedure, taking the upper part of the rod of the length r*L as Subsystem. Applying (4) to the Subsystem we get

$$x1= x2^*[1 + f1(r^*L,\omega)] \qquad (6)$$

or, considering (5),

$$x1 = xo^*[1 + f1(r^*L,\omega)]^*[1 + (1-r)^*f1(L,\omega)] \qquad (7)$$

Thus x1 is determined by (4) and (7). It seems that (4) and (7) must give similar results independent of r. It turns out, however, that this is not true.

Fig.1 shows the absolute values of the ratio x1/xo from (7) with three different values of r: r=0 (in this case (7) is similar to (4)), r=0.25 and r=0.5. The parameters of the System are: L=1 m, M=1 kg, F=1 m2, e=0.1. Elastic modulus E is set so as to get the SDOF eigenfrequency 1 Hz. Thus E=4*π2 N/ m2.

The results show that the first resonance for the System is well modeled in all cases. There appears however the second resonance at the eigenfrequency of the Subsystem. The less is r, the greater is this partial eigenfrequency and the greater is the discrepancy in the absolute value of the transfer function at this eigenfrequency. For r=0.25 partial eigenfrequency is 2 Hz and the discrepancy is 100%.

This example shows main features of “zooming paradox”. The results of the two-step analysis differ from those of one-step analysis, maximum discrepancies arising on the eigenfrequencies of the Subsystem. The greater is the difference in the eigenfrequencies between System and Subsystem, the greater is discrepancy in results.

The consequences of “zooming paradox” are obvious: the results of the “zooming procedure” depend on the choice of Subsystem, thus being incorrect. In our example with SDOF the error is conservative, but in the multi-degrees-of-freedom system they may be non-conservative.

The frequency range of discrepancies is different from the resonant range of the System, but it may be important for the equipment, placed inside the structure. “Zooming paradox” was discovered in the process of analyses of large systems , appearing as high-frequency “plato” in floor response spectra. For a long time these “plato” were considered to be a result of local effects in sophisticated models of Subsystems (usually these models are more sophisticated than Subsystems inside the System), and only a year ago special investigation was performed to make the question clear. The main results are as follows.

“Zooming paradox” does not deny the principal advantages of “zooming procedure”. The main reason of discrepancies is that the Subsystem model is in fact different from the Subsystem part of the System model. The

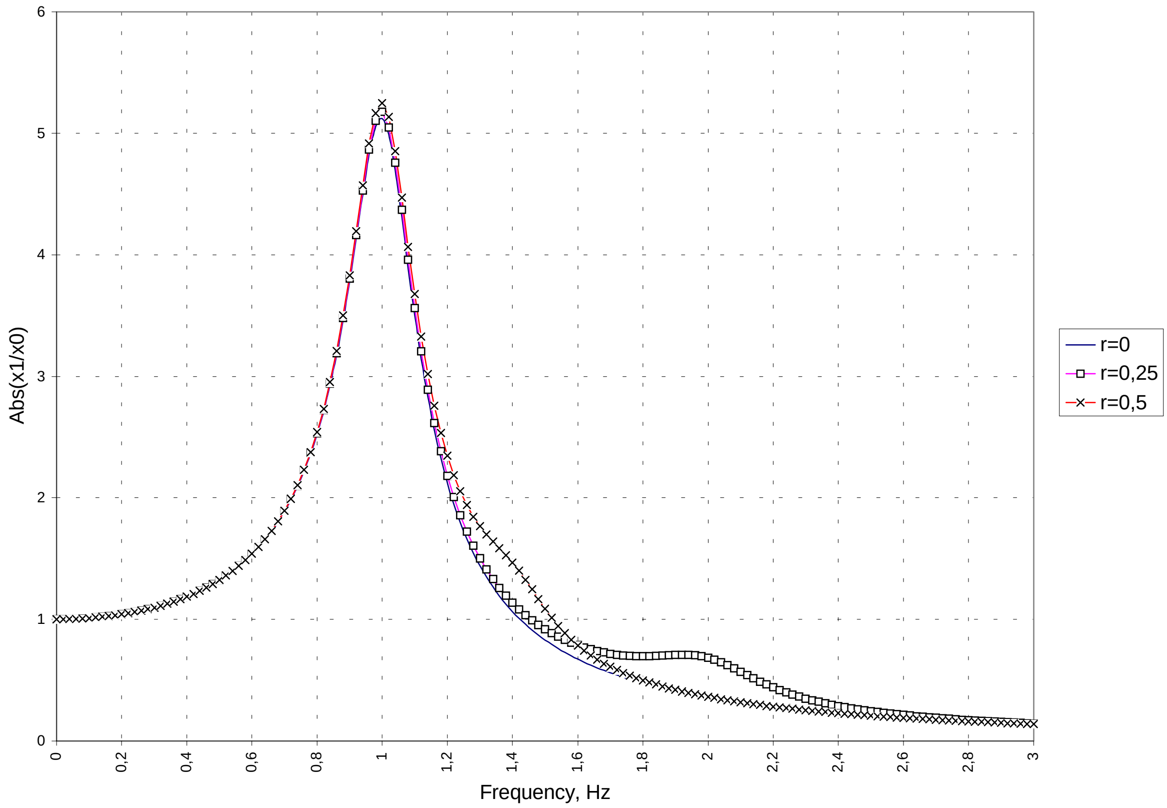

Fig.1

difference arise from basic differences in the technique of the damping modeling, meaning viscous modal damping and material damping (let us take it frequency-independent). In both cases some damping coefficients are used, but their meaning is considerably different. For the material damping we use the ratio of the imaginary and real parts of the complex elastic modules. It is true material local characteristic independent from inertial properties of the structure. Mathematical description of material damping in the frequency domain is well known. For the above mentioned example we get instead of (1)

-ω2*y1 + i*2e*ωo2*y1 + ωo2*y1 = ω2*xo (8)

Instead of (3)

y1=xo*f2(L,ω), f2(L,ω)=1/(ωo2/ω2 - 1 + 2*i*e*ωo2/ω2) (9)

Placing f2 instead of f1 into (4) and (7) we get similar results for x1 . No “zooming paradox” occurs.

Let us return to the viscous damping. Local characteristic of damping is the ratio of damping force and velocity

b=2*e*ωo*M (10)

Taking (2) into consideration

$$e=0.5*b*\sqrt{(L/E\ F\ M)} \quad (11)$$

Inertial characteristic M is used here together with local characteristics of the rod. Thus e is not local for viscous damping, it depends on System as a whole. Keeping e for Subsystem in (7) means in fact changing of material. If physical damping is true viscous, parameter b must be kept for Subsystem instead of e.

It is well known that common materials used in engineering practice (e.g., concrete, steel, soil, etc.) show frequency-independent damping. That is why in FEM codes dimensionless damping coefficients are used as material constants. However modal damping in dynamic analysis is physically viscous (see (1)). Viscous damping equivalent to the material one is set from the equivalent resonant peaks: f2 is similar to f1 at the resonant frequency ωo It may be shown, that transfer functions from xo to x1, equal to [1+f2] and [1+f1], are almost similar (with several percent difference). This well-known fact is often misunderstood by engineers as the absence of considerable difference between material and viscous damping.

As $[1+f1(L,\omega)]$ is almost similar to $[1+f2(L,\omega)]$, the value of $[1+f1(r*L,\omega)]$ is almost similar to $[1+f2(r*L,\omega)]$. Thus the main reason of 100% discrepancies in our example are in the transfer function (5) from xo to x2. At the eigenfrequency of the Subsystem this transfer function goes down in absolute value (as a “compensation” to the resonance in Subsystem to get smooth transfer function from xo to x1). At this very frequency comparatively small absolute difference between f1 and f2 lead to the significant relative difference in transfer functions of the type (5). It means that the discrepancies of “zooming paradox” appear not on the second step of analysis, but mostly on the first one.

The same mechanism works for large systems. The eigenfrequencies of the Subsystem are not the eigenfrequencies of the System, that is why partial resonance in the Subsystem must be “compensated” by the fall in the transfer function from the external excitation to the exciting borders of Subsystem in order to get smooth resulting transfer function. This effect is often used for “dynamic” protection from vibrations. But at the same time these “falls” in transfer functions let the difference between material and viscous damping lead to the significant discrepancies of “zooming paradox

CONCLUSIONS

The main recommendation to avoid problems of “zooming procedure” is to perform at least the first step of the procedure using correct techniques, e.g. analysis in the frequency domain together with Fourier trasform.

The next question arising is the application of “zooming procedure” to the systems with true viscous-type damping (e.g., radiation damping in the soil halfspace acts very much like viscous damping, thus this question is important for SSI analysis) out of the Subsystem. It may seem that in such case the discrepancies must be less, because viscous damping may be modeled in modal scheme properly. The author investigated model with two consequent vertical rods: the lower one with viscous damping, the upper - with material one. Two masses were placed on the top of each rod/ In the “zooming” procedure the second rod with the second mass was considered to be Subsystem. It turned out that the improvement of the situation with discrepancies was very little.

Thus the conclusions made above are still valid for systems combining different types of damping.

One more conclusion was made. If modal seismic analysis is applied to the system with material damping, the results (e.g., response spectra) in the internal points of the system at some frequencies are incorrect. These frequencies are those of the “fall” of transfer functions and at the same time they are partial eigenfrequencies of the upper laying Subsystem. Even if no “zooming” is used in analysis, these discrepancies should be taken into consideration.

SSI analysis of NPP in Moscow Atomenergoproject is performed now using frequency-domain procedures and Fourier transform.

ACKNOWLEDGEMENTS

The author should like to express gratitude to Mr. N.Krutzik (SIEMENS KWU, Germany) and K.Laimbach (Ruhr University, Bochum, Germany) for very useful discussions.