Complexity Analysis of REKF-based Joint Symbol Detection and Channel Estimation in Fast Fading STTC MIMO Systems

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1 STTC MIMO System Model for REKF-based Joint Estimation

In this section, we first present a brief description of a baseband space-time trellis coded (STTC) wireless system and then demonstrate the performance comparison of the proposed random parameter EM-based Kalman Filter (REKF) technique with respect to some of the existing schemes [1][2] in literature.

A conventional STTC system comprises of n transmit and m receive antennas as depicted in Fig.1. The raw data to be transmitted is generated by the information source and subsequently fed into the space-time encoder which generates the code vector at the k^{th} time instant $\mathbf{c}(k) = [c_1(k), c_2(k), \ldots, c_n(k)]^T \in \mathbb{C}^{n\times 1}$. Each $c_i(k)$ denotes the symbol to be transmitted from the i^{th} transmit antenna. The space-time codeword $\mathcal{C} = [\mathbf{c}(1), \mathbf{c}(2), \ldots, \mathbf{c}(N)] \in \mathbb{C}^{n\times N}$ comprises of code vectors $\mathbf{c}(k)$ concatenated for a frame length N. An example of a 4-Phase Shift Keying (PSK) four-state STTC for an $n = 2$ transmit antenna wireless system has been illustrated in Fig.2. The MIMO channel $\mathbf{H}(k) \in \mathbb{C}^{m\times n}$ matrix at the k^{th}

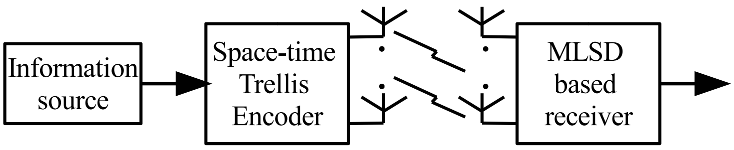

Figure 1: Block diagram representation of a space-time trellis coded (STTC) MIMO system

time instant is given by,

$$\mathbf{H}(k) = \begin{bmatrix} h_{1,1}(k) & h_{1,2}(k) & \ldots & h_{1,n}(k) \\ \vdots & \vdots & \ddots & \vdots \\ h_{m,1}(k) & h_{m,2}(k) & \ldots & h_{m,n}(k) \end{bmatrix},$$

where $h_{j,i}(k)$ denotes the channel coefficient between the j^{th} receive and the i^{th} transmit antenna. The received signal vector $\mathbf{r}(k) \in \mathbb{C}^{m\times 1}$ can be expressed as,

$$\mathbf{r}(k) = \mathbf{H}(k)\mathbf{c}(k) + \boldsymbol{\eta}(k),$$

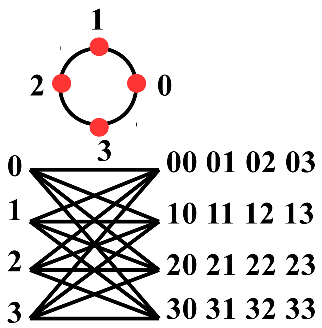

Figure 2: 4-PSK four-state STTC for transmission of 2bits/s/Hz using $n = 2$ transmit antennas.

where $\boldsymbol{\eta}(k) \in \mathbb{C}^{m\times 1}$ denotes the zero mean additive white Gaussian noise vector. Corresponding to the STTC trellis depicted in Fig.2, the decoder uses the maximum likelihood (ML) Viterbi decoding algorithm to estimate the transmitted information sequence $\mathbf{e}(k)$ based on the minimum decision metric given by,

$$\sum_{k=1}^{N}\sum_{j=1}^{m}\left|r_j(k) - \sum_{i=1}^{n} h_{j,i}(k)e_i(k)\right|^2 .$$

We now plot and compare the bit error rate (BER) and frame error rate (FER) performances of the proposed REKF and the existing schemes in [1][2] for joint channel estimation and symbol detection in space-time trellis coded (STTC) multiple-input multiple-output (MIMO) systems. A 2×2, 4 state and 4-PSK conventional STTC MIMO system as depicted in Fig. 2 is considered for the simulation study.

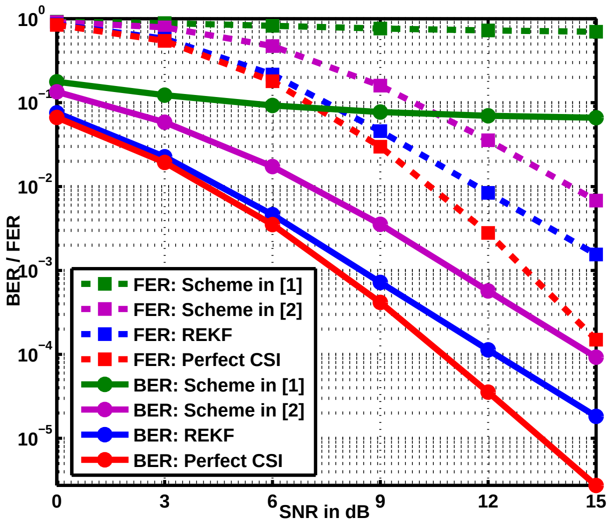

Figure 3: FER/BER performance comparison of various estimation schemes for $\alpha = 0.99$

Fig. 3 illustrates the FER and BER performance comparison for the proposed REKF-based joint estimation scheme and the existing techniques in [1] and [2]. It can be inferred that the proposed scheme has an improved performance in comparison to the technique in [1] due to the optimal Kalman filter-based channel estimates, unlike the conventional MMSE-based channel estimates in [1]. Further, the proposed scheme technique outperforms the scheme in [2] due to the modified metric-based maximum likelihood trellis decoder and suffers only a marginal performance degradation in comparison to the receiver with perfect CSI.

2 Complexity Analysis

We present a detailed analysis of the complexity in terms of the various mathematical operations involved in joint channel estimation and symbol detection in STTC MIMO systems for the proposed and the existing scheme in [2]. We derive the computational complexity of both the schemes per EM iteration.

2.1 REKF-based Joint Estimation Scheme

E-step: The E-step of the proposed REKF technique employs a data-aided KF to evaluate the optimal MMSE-based MIMO channel estimates $\mathbf{H}^{(j)}_{k|k}$. The various operations involved in the different steps of the KF are summarized in Table 1. It also involves one $m \times m$

Table 1: Operations in the computation of $\mathbf{H}^{(j)}_{k|k}$ using KF

Equation	Multiplications	Additions	Conjugate
$\mathbf{E}^{(j)}(k) = \mathbf{I}_{m\times m} \otimes \left(\mathbf{e}^{(j)}(k)\right)^T$	m^2n	-	-
$\mathbf{h}^{(j)}_{k\|k-1} = \mathbf{F}\mathbf{h}^{(j)}_{k-1\|k-1}$	mn	-	-
$\mathbf{M}^{(j)}_{k\|k-1} = \mathbf{F}\mathbf{M}^{(j)}_{k-1\|k-1}\mathbf{F}^H + \mathbf{G}\mathbf{R}_w\mathbf{G}^H$	$2m^2n^2$	m^2n^2	-
$\left\{\mathbf{E}^{(j)}(k)\mathbf{M}^{(j)}_{k\|k-1}\left(\mathbf{E}^{(j)}(k)\right)^H + \mathbf{R}_\eta\right\}^{-1}$	$m^3n^2 + m^3n$ $+m^2$	$m^3n^2 + m^3n$ $-m^2n$	m^2n
$\mathbf{K}^{(j)}(k)$	$m^3n + m^3n^2$	$m^3n^2 + m^3n$ $-2m^2n$	m^2n
$\mathbf{h}^{(j)}_{k\|k} = \mathbf{h}^{(j)}_{k\|k-1} + \mathbf{K}^{(j)}(k)\left(\mathbf{r}(k) - \mathbf{E}^{(j)}(k)\mathbf{h}^{(j)}_{k\|k-1}\right)$	$2m^2n$	$2m^2n$	-
$\mathbf{M}^{(j)}_{k\|k} = \left(\mathbf{I}_{mn\times mn} - \mathbf{K}^{(j)}(k)\ \mathbf{E}^{(j)}(k)\right)\mathbf{M}^{(j)}_{k\|k-1}$	$m^3n^2 + m^3n^3$	$m^3n^2 + m^3n^3$ $-m^2n^2$	-

Matrix Inversion of $\mathcal{O}(m^3)$. The total number of operations involved in the E-step of the REKF scheme for a frame length N is illustrated in Table 2.

M-step: The M-step of the REKF scheme computes the ML codeword which minimizes the modified decision metric using the trellis-based Viterbi decoder and is given as,

$$\mathcal{E}^{(j+1)} = \arg\min \sum_{k=1}^{N} \xi\left(\mathbf{r}(k), \mathbf{e}^{(j+1)}(k); \mathbf{H}^{(j)}_{k|k}, \mathbf{R}^{(j)}_{(k|k)}\right),$$

Table 2: Operations in the E-step of the REKF scheme

Multiplications	$N(m^3n^3 + 3m^3n^2 + 2m^3n + 3m^2n + 2m^2n^2 + mn) + m^2$
Additions	$N(m^3n^3 + 3m^3n^2 + 2m^3n - m^2n)$
Complex Conjugate	$2Nm^2n$
Matrix Inversion $\mathcal{O}(m^3)$	N

where the modified metric $\xi\left(\mathbf{r}(k), \mathbf{e}^{(j+1)}(k); \mathbf{H}^{(j)}_{k|k}, \mathbf{R}^{(j)}_{(k|k)}\right)$ is given by,

$$\xi\left(\mathbf{r}(k), \mathbf{e}(k); \mathbf{H}^{(j)}_{k|k}, \mathbf{R}^{(j)}_{k|k}\right) = \left\| \begin{bmatrix} \mathbf{r}(k) \\ \mathbf{0}_{n\times 1} \end{bmatrix} - \begin{bmatrix} \mathbf{H}^{(j)}_{k|k} \\ \left(\mathbf{R}^{(j)}_{k|k}\right)^{1/2} \end{bmatrix} \mathbf{e}(k) \right\|^2.$$

We consider a STTC MIMO system with S states and B branches emerging from each state. As illustrated in the manuscript, each symbol $c_i(k)$, the i^{th} component of the codeword vector $\mathbf{c}(k)$, is drawn from the constellation Γ. The major steps involved in the evaluation of the codeword $\mathcal{E}^{(j+1)}$ are,

(a) Path metric computation

This step involves computation of the path metric $\xi\left(\mathbf{r}(k), \mathbf{e}(k); \mathbf{H}^{(j)}_{k|k}, \mathbf{R}^{(j)}_{k|k}\right)$ corresponding to each branch and each state at the k^{th} time instant. The various operations involved are,

Equation	Multiplications	Additions	Conjugate
$\mathbf{R}^{(j)}_{k\|k} = \sum_{i=1}^{m} \mathbf{M}^{(j)}_{k\|k}\left[(i-1)n+1 : in\right]$	-	n^2m	-
$\left\| \begin{bmatrix} \mathbf{r}(k) \\ 0_{n\times 1} \end{bmatrix} - \begin{bmatrix} \mathbf{H}^{(j)}_{k\|k} \\ \left(\mathbf{R}^{(j)}_{k\|k}\right)^{1/2} \end{bmatrix} \mathbf{e}(k) \right\|^2$	$n^2 + mn + m + n$	$n^2 + mn + m + n - 1$	$m + n$

It also involves one $n \times n$ matrix square root operation. A total SB number of path metrics are computed corresponding to every probable state transition. Finally, the total number of path metric operations for a frame length N are shown in Table 3.

Table 3: Operations in the path metric computation

Multiplications	$SBN(n^2 + mn + m + n)$
Additions	$SBN(n^2m + n^2 + mn + m + n - 1)$
Complex Conjugate	$SBN(m + n)$
Matrix Square Root	N

(b) Branch Metric Computation

In order to evaluate the surviving path at the k^{th} time instant for each state in the

Table 4: Operations in the branch metric computation

Additions	SBN

trellis, the branch metrics are computed corresponding to each state. The operations involved for an entire coded frame are listed in Table 4.

(c) Final Decoding
The final step in the Viterbi-based space-time trellis decoder involves retracing the path corresponding the codeword which minimizes $\sum_{k=1}^{N} \xi\left(\mathbf{r}(k), \mathbf{e}^{(j+1)}(k); \mathbf{H}_{k|k}^{(j)}, \mathbf{R}_{(k|k)}^{(j)}\right)$ and thus involves no mathematical operations.

In the Table 5 below, we summarize the number of operations of the various decoding steps above for obtaining the ML symbol estimate in the M-step of the proposed REKF-based joint estimation scheme.

Table 5: Operations in the M-step of the REKF scheme

Multiplications	$SBN(n^2 + mn + m + n)$
Additions	$SBN(n^2m + n^2 + mn + m + n - 1) + SBN$
Complex Conjugate	$SBN(m + n)$
Matrix Square Root	N

A brief summary of the mathematical operations employed in the E and M-steps of the proposed REKF scheme is given in Table 6.

Table 6: Different operations in the proposed REKF-based joint estimation technique

	E-step	M-Step
Multiplications	$N(m^3n^3 + 3m^3n^2 + 2m^3n$ $+3m^2n + 2m^2n^2 + mn) + m^2$	$SBN(n^2 + mn + m + n)$
Additions	$N(m^3n^3 + 3m^3n^2 + 2m^3n$ $-m^2n)$	$SBN(n^2m + n^2$ $+mn + m + n)$
Complex Conjugate	$2Nm^2n$	$SBN(m + n)$
Matrix Inversion $\mathcal{O}(m^3)$	N	-
Matrix Square Root	-	N

2.2 Existing Scheme [2] for Joint Symbol Detection and Channel Estimation

E-step: The E-step of the existing technique [2] evaluates the *a posteriori*-based soft symbol mean and covariance estimates of the effective code matrices $\mathbf{C}(k)$. The mean and covariance estimates are given by,

$$\begin{aligned}\mathrm{E}\left\{\mathbf{C}(k)|\mathbf{r}(k),\mathbf{H}(k)\right\} &= \mathbf{I}_{m\times m}\otimes \mathrm{E}\left\{\mathbf{c}^T(k)|\mathbf{r}(k),\mathbf{H}(k)\right\}\\ \mathrm{Cov}\left\{\mathbf{C}^H(k)|\mathbf{r}(k),\mathbf{H}(k)\right\} &= \mathbf{I}_{m\times m}\otimes \mathrm{Cov}\left\{\left(\mathbf{c}^T(k)\right)^H|\mathbf{r}(k),\mathbf{H}(k)\right\},\end{aligned}$$

where $\mathrm{Cov}\left\{\mathbf{C}^H(k)\right\} = \mathrm{E}\left\{\mathbf{C}^H(k)\mathbf{C}(k)\right\} - \mathrm{E}\left\{\mathbf{C}^H(k)\right\}\mathrm{E}\left\{\mathbf{C}(k)\right\}$. Further, the *a posteriori* mean $\mathrm{E}\left\{\mathbf{c}^T(k)|\mathbf{r}(k),\mathbf{H}(k)\right\}$ is evaluated as,

$$\mathrm{E}\left\{\mathbf{c}^T(k)|\mathbf{r}(k),\mathbf{H}(k)\right\} = \frac{\sum_{j=1}^{|\Gamma|^n}\mathbf{c}_j^T e^{\frac{\left\|\mathbf{r}(k)-\mathbf{H}(k)\mathbf{c}_j\right\|^2}{\sigma_\eta^2}}}{\sum_{j=1}^{|\Gamma|^n} e^{\frac{\left\|\mathbf{r}(k)-\mathbf{H}(k)\mathbf{c}_j\right\|^2}{\sigma_\eta^2}}}, \tag{1}$$

where $\mathbf{c}_j \in \left\{\mathbf{c}_1,\mathbf{c}_2,\ldots,\mathbf{c}_{|\Gamma|^n}\right\}$ denotes a transmit symbol vector drawn from the vector constellation Γ^n and $|\cdot|$ denotes the cardinality of the set. The second moment of $\left(\mathbf{c}^T(k)\right)^H$ is given by,

$$\mathrm{E}\left\{\left(\mathbf{c}^T(k)\right)^H\mathbf{c}^T(k)\mid \mathbf{r}(k),\mathbf{H}(k)\right\} = \frac{\sum_{j=1}^{|\Gamma|^n}\left(\mathbf{c}_j^T\right)^H\mathbf{c}_j^T e^{\frac{\left\|\mathbf{r}(k)-\mathbf{H}(k)\mathbf{c}_j\right\|^2}{\sigma_\eta^2}}}{\sum_{j=1}^{|\Gamma|^n} e^{\frac{\left\|\mathbf{r}(k)-\mathbf{H}(k)\mathbf{c}_j\right\|^2}{\sigma_\eta^2}}}, \tag{2}$$

which is further utilized to compute the *a posteriori* covariance $\mathrm{Cov}\left\{\left(\mathbf{c}^T(k)\right)^H|\mathbf{r}(k),\mathbf{H}(k)\right\}$,

$$\begin{aligned}\mathrm{Cov}\left\{\left(\mathbf{c}^T(k)\right)^H|\mathbf{r}(k),\mathbf{H}(k)\right\} &= \mathrm{E}\left\{\left(\mathbf{c}^T(k)\right)^H\mathbf{c}^T(k)\mid\mathbf{r}(k),\mathbf{H}(k)\right\} -\\ &\quad \mathrm{E}\left\{\left(\mathbf{c}^T(k)\right)^H|\mathbf{r}(k),\mathbf{H}(k)\right\}\mathrm{E}\left\{\mathbf{c}^T(k)|\mathbf{r}(k),\mathbf{H}(k)\right\}. \quad (3)\end{aligned}$$

The operations involved in the E-step of the existing technique [2] for the STTC frame of length N are summarized in Table 7.

Table 7: Operations in the E-step of the Existing Scheme [2]

Equations (1),(2) and (3)	Multiplications	$N\{\|\Gamma\|^n(2n^2+mn$ $+m+2n+2)+n^2\}$
	Additions	$N\{\|\Gamma\|^n(n^2+mn+m+2n)$ $+n^2-1\}$
	Exponential	$N\|\Gamma\|^n$
	Complex Conjugate	$mN\|\Gamma\|^n$
$\mathrm{E}\left\{\mathbf{C}(k)\|\mathbf{r}(k),\mathbf{H}(k)\right\}$	Multiplications	m^2n
$\mathrm{Cov}\left\{\mathbf{C}^H(k)\|\mathbf{r}(k),\mathbf{H}(k)\right\}$	Multiplications	m^2n^2

M-step: The M-step employs the soft mean and covariance estimates of the effective code

Table 8: Operations in the M-step of the existing scheme [2] employing the KF

Equation	Multiplications	Additions	Conjugate
$\mathbf{h}^{(j)}_{k\|k-1} = \mathbf{F}\mathbf{h}^{(j)}_{k-1\|k-1}$	mn	-	-
$\mathbf{M}^{(j)}_{k\|k-1} = \mathbf{F}\mathbf{M}^{(j)}_{k-1\|k-1}\mathbf{F}^H + \mathbf{G}\mathbf{R}_w\mathbf{G}^H$	$2m^2n^2$	m^2n^2	-
$\{\overline{\mathbf{C}}(k)\mathbf{M}^{(j)}_{k\|k-1}\left(\overline{\mathbf{C}}(k)\right)^H + \mathbf{R}_{\overline{\eta}}\}^{-1}$	$m^2n^2(m+mn) + mn(m+mn)^2$	$mn(m+mn)\{m + 2mn - 1\}$	m^2n
$\mathbf{K}^{(j)}(k)$	$mn(m+mn)^2 + m^2n^2(m+mn)$	$mn(m+mn)\{m + 2mn - 2\}$	$m^2n^2(m+mn)^2$
$\mathbf{h}^{(j)}_{k\|k} = \mathbf{h}^{(j)}_{k\|k-1} + \mathbf{K}^{(j)}(k)\left(\mathbf{r}(k) - \mathbf{E}^{(j)}(k)\mathbf{h}^{(j)}_{k\|k-1}\right)$	$2m^2n$	$2m^2n$	-
$\mathbf{M}^{(j)}_{k\|k} = \boldsymbol{\beta}\mathbf{M}^{(j)}_{k\|k-1}$, $\boldsymbol{\beta} = \left(\mathbf{I}_{mn} - \mathbf{K}^{(j)}(k)\ \mathbf{E}^{(j)}(k)\right)$	$m^3n^2 + m^3n^3$	$m^3n^2 + m^3n^3 - m^2n^2$	-

Table 9: Operations in the M-step of the existing scheme [2]

Multiplications	$N\{m^3n^3 + m^2n^2(3(m+mn)+2) + mn(2(m+mn)^2 + 2(m+mn)+1)\} + (m+mn)^2$
Additions	$N\{m^3n^3 + 3m^2n^2(m+mn) + mn(m+mn)^2\}$
Complex Conjugate	$2Nm^2n^2(m+mn)^2$
Matrix Inversion $\mathcal{O}((m+mn)^3)$	N
Matrix Square Root $(n \times n)$	N

matrix $\mathbf{C}(k)$ evaluated in the E-step and applies the KF to obtain the time-varying MIMO channel estimates $\hat{\mathbf{H}}(k)$. As illustrated by the authors in [2], the state and the observation equations for the KF in the M-step are,

$$\mathbf{h}(k) = \mathbf{F}\mathbf{h}(k-1) + \mathbf{G}\mathbf{w}(k), \tag{4}$$

$$\underbrace{\begin{bmatrix}\mathbf{r}(k)\\ 0_{n\times 1}\end{bmatrix}}_{\overline{\mathbf{r}}(k)} = \underbrace{\begin{bmatrix}\mathrm{E}\left\{\mathbf{C}(k)\vert\mathbf{r}(k),\mathbf{H}(k)\right\}\\ \left(\mathrm{Cov}\left\{\mathbf{C}^H(k)\vert\mathbf{r}(k),\mathbf{H}(k)\right\}\right)^{1/2}\end{bmatrix}}_{\overline{\mathbf{C}}(k)}\mathbf{h}(k) + \underbrace{\begin{bmatrix}\boldsymbol{\eta}(k)\\ \boldsymbol{\eta}'(k)_{n\times 1}\end{bmatrix}}_{\overline{\boldsymbol{\eta}}(k)}. \tag{5}$$

The additive Gaussian noise $\overline{\boldsymbol{\eta}}(k)$ above denotes the zero mean concatenated noise vector with covariance $\mathbf{R}_{\overline{\boldsymbol{\eta}}} = \sigma^2_{\boldsymbol{\eta}}\mathbf{I}_{(m+n)}$. The various operations required for computing the channel estimates in the M-step of the existing scheme [2] along with the associated computational complexities are given in Table 8. The M-step also includes a matrix inversion operation of

Table 10: Different operations in the existing joint estimation technique in [2]

	E-step	M-Step
Multiplications	$N[\{\|\Gamma\|^n(2n^2+mn+m$ $+2n+2)+n^2\}+m^2n+m^2n^2]$	$N\{m^3n^3+m^2n^2(3(m+mn)$ $+2)+mn(2(m+mn)^2$ $+2(m+mn)+1)\}+(m+mn)^2$
Additions	$N\{\|\Gamma\|^n(n^2+mn+m+2n)$ $+n^2-1\}$	$N\{m^3n^3+3m^2n^2(m+mn)$ $+mn(m+mn)^2\}$
Complex Conjugate	$mN\|\Gamma\|^n$	$2Nm^2n^2(m+mn)^2$
Exponential	$N\|\Gamma\|^n$	-
Matrix Inversion $\mathcal{O}((m+mn)^3)$	-	N
Matrix Square Root $(n\times n)$	-	N

$\mathcal{O}(m+mn)^3$ and a $n\times n$ matrix square root operation. The list of various mathematical operations involved in the M-step of the existing scheme [2] is given in Table 9. The different mathematical operations employed in the E and M-steps of the existing technique [2] are also summarized in Table 10.

2.3 Complexity Comparison

We plot and compare the complexities of the operations involved in the REKF-based joint estimation scheme and the existing technique in [2] for the simulation scenario in the paper. As illustrated in the paper, we consider a $m=n=2$, $S=4$ state and $|\Gamma|=4$ STTC MIMO system with $B=4$ branches emerging from each state. We set the frame length $N=100$ symbols and $N_i=3$ EM iterations for both the schemes. A total number of 10^4 blocks is considered for comparison of the complexity of the schemes. Fig. 4 demonstrates the complexity of the various mathematical operations involved in the simulation of the proposed and the existing technique in [2]. It can be clearly seen that the proposed REKF-based joint estimation technique results in a substantial reduction in the number of multiplications, additions and complex conjugate operations in comparison to the existing scheme in [2]. The total number of matrix inversions for both the schemes is the same. However, the REKF scheme computes matrix inverses of complexity $\mathcal{O}(m^3)=\mathcal{O}(8)$ whereas the existing scheme [2] involves matrix inversions of complexity $\mathcal{O}((m+mn)^3)=\mathcal{O}(216)$. Both the techniques involve an equal number of $n\times n$ matrix square root operations. Additionally, the existing scheme [2] requires a large number of exponential operations which is not necessary in the proposed REKF-based joint estimation scheme.

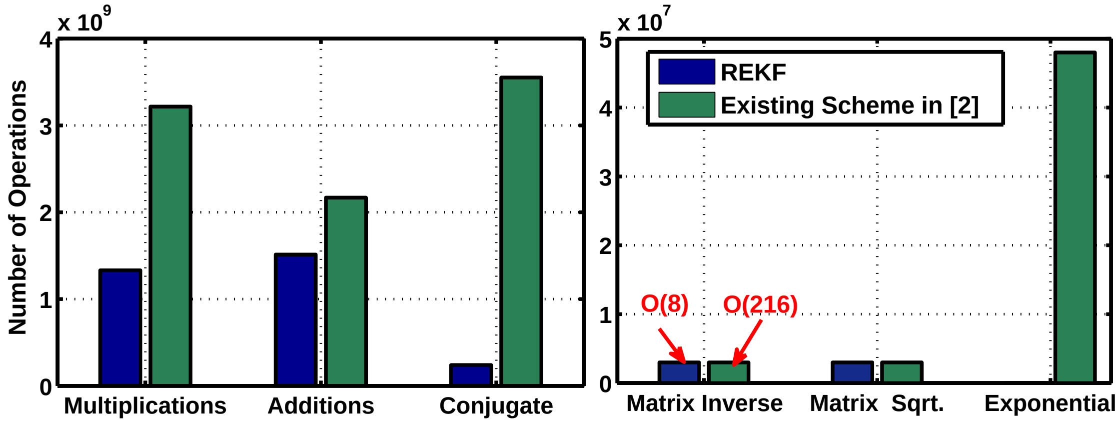

Figure 4: Complexity comparison of the proposed and existing scheme in [2].

References

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