Technical Report: MVDR based Multi-Cell Cooperative Beamforming Techniques for Unicast/ Multicast MIMO Networks with Perfect/Imperfect CSI

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1 SCEB, White and Proportional Power Allocation schemes

In this section, we present other sub-optimal transmission schemes such as SCEB, White and Proportional Power Allocation for performance comparison with the optimal successive minimum variance beamforming (SMVB) schemes described in section III. A, titled "Successive Minimum Variance Beamforming (SMVB)", in the main paper . One can also employ a suboptimal successive constrained eigenbeamforming (SCEB) [1], similar to maximum ratio combining for cooperative multi-cellular scenarios. Employing maximum ratio combining, the optimal beamformer $\mathbf{b}_k$ for user k, for a particular subcarrier, satisfying the zero interference condition to users $1, 2, \ldots, k-1$ can be computed as the solution to following optimization problem,

$$\begin{aligned} \max_{\mathbf{b}_k}. \quad & \mathbf{b}_k^H \left(\mathbf{H}_k^H \mathbf{H}_k\right) \mathbf{b}_k \\ \text{s.t.} \quad & \|\mathbf{b}_k\|_2^2 \leq P, \\ & \mathbf{w}_j^H \mathbf{H}_j \mathbf{b}_k = 0,\ j = 1, 2, \ldots, k-1, \end{aligned} \tag{1}$$

where P is the power constraint for each beamformer. The maximum ratio combining formulation above ignores the interference from the previously scheduled users $1, 2, \ldots, k-1$ at user k. The constraint $\mathbf{w}_j^H \mathbf{H}_j \mathbf{b}_k = 0$ for $j = 1, 2, \ldots, k-1$, similar to SMVB, is satisfied by $\mathbf{b}_k = \mathbf{G}_{k-1}^{\perp} \mathbf{c}_k$, where $\mathbf{c}_k \in \mathbb{C}^{(NN_b - k+1)\times 1}$. Thus, the modified SINR maximization problem for user k is expressed as,

$$\begin{aligned} \max_{\mathbf{c}_k}. \quad & \mathbf{c}_k^H (\mathbf{G}_{k-1}^{\perp})^H \left(\mathbf{H}_k^H \mathbf{H}_k\right) \mathbf{G}_{k-1}^{\perp} \mathbf{c}_k \\ \text{s.t.} \quad & \|\mathbf{c}_k\|_2^2 \leq P. \end{aligned} \tag{2}$$

The second constraint follows from $(\mathbf{G}_{k-1}^{\perp})^H \mathbf{G}_{k-1}^{\perp} = \mathbf{I}$. The solution, $\mathbf{c}_k$, to the above problem (2) is the eigenvector corresponding to the largest eigenvalue of $(\mathbf{G}_{k-1}^{\perp})^H \left(\mathbf{H}_k^H \mathbf{H}_k\right) \mathbf{G}_{k-1}^{\perp}$

and the corresponding transmit beamforming vector is obtained as $\mathbf{b}_k = \sqrt{P}\mathbf{G}_{k-1}^{\perp}\mathbf{c}_k$. Similarly, for the multicast scenario, a sub-optimal beamforming vector for the ith multicast group is $\mathbf{b}^{(i)} = \left(\mathbf{D}^{(i)}\right)^{\perp}\mathbf{c}^{(i)}$, where $\mathbf{c}^{(i)}$ is given as the solution of the optimization problem,

$$\begin{aligned} \max_{\mathbf{c}^{(i)}}. \quad & \left(\mathbf{c}^{(i)}\right)^H \left(\mathbf{H}^{(i)}\left(\mathbf{D}^{(i)}\right)^{\perp}\right)^H \mathbf{H}^{(i)}\left(\mathbf{D}^{(i)}\right)^{\perp}\mathbf{c}^{(i)} \\ \text{s.t.} \quad & \|\mathbf{c}^{(i)}\|_2^2 = K_i P. \end{aligned} \tag{3}$$

It can be seen that $\mathbf{c}^{(i)}$ is obtained as the eigenvector $\boldsymbol{\nu}_{max}(\boldsymbol{\Phi})$ corresponding to the largest eigenvalue $\lambda_{max}(\boldsymbol{\Phi})$ of the matrix $\boldsymbol{\Phi} = \left(\mathbf{H}^{(i)}\left(\mathbf{D}^{(i)}\right)^{\perp}\right)^H \mathbf{H}^{(i)}\left(\mathbf{D}^{(i)}\right)^{\perp}$. The multicast SCEB-M beamforming vector is given as $\sqrt{K_i P}\left(\mathbf{D}^{(i)}\right)^{\perp}\mathbf{c}^{(i)}$. In White Power Allocation, the power allocated by each of the base stations to the kth user is given as $\frac{P}{N}$. Let $\mathbf{v}_{k,i} \in \mathbb{C}^{N_b \times 1}$ denote the dominant right singular vector of $\mathbf{H}_{k,i}$. The white power allocation transmit beamformer $\mathbf{b}_{k,\mathrm{WPA}} \in \mathbb{C}^{NN_b \times 1}$ is therefore given as,

$$\mathbf{b}_{k,\mathrm{WPA}} = \sqrt{\frac{P}{N}}\left[\mathbf{v}_{k,1}^T, \mathbf{v}_{k,2}^T, \ldots, \mathbf{v}_{k,N}^T\right]^T. \tag{4}$$

Similarly, in Proportional Power allocation, the power allocated by base station i to user k is proportional to $\sigma_{k,i}^2$, where $\sigma_{k,i}$ is the dominant singular value of $\mathbf{H}_{k,i}$. Let the quantity $\alpha_{k,i}$ be defined as $\alpha_{k,i} = \sqrt{\sigma_{k,i}^2 / \sum_{j=1}^{N}\sigma_{k,j}^2}$. The corresponding transmit beamformer, $\mathbf{b}_{k,\mathrm{PPA}} \in \mathbb{C}^{NN_b \times 1}$ is given as,

$$\mathbf{b}_{k,\mathrm{PPA}} = \sqrt{P}\left[\alpha_{k,1}\mathbf{v}_{k,1}^T, \alpha_{k,2}\mathbf{v}_{k,2}^T, \ldots, \alpha_{k,N}\mathbf{v}_{k,N}^T\right]^T. \tag{5}$$

2 Proof of Lemma 2

Lemma 2. *To the first order of approximation, $\hat{\mathbf{v}}_{k,1}$, the perturbed dominant eigenvector $\mathbf{v}_{k,1}$ of $\mathbf{H}_k^H\mathbf{H}_k$ can be expressed as,*

$$\mathbf{v}_{k,1} = \hat{\mathbf{v}}_{k,1} + \sum_{i=2}^{NN_b}\frac{\beta_{k,i1}\hat{\mathbf{v}}_{k,i}}{(\hat{\lambda}_{k,1} - \hat{\lambda}_{k,i})},$$

where $\beta_{k,ij} = \hat{\mathbf{v}}_{k,i}^H\mathbf{U}_k^e\hat{\mathbf{v}}_{k,j}$, $\hat{\lambda}_{k,i} = \hat{\sigma}_{k,i}^2$ and $\hat{\sigma}_{k,i}$ denotes the ith singular value of the nominal channel estimate $\hat{\mathbf{H}}_k$ of user k.

Proof. In the presence of uncertainty in CSI, the channel matrix of the kth user can be expressed as $\mathbf{H}_k = \hat{\mathbf{H}}_k + \mathbf{P}_k\mathbf{U}_k$, where $\mathbf{U}_k = [\mathbf{u}_{k,1}, \mathbf{u}_{k,2}, \ldots, \mathbf{u}_{k,NN_b}]$ and $\|\mathbf{u}_{k,l}\|_2 \leq 1$, $1 \leq l \leq NN_b$, which captures the uncertainty in each of the MIMO channel matrices of the cooperative cellular scenario. Based on the uncertainty model above, the matrix $\mathbf{H}_k^H\mathbf{H}_k$ can be simplified as,

$$\begin{aligned} \mathbf{H}_k^H\mathbf{H}_k &= (\hat{\mathbf{H}}_k + \mathbf{P}_k\mathbf{U}_k)^H(\hat{\mathbf{H}}_k + \mathbf{P}_k\mathbf{U}_k) \\ &= \hat{\mathbf{H}}_k^H\hat{\mathbf{H}}_k + \underbrace{(\hat{\mathbf{H}}_k^H\mathbf{P}_k\mathbf{U}_k + \mathbf{U}_k^H\mathbf{P}_k^H\hat{\mathbf{H}}_k + (\mathbf{P}_k\mathbf{U}_k)^H\mathbf{P}_k\mathbf{U}_k)}_{\mathbf{U}_k^e}, \end{aligned}$$

where $\mathbf{U}_k^e$ is the effective uncertainty matrix corresponding to beamforming for user k. The eigenvector perturbation theory [2] can be used to model the uncertainty in the principal eigenvector $\mathbf{v}_{k,1}$ of $\mathbf{H}_k^H\mathbf{H}_k$ as shown in the following discussion. In the presence of uncertainty $\mathbf{U}_k^e$, imperfect CSI can be expressed as, $\mathbf{H}_k^H\mathbf{H}_k = \hat{\mathbf{H}}_k^H\hat{\mathbf{H}}_k + \epsilon\mathbf{U}_k^e$, where $\mathbf{H}_k^H\mathbf{H}$ represents perfect CSI and $\epsilon\mathbf{U}_k^e$ represents the uncertainty in the channel knowledge. It can be noted that as $\epsilon \to 0$, the matrix $\hat{\mathbf{H}}_k^H\hat{\mathbf{H}}_k \to \mathbf{H}_k^H\mathbf{H}_k$.

Let $\hat{\lambda}_{k,i}$, $1 \leq i \leq NN_b$, denote the eigenvalues of the matrix $\hat{\mathbf{H}}_k^H\hat{\mathbf{H}}_k$ and $\hat{\mathbf{v}}_{k,i}$ denote the perturbed eigenvector corresponding to the eigenvalue $\hat{\lambda}_{k,i}$ of the matrix $\hat{\mathbf{H}}_k^H\hat{\mathbf{H}}_k$. Let $\lambda_{k,i}$ denote the eigenvalues of the matrix $\mathbf{H}_k^H\mathbf{H}_k$ and $\mathbf{v}_{k,i}$ denote the eigenvector corresponding to the eigenvalue $\lambda_{k,i}$ of the matrix $\mathbf{H}_k^H\mathbf{H}_k$. From the eigenvector perturbation theory [2], the true principal eigenvalue $\lambda_{k,1}$ can be expressed as the convergent power series,

$$\lambda_{k,1} = \hat{\lambda}_{k,1} + a_{k,1}\epsilon + a_{k,2}\epsilon^2 + \cdots , \tag{6}$$

where $a_{k,i}$ is the multiplicative error factor due to the matrix $\mathbf{U}_k^e$. It can be observed that $\hat{\lambda}_{k,1} \to \lambda_{k,1}$ as $\epsilon \to 0$ and $|\hat{\lambda}_{k,1} - \lambda_{k,1}| = O(\epsilon)$. Since $(\mathbf{H}_k^H\mathbf{H}_k)\mathbf{v}_{k,1} = \lambda_{k,1}\mathbf{v}_{k,1}$, we can also express the principal eigenvector as the convergent power series [2],

$$\mathbf{v}_{k,1} = \hat{\mathbf{v}}_{k,1} + \epsilon\mathbf{z}_{k,1} + \epsilon^2\mathbf{z}_{k,2} + \cdots , \tag{7}$$

where $\mathbf{z}_{k,i}$ is the error vector due to the matrix $\mathbf{U}_k^e$. Each of the vectors $\mathbf{z}_{k,i}$ can be expressed in terms of $\{\hat{\mathbf{v}}_{k,i}\}_{i=1}^{NN_b}$ as the set of eigenvectors $(\hat{\mathbf{v}}_{k,1}, \hat{\mathbf{v}}_{k,2}, \cdots , \hat{\mathbf{v}}_{k,NN_b})$ forms a basis for the NN_b dimensional complex space $\mathbb{C}^{NN_b}$. Therefore, the vectors $\mathbf{z}_{k,i}$ can be expressed as the span of the basis vectors $\hat{\mathbf{v}}_{k,j}$,

$$\mathbf{z}_{k,i} = \sum_{j=1}^{NN_b} s_{k,ji}\hat{\mathbf{v}}_{k,j}, \tag{8}$$

where $s_{k,ji} \in \mathbb{R}$ are scalars. By substituting the above expression for $\mathbf{z}_{k,i}$ in (7), the true principal eigenvector $\mathbf{v}_{k,1}$ is given as,

$$\mathbf{v}_{k,1} = \hat{\mathbf{v}}_{k,1} + \epsilon \sum_{j=1}^{NN_b} s_{k,j1}\hat{\mathbf{v}}_{k,j} + \epsilon^2 \sum_{j=1}^{NN_b} s_{k,j2}\hat{\mathbf{v}}_{k,j} + \cdots , \tag{9}$$

which can be in turn expressed as,

$$\begin{aligned}\mathbf{v}_{k,1} = (1 + \epsilon s_{k,11} + \epsilon^2 s_{k,12} + \cdots)\hat{\mathbf{v}}_{k,1} + (\epsilon s_{k,21} + \epsilon^2 s_{k,22} + \cdots)\hat{\mathbf{v}}_{k,2} + \cdots \\ + (\epsilon s_{k,NN_b1} + \epsilon^2 s_{k,NN_b2} + \cdots)\hat{\mathbf{v}}_{k,NN_b}.\end{aligned}$$

We normalize the above equation in order to eliminate the scaling factor $(1+\epsilon s_{k,11}+\epsilon^2 s_{k,12}+\cdots)$ in $\hat{\mathbf{v}}_{k,1}$ so that true and the perturbed principal eigenvectors are related as,

$$\mathbf{v}_{k,1} = \hat{\mathbf{v}}_{k,1} + (\epsilon t_{k,21} + \epsilon^2 t_{k,22} + \cdots)\hat{\mathbf{v}}_{k,2} + (\epsilon t_{k,NN_b1} + \epsilon^2 t_{k,NN_b2} + \cdots)\hat{\mathbf{v}}_{k,NN_b}, \tag{10}$$

where $t_{k,ji} \in \mathbb{R}$ are scalars and the expressions in the brackets represent convergent power series. Considering only the first order perturbation in the above expression, the principal eigenvector is given as,

$$\mathbf{v}_{k,1} = \hat{\mathbf{v}}_{k,1} + \epsilon \sum_{i=2}^{NN_b} t_{k,i1}\hat{\mathbf{v}}_{k,i}, \tag{11}$$

and the corresponding eigenvalue is given as $\lambda_{k,1} = \hat{\lambda}_{k,1} + a_{k,1}\epsilon$. By substituting the above expressions for $\lambda_{k,1}$ and $\mathbf{v}_{k,1}$ in the system of linear equations $(\mathbf{H}_k^H \mathbf{H}_k)\mathbf{v}_{k,1} = \lambda_{k,1}\mathbf{v}_{k,1}$, we have

$$\left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \epsilon \mathbf{U}_k^e\right)\left(\hat{\mathbf{v}}_{k,1} + \epsilon \sum_{i=2}^{NN_b} t_{k,i1}\hat{\mathbf{v}}_{k,i}\right) = \left(\hat{\lambda}_{k,1} + a_{k,1}\epsilon\right)\left(\hat{\mathbf{v}}_{k,1} + \epsilon \sum_{i=2}^{NN_b} t_{k,i1}\hat{\mathbf{v}}_{k,i}\right). \quad (12)$$

Equating the first order error terms in the above expression, we have

$$\sum_{i=2}^{NN_b} t_{k,i1}\left(\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k\right)\hat{\mathbf{v}}_{k,i} + \mathbf{U}_k^e \hat{\mathbf{v}}_{k,1} = a_{k,1}\hat{\mathbf{v}}_{k,1} + \hat{\lambda}_{k,1}\left(\sum_{i=2}^{NN_b} t_{k,i1}\hat{\mathbf{v}}_{k,i}\right), \quad (13)$$

which can be further simplified as,

$$\sum_{i=2}^{NN_b} t_{k,i1}(\hat{\lambda}_{k,i} - \hat{\lambda}_{k,1})\hat{\mathbf{v}}_{k,i} + \mathbf{U}_k^e \hat{\mathbf{v}}_{k,1} = a_{k,1}\hat{\mathbf{v}}_{k,1}. \quad (14)$$

Multiplying by $\hat{\mathbf{v}}_{k,1}^T$ on both sides of the above equation we obtain $a_{k,1} = \hat{\mathbf{v}}_{k,1}^T \mathbf{U}_k^e \hat{\mathbf{v}}_{k,1}$, since $\hat{\mathbf{v}}_{k,1}^T \hat{\mathbf{v}}_{k,i} = 0$, $i \neq 1$ due to the orthogonality of the eigenvectors corresponding to distinct eigenvalues of a symmetric matrix and the norm of the eigenvector $\|\hat{\mathbf{v}}_{k,1}\|_2^2 = 1$. Now multiplying $\hat{\mathbf{v}}_{k,j}^T$, $j \neq 1$ on both the sides of the equation (14), we have

$$\sum_{i=2}^{NN_b} t_{k,i1}(\hat{\lambda}_{k,i} - \hat{\lambda}_{k,1})\hat{\mathbf{v}}_{k,j}^T \hat{\mathbf{v}}_{k,i} + \hat{\mathbf{v}}_{k,j}^T \mathbf{U}_k^e \hat{\mathbf{v}}_{k,1} = a_{k,1}\hat{\mathbf{v}}_{k,j}^T \hat{\mathbf{v}}_{k,1}, \quad (15)$$

and the above expression reduces to $t_{k,j1} = \frac{\beta_{k,j1}}{\hat{\lambda}_{k,1} - \hat{\lambda}_{k,j}}$, where $\beta_{k,ji} = \hat{\mathbf{v}}_{k,j}^T \mathbf{U}_k^e \hat{\mathbf{v}}_{k,i}$. Substituting the expression for $\beta_{k,ji}$ in (11), with $\epsilon = 1$, which implies that $\mathbf{H}_k^H \mathbf{H}_k = \hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k + \mathbf{U}_k^e$ as in (6), the true principal eigenvector $\mathbf{v}_{k,1}$ can be expressed as,

$$\mathbf{v}_{k,1} = \hat{\mathbf{v}}_{k,1} + \sum_{j=2}^{NN_b} \frac{\beta_{k,j1}\hat{\mathbf{v}}_{k,j}}{\hat{\lambda}_{k,1} - \hat{\lambda}_{k,j}}. \qquad \square$$

3 Minimum Variance Distortionless Response Beamforming

Consider the beamforming system with the received signal $\mathbf{y} = \mathbf{H}\mathbf{b}x + \mathbf{n} \in \mathbb{C}^{N_u \times 1}$ where $\mathbf{b} \in \mathbb{C}^{NN_b \times 1}$ is the transmit beamforming vector, $\mathbf{H} \in \mathbb{C}^{N_u \times NN_b}$ is the channel matrix, and $\mathbf{n}$ is the interference plus noise with covariance $\mathrm{E}\left[\mathbf{n}\mathbf{n}^H\right] = \mathbf{R}_n \in \mathbb{C}^{N_u \times N_u}$. Let $\mathbf{w} \in \mathbb{C}^{N_u \times 1}$ be the receive beamformer employed. The estimate of the symbol x at the receiver can be expressed as $\hat{x} = \mathbf{w}^H \mathbf{H}\mathbf{b}x + \mathbf{w}^H \mathbf{n}$. Therefore, the SINR at the receiver is

$$\mathrm{SINR} = \frac{|\mathbf{w}^H \mathbf{H}\mathbf{b}|^2}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}}. \quad (16)$$

The SINR maximization problem described above can be formulated as,

$$\begin{aligned} \min_{\mathbf{w}}. \quad & \mathbf{w}^H \mathbf{R}_n \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{H}\mathbf{b} = 1. \end{aligned} \quad (17)$$

(a) Akiyo

(b) City

(c) Crew

(d) Football

(e) Foreman

Figure 1: Test Video Sequences [6].

The above problem is known as Capon beamforming or minimum variance distortionless (MVDR) beamforming and a detailed proof can be found in standard references such as [3,4]. However, a brief outline of the proof is provided below. The Lagrangian $L(\mathbf{w}, \mu)$ of the convex optimization problem in (17) can be expressed as

$$L(\mathbf{w}, \mu) = \mathbf{w}^H \mathbf{R}_n \mathbf{w} + \mu(\mathbf{w}^H \mathbf{H}\mathbf{b} - 1), \tag{18}$$

where μ is the Lagrange multiplier. Employing the Karush-Kuhn-Tucker (KKT) conditions [5], we have, $\nabla_{\mathbf{w}} L(\mathbf{w}, \mu) = 2\mathbf{R}_n\mathbf{w} + \mu\mathbf{H}\mathbf{b} = 0$, which yields $\mathbf{w}^* = -\frac{1}{2}\mathbf{R}_n^{-1}\mathbf{H}\mathbf{b}$. The Lagrange multiplier μ can be determined as $\mu = -2\left(\mathbf{b}^H\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\mathbf{b}\right)^{-1}$. Therefore, the optimal beamformer $\mathbf{w}^*$ is given as,

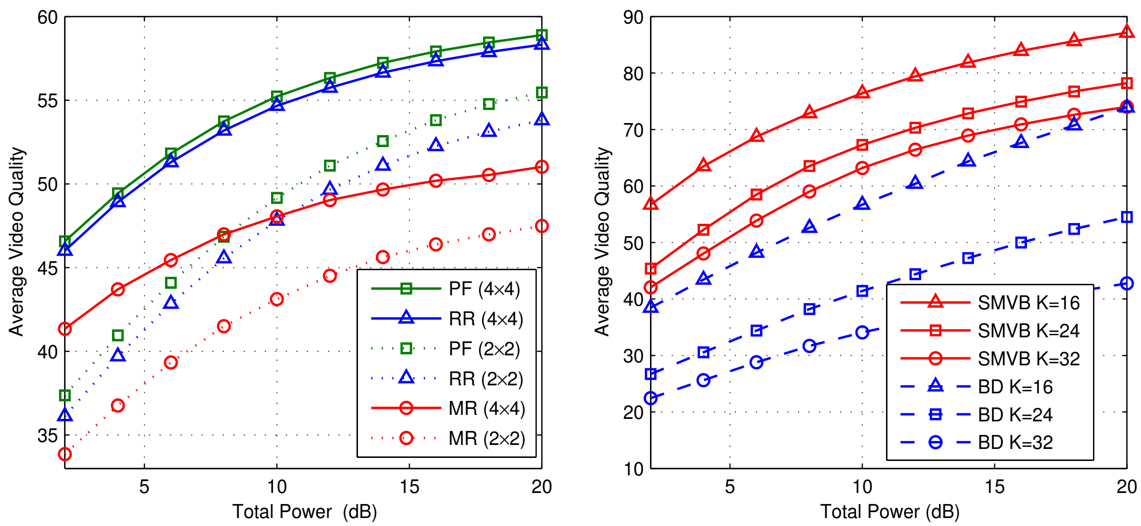
$$\mathbf{w}^* = \frac{\mathbf{R}_n^{-1}\mathbf{H}\mathbf{b}}{\mathbf{b}^H\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}\mathbf{b}}. \tag{19}$$

4 Video Quality

We now present the video quality performance to illustrate the suitability of the proposed SMVB beamforming schemes, described in section III. A and section III. B of the main paper, in the context of unicast, multicast and broadcast transmission of multimedia content in 4G wireless networks. It has been shown that H.264 scalable video transmission is ideally suited for 4G wireless systems due to its ease of rate adaptation with respect to the time-varying wireless channel [7]. We employ parametric models to characterize the quality of the encoded video stream. The rate and quality models corresponding to H.264 scalable video codec have been derived employing the Joint Scalable Video Model software developed by the Joint Video Team [7]. The scalable video rate function $R(q, t)$ in terms of the

Table 1: H.264 SVC model parameters for various video sequences

Sequence	a_i	c_i	d_i	β_i	γ_i	$R^i_{\mathbf{max}}$
Foreman CIF	7. 70	2. 057	2. 207	-0. 0298	1. 4475	3046. 3
Akiyo CIF	8. 03	3. 491	2. 252	-0. 0316	1. 4737	612. 85
Football CIF	5. 38	1. 395	1. 490	-0. 0258	1. 3872	5248. 9
Crew CIF	7. 34	1. 627	1. 854	-0. 0393	1. 5898	4358. 2
City CIF	7. 35	2. 044	2. 326	-0. 0346	1. 5196	2775. 5
Akiyo QCIF	5. 56	4. 019	1. 832	-0. 0316	1. 4737	139. 63
Foreman QCIF	7. 10	2. 590	1. 785	-0. 0298	1. 4475	641. 73
City 4CIF	8. 40	1. 096	2. 367	-0. 0346	1. 5196	20899. 0
Crew 4CIF	7. 34	1. 153	2. 405	-0. 0393	1. 5898	18021. 0



(a) Multicast scenario with $N_G = 10$ user groups with $K_i = 2$ users per user group employing different scheduling schemes.

(b) Unicast scenario with $N_b = 4$ transmit antennas, $N_u = 4$ receive antennas, and $K \in \{16, 24, 32\}$ users.

Figure 2: Average video quality vs power for various beamforming and scheduling schemes with $N = 4$ base station, $M = 4$ sub-channels.

quantization parameter q and frame rate t of the video is given as,

$$R(q,t) = R_{\max} \underbrace{\left(\frac{1 - e^{(-ct/t_{\max})}}{1 - e^{-c}} \right)}_{R_t(t)} \underbrace{e^{d(1-q/q_{\min})}}_{R_q(q)},$$

where $R_{\max} = R(q_{\min}, t_{\max})$ is the maximum bit rate of the coded video stream of the highest quality video corresponding to the maximum frame rate $t_{\max}$ and minimum quantization parameter $q_{\min}$. The quantities $R_q(q)$ and $R_t(t)$ are the normalized rate function versus quantization parameter and frame rate respectively. The scalable video joint quality function is given as,

$$Q(q,t) = \underbrace{Q_{\max}\left(\frac{1-e^{(-at/t_{\max})}}{1-e^{-a}}\right)}_{Q_t(t)}\underbrace{(\beta q+\gamma)}_{Q_q(q)},$$

where $Q_{\max} = Q(q_{\min}, t_{\max})$ is the highest quality of the video sequence corresponding to the maximum frame rate $t_{\max}$ and minimum quantization parameter $q_{\min}$. The quantities a, c, d, β, and γ are the scalable video rate and quality parameters [8]. In the simulations, the video quality parameter $q_{\min}$ is set as $q_{\min} = 15$, the frame rate is fixed at $t_{\max} = t = 30$fps, and the normalized value of the parameter $Q_{\max}$ is set to 100 for all the video sequences to evaluate the average video quality.

The simulation setup to evaluate the video quality performance of the proposed MU-MIMO beamforming schemes in the context of video transmission in a 4G wireless network is described below. The standard video sequences *Akiyo*, *Foreman*, *Football*, *City*, and *Crew*, [6] shown in Fig. 1 are considered for the simulations. The scalable video rate and quality parameters a, c, d, β, and γ for these video sequences [8] are presented in Table 1. The video quality parameter $q_{\min}$ is set as $q_{\min} = 15$ while the frame rate is fixed at $t_{\max} = t = 30$fps. The normalized value of the parameter $Q_{\max}$ is set to 100. The average video quality performance achieved by employing the proposed SMVB-M beamforming in a multicast multi-cell scenario is shown in Fig. 2a. It is interesting to note that although the maximum rate scheduling scheme provides a higher sum-rate compared to the proportional fair and round robin scheduling schemes, it yields the worst performance in terms of the average video quality. This is because the maximum rate scheduling scheme does not consider fairness while allocating the resources, thereby leading to a poor performance. Hence, while some users experience a very high data rate with the maximum rate scheduling scheme, which leads to a saturation of the video quality experienced by them, the other users are starved, thus resulting in no contribution to the net video quality. This reduces the average video quality of the multicast group. Naturally, proportional fair scheduling, which ensures user/group fairness while compromising marginally on the sum-rate, provides the best video quality. Fig. 2b illustrates the average video quality performance achieved by employing the proposed SMVB beamforming and block diagonalization schemes in a unicast multi-cell scenario. It can be observed that the SMVB beamforming scheme can achieve a better video quality performance in comparison to the block diagonalization scheme. This is because the users are served more frequently in SMVB whereas the users are starved when block diagonalization is employed, as it can only support a fewer number of users in each slot. The higher video quality achieved by the SMVB scheme demonstrates the suitability of the proposed optimal beamforming schemes in the context of video content based unicast/ multicast services in 4G wireless networks.

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