Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

MTH 101N, Section C : Quiz - 2A

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Consider the surface given by $z = x^2 + y^2 - 2$.
   Find the tangent plane to this surface at the point $(1, 0, -1)$. [5]

   Solution : The given surface is $f(x, y, z) = x^2 + y^2 - z - 2$.
   $\nabla f = (2x, 2y, -1)$. [1]
   $\nabla f(1, 0, -1) = (2, 0, -1)$. [1]
   This vector is a vector normal to the tangent plane at $(1, 0, -1)$.
   Hence, the equation of the tangent plane to the surface at $(1, 0, -1)$ is
   $\nabla f(1, 0, -1).((x, y, z) - (1, 0, -1)) = 0$. [2]
   The equation of the tangent plane is $2x - z = 3$. [1]

2. Find the volume of the solid bounded by $r = 2a\cos\theta$, $z = r$ and $z = 0$. [5]

   Solution : The volume is
   $$V = 2\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta}\int_0^{r} r\,dz\,dr\,d\theta.$$ [3]
   Hence $V = \dfrac{32a^3}{9}$. [2]

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MTH 101N, Section C : Quiz - 2B

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Consider the surface given by $x^2 + y^2 - z^2 - 2xy + 4xz = 4$.
   Find the tangent plane to this surface at the point $(1, 0, 1)$. [5]

   Solution : The given surface is $f(x, y, z) = x^2 + y^2 - z^2 - 2xy + 4xz - 4$.
   $\bigtriangledown f = (2x - 2y + 4z, 2y - 2x, -2z + 4x)$. [1]
   $\bigtriangledown f(1, 0, 1) = (6, -2, 2)$. [1]
   This vector is a vector normal to the tangent plane at $(1, 0, 1)$. Hence, the equation of the tangent plane to the surface at $(1, 0, 1)$ is $\bigtriangledown f(1, 0, 1).((x, y, z) - (1, 0, 1)) = 0$. [2]
   The equation of the tangent plane is $6x - 2y + 2z = 8$. [1]

2. Find the volume of the region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$. [5]

   Solution : The volume is
   $$V = \int_0^{2\pi} \int_0^1 \int_{r^2}^{r} r\,dz\,dr\,d\theta.$$ [3]
   Hence $V = \dfrac{\pi}{6}$. [2]

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MTH 101N, Section D : Quiz - 2A

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Let the temperature at a point (x, y, z) is space be given by $x + 2y + 3z$. Find the points at which the extremal temperatures are attained on the surface of a sphere of radius 1 centered at the origin. Find the values of the extremal temperatures. [5]

   Solution : It is required to maximize and minimize the function $f(x, y, z) = x+2y+3z$, subject to the constraint $g(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$.

   Using the method of Lagrange multipliers: Let $\bigtriangledown f = \lambda \bigtriangledown g$, where $\lambda \in \mathbb{R}$.

   Solving the above we get $\lambda = \frac{\sqrt{14}}{2}$ or $\lambda = -\frac{\sqrt{14}}{2}$. [2]

   The corresponding points are $X = (\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}})$ and
   $Y = (-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}})$. [2]

   $f(X) = \sqrt{14}$ and $f(Y) = -\sqrt{14}$. Therefore the maximum temperature is $\sqrt{14}$ attained at X and the minimum temperature is $-\sqrt{14}$ attained at Y. [1]

2. Evaluate $\int\int\int_S \frac{dxdydz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}$ where S is the solid bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, where $a > b > 0$. [5]

   Solution : $\int\int\int_S \frac{dxdydz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} = \int_0^{\pi}\int_0^{2\pi}\int_b^{a} \frac{1}{\rho^3}\rho^2 \sin\phi d\rho d\theta d\phi$. [3]

   Hence $\int\int\int_S \frac{dxdydz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} = 4\pi \ln\frac{a}{b}$. [2]

Department of Mathematics and Statistics
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MTH 101N, Section D : Quiz - $2B$

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Find the maximal and minimal values of the function $x + 2y + 4z$ among all points on the surface of a sphere of radius 1 centered at the origin. Further, find the points at which these extremal values are attained. [5]

   Solution : It is required to maximize and minimize the function $f(x, y, z) = x+2y+4z$, subject to the constraint $g(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$.

   Using the method of Lagrange multipliers: Let $\bigtriangledown f = \lambda \bigtriangledown g$, where $\lambda \in \mathbb{R}$.

   Solving the above we get $\lambda = \frac{\sqrt{21}}{2}$ or $\lambda = -\frac{\sqrt{21}}{2}$. [2]

   The corresponding points are $X = (\frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}})$ and
   $Y = (-\frac{1}{\sqrt{21}}, -\frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}})$. [2]

   $f(X) = \sqrt{21}$ and $f(Y) = -\sqrt{21}$. Therefore the maximum value is $\sqrt{21}$ attained at X and the minimum value is $-\sqrt{21}$ attained at Y. [1]

2. Find the volume of the region inside the sphere $x^2+y^2+z^2 = 1$ and outside the cylinder $x^2 + y^2 = \frac{1}{4}$. [5]

   Solution : The volume is

   $$V = 2\int_0^{2\pi}\int_{\frac{1}{2}}^{1}\int_0^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta. \qquad [3]$$

   Hence $V = \frac{\sqrt{3}\pi}{2}$. [2]