Department of Mathematics and Statistics
Indian Institute of Technology Kanpur

MTH 101N, Quiz - IA (Section C)

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Determine whether the series $\sum(-1)^k \frac{k}{k^2+5}$ converges, converges absolutely, or diverges. Justify your arguments. [5]

   Solution : $a_k > 0$, $a_{k+1} \leq a_k$ (therefore a_k is decreasing) [1]

   $\lim_{k\to\infty} a_k = 0$ [1]

   Therefore by Leibnitz Test, $\sum(-1)^k \frac{k}{k^2+5}$ converges. [1]

   Now, $|(-1)^k a_k| = a_k > 0$. Let $b_k = \frac{1}{k} > 0$. $\lim_{k\to\infty} \frac{a_n}{b_n} = \lim_{k\to\infty} \frac{k^2}{k^2+5} = 1$ [1]

   Therefore, by the limit comparison test, $\sum a_k$ and $\sum b_k$ converge or diverge together. Since $\sum b_k$ is divergent, $\sum a_k$ is also divergent. [1]

2. Show that a decreasing function on $[a, b]$ is integrable. [5]

   Solution : Let $P = \{a = x_0, \ldots, x_n = b\}$ be a partition such that $\triangle x_i = \frac{b-a}{n}$ [2]

   $U(P, f) - L(P, f) = \sum_{i=0}^{n-1}(f(x_i) - f(x_i + 1))\triangle x_i$ [2]

   $= \frac{b-a}{n}(f(a) - f(b)) \to 0$ [1]

   Therefore $f \in R[a, b]$.

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1. Show that an increasing function on $[a, b]$ is integrable. [5]

   Solution : Let $P = \{a = x_0, \ldots, x_n = b\}$ be a partition such that $\triangle x_i = \frac{b-a}{n}$ [2]

   $U(P, f) - L(P, f) = \sum\limits_{i=0}^{n-1} (f(x_{i+1}) - f(x_i))\triangle x_i$ [2]

   $= \frac{b-a}{n}(f(b) - f(a)) \to 0$ [1]

   Therefore $f \in R[a, b]$.

2. Determine whether the series $\sum(-1)^k \frac{k}{k^2+15}$ converges, converges absolutely, or diverges. Justify your arguments. [5]

   Solution :

   $a_k > 0$, $a_{k+1} \leq a_k$ (therefore a_k is decreasing) [1]

   $\lim\limits_{k\to\infty} a_k = 0$ [1]

   Therefore by Leibnitz Test, $\sum(-1)^k \frac{k}{k^2+15}$ converges. [1]

   Now, $|(-1)^k a_k| = a_k > 0$. Let $b_k = \frac{1}{k} > 0$. $\lim\limits_{k\to\infty} \frac{a_n}{b_n} = \lim\limits_{k\to\infty} \frac{k^2}{k^2+15} = 1$ [1]

   Therefore, by the limit comparison test, $\sum a_k$ and $\sum b_k$ converge or diverge together. Since $\sum b_k$ is divergent, $\sum a_k$ is also divergent. [1].

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MTH 101N, Quiz - IA, (Section D)

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Determine the radius of convergence of the power series $\sum a_n(x-2)^n$, where a_n is given by $\frac{n^n}{n!}$. [5]

   Solution:

   By the ratio test $\lim\limits_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim\limits_{n\to\infty} |\frac{(n+1)^{n+1}}{(n+1)!}\frac{n!}{n^n}| = \lim\limits_{n\to\infty}(1+\frac{1}{n})^n = e$ [3]

   Therefore, radius of convergence $=\frac{1}{e}$ [2]

2. Let

$$f(x) = \begin{cases} 1 & -1 \le x < 0 \\ 2 & 0 \le x < 1 \\ 0 & 1 \le x \le 2 \end{cases}$$

   Using Riemann's criterion for integrability, determine whether f is integrable on $[-1,2]$. What is $\int\limits_{-1}^{2} f(x)dx$? [5]

   Solution:

   Let $P = \{x_0 = -1, x_1 = -\epsilon, x_2 = \epsilon, x_3 = 1-\epsilon, x_4 = 1+\epsilon, x_5 = 2\}$ be a partition of $[-1,2]$. [2]

   $U(P,f) - L(P,f) = 2\epsilon(2-1) + 2\epsilon(2-0) = 6\epsilon$. Thus f is integrable on $[-1,2]$. [2]

   $\int\limits_{-1}^{2} f(x)dx = \int\limits_{-1}^{0} f(x)dx + \int\limits_{0}^{1} f(x)dx + \int\limits_{1}^{2} f(x)dx = 3.$ [1]

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MTH 101N, Quiz - IB (Section D)

Maximum Marks: 10
Time: 20 Minutes

Name: **Roll No:**

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1. Determine the radius of convergence of the power series $\sum a_n(x-1)^n$, where a_n is given by $(\frac{n+1}{n})^{n^2}$. [5]

   Solution:

   By the root test $\lim\limits_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim\limits_{n\to\infty} (1+\frac{1}{n})^n = e$. [3]

   Therefore, radius of convergence $=\frac{1}{e}$ [2]

2. Let

$$f(x) = \begin{cases} 2 & -1 \le x < 0 \\ 1 & 0 \le x < 1 \\ 2 & 1 \le x \le 2 \end{cases}$$

   Using Riemann's criterion for integrability, determine whether f is integrable on $[-1, 2]$. What is $\int\limits_{-1}^{2} f(x)dx$? [5]

   Solution:

   Let $P = \{x_0 = -1, x_1 = -\epsilon, x_2 = \epsilon, x_3 = 1-\epsilon, x_4 = 1+\epsilon, x_5 = 2\}$ be a partition of $[-1, 2]$. [2]

   $U(P, f) - L(P, f) = 2\epsilon(2-1) + 2\epsilon(2-1) = 4\epsilon$. Thus f is integrable on $[-1, 2]$. [2]

   $\int\limits_{-1}^{2} f(x)dx = \int\limits_{-1}^{0} f(x)dx + \int\limits_{0}^{1} f(x)dx + \int\limits_{1}^{2} f(x)dx = 5$. [1]