

Answer all the questions.

1. For what values of  $x$  does the series  $\sum_2^{\infty} \frac{x^n}{n(\ln n)^{\frac{1}{2}}}$  converge? [6]

2. (a) Plot the curves  $r = \sin 2\theta$  and  $r = \cos \theta$ . Consider the region that lies between both the curves. Express the area as an integral.  
Now revolve this region around the  $x$ -axis. Express the area of the surface generated as an integral. [6]

(b) Does the integral  $\int_0^1 \frac{\tan \sqrt{x}}{x + x^2} dx$  converge? Give reasons. [5]

3. (a) Let

$$f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Is  $f$  differentiable at  $(0, 0)$ ? Supply the necessary arguments. [6]

- (b) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at an angle of  $60^\circ$  at the center of the cylinder. Find the volume of the wedge. [5]

4. (a) Let  $h$  be continuous and  $f, g$  be differentiable on  $\mathbb{R}$ . If

$$F(x) = \int_{f(x)}^{g(x)} h(t) dt,$$

then prove that  $F$  is differentiable. Find  $F'(x)$ . [6]

- (b) A particle moves along a circle of radius 3 with uniform speed  $t^2$ , at time  $t$ . Compute the tangential and the normal components of the acceleration at time  $t = 3$ . Also, find the time required to complete the full circle. [6]