

Department of Mathematics & Statistics, I.I.T. Kanpur

MTH101N - 1st Mid Semester Exam-30 August 2007

8:00 - 9:00 a.m.

Maximum Marks : 40

Answer all questions.

1. (a) Sketch the graph $f(x) = \frac{x^2-1}{x+2}$, providing all the appropriate arguments.
(b) Define the sequence (x_n) as follows: $x_1 \in (0, 1)$; $x_{n+1} = \frac{1}{7}(x_n^2 + 2)$. Determine whether the sequence (x_n) is convergent. If it is, then find the limit of the sequence. [5+5]
2. (a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{2+\sin^3(n+1)}{2^n+n^2}$.
(b) Show that $0 < \frac{1}{x} \log\left(\frac{e^x-1}{x}\right) < 1$ for $x > 0$. [5+5]
3. (a) Let $[x]$ denote the integral part of x , *i.e.*, the largest of the integers not exceeding x . Examine the continuity of the function $f(x) = x - [x]$ on \mathbb{R} .
(b) Show that there are exactly two real values of x which satisfy the equation $x^2 = x \sin x + \cos x$. [5+5]
4. (a) Let f be a twice differentiable function on $(0, \infty)$ such that $f''(x)$ is bounded on $(0, \infty)$ and $\lim_{x \rightarrow \infty} f(x) = 0$. Show that $\lim_{x \rightarrow \infty} f'(x) = 0$.
(b) Let $f : [0, 1] \rightarrow \mathbb{R}$ be a differentiable function. Suppose f has a local maximum at $c \in [0, 1]$. Show that $f'(c) = 0$ if $c \in (0, 1)$. What happens if c is either 0 or 1? Justify your argument. [5+5]