

Assignment 9 : Vectors, Curves, Surfaces and Vector Functions

1. Consider the planes $x - y + z = 1$, $x + ay - 2z + 10 = 0$ and $2x - 3y + z + b = 0$, where a and b are parameters. Determine the values of a and b such that the three planes
 - (a) intersect at a single point,
 - (b) intersect in a line,
 - (c) intersect (taken two at a time) in three distinct parallel lines.
2. Determine the equation of the cylinder generated by a line through the curve $(x - 2)^2 + y^2 = 4$, $z = 0$ moving parallel to the vector $\vec{i} + \vec{j} + \vec{k}$.
3. Determine the equation of a cone with vertex $(0, -a, 0)$ generated by a line passing through the curve $x^2 = 2y$, $z = h$.
4. The velocity of a particle moving in space is $\frac{d}{dt}c(t) = (\cos t)\vec{i} - (\sin t)\vec{j} + \vec{k}$. Find the particle's position as a function of t if $c(0) = 2\vec{i} + \vec{k}$. Also find the angle between its position vector and the velocity vector.
5. Show that $c(t) = \sin t^2\vec{i} + \cos t^2\vec{j} + 5\vec{k}$ has constant length and is orthogonal to its derivative. Is the velocity vector of constant magnitude?
6. Find the point on the curve $c(t) = (5 \sin t)\vec{i} + (5 \cos t)\vec{j} + 12t\vec{k}$ at a distance 26π units along the curve from the origin in the direction of increasing arc length.
7. If a plane curve has the Cartesian equation $y = f(x)$, show that the curvature at the point $(x, f(x))$ is $\frac{|f''(x)|}{[1 + f'(x)^2]^{3/2}}$.
8. For the curve $c(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + 4t\vec{k}$ find the equations of the tangent, principal normal and binormal. Also calculate the curvature of the curve.
9. Reparametrise the curves
 - (a) $c(t) = \frac{t^2}{2}\vec{i} + \frac{t^3}{3}\vec{k}$, $0 \leq t \leq 2$,
 - (b) $c(t) = 2 \cos t\vec{i} + 2 \sin t\vec{j}$, $0 \leq t \leq 2\pi$in terms of arc length.
10. Show that the parabola $y = ax^2$, $a \neq 0$ has its largest curvature at its vertex and has no minimum curvature.