Assignment- 14 : Green's /Stoke's /Gauss's Theorems

1. Use Green's Theorem to compute the area inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

2. Use Green's Theorem to compute $\int\limits_C (2x^2 - y^2)\, dx + (x^2 + y^2)\, dy$ where C is the boundary of the region $\{(x, y) : x, y \geq 0 \;\; \& \;\; x^2 + y^2 \leq 1\}$.

3. Evaluate $\int\limits_C \frac{xdy - ydx}{x^2 + y^2}$ along any simple closed curve in the xy plane not passing through the origin. Distinguish the cases where the region R enclosed by C: (a) includes the origin (b) does not include the origin.

4. Use Stoke's Theorem to evaluate the line integral $\int\limits_C -y^3\, dx + x^3\, dy - z^3\, dz$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$ and the orientation of C corresponds to counterclockwise motion in the xy-plane.

5. Let $\overrightarrow{F} = \frac{\overrightarrow{r}}{|\overrightarrow{r}|^3}$ where $\overrightarrow{r} = x\,\overrightarrow{i} + y\,\overrightarrow{j} + \overrightarrow{k}$ and let S be any surface that surrounds the origin. Prove that $\iint\limits_S \overrightarrow{F}.n\, d\sigma = 4\pi$.

6. Let D be the domain inside the cylinder $x^2 + y^2 = 1$ cut off by the planes $z = 0$ and $z = x + 2$. If $\vec{F} = (x^2 + ye^z, y^2 + ze^x, z + xe^y)$, use the divergence theorem to evaluate $\iint\limits_{\partial D} F \cdot \mathbf{n}\, dS$.