Assignment- 12 : Double Integrals

1. Let $f(x,y) = x^2 + y^2$ and let $R = [-1,\ 1] \times [0,\ 1]$. Evaluate the integral $\iint\limits_R (x^2+y^2)dxdy$.

2. Evaluate the following integrals:

   $i)\ \int\limits_0^1 \int\limits_0^{\sqrt{1-x^2}} \sqrt{1-y^2}dydx \qquad ii)\ \int\limits_0^{\pi}\int\limits_x^{\pi} \frac{\sin y}{y}dydx \qquad iii)\ \int\limits_0^1\int\limits_y^1 x^2 \exp^{xy} dxdy.$

3. Evaluate the integral $\iint\limits_R (x+y)^2 dxdy$ over the triangle R with vertices $(0,0),\ (2,2)$ and $(0,1)$.

4. Evaluate the integral $\iint\limits_R (x^2-y^2)dxdy$ over the square R with vertices $(0,0),\ (1,-1),\ (2,0)$ and $(1,1)$.

5. Determine the volume of the region bounded by the surface $z = x^2$ and $z = 4 - x^2 - y^2$.

6. Compute $\lim\limits_{a\longrightarrow\infty} \int\int\limits_{D(a)} \exp^{-(x^2+y^2)} dxdy$, where

   $i)\ D(a) = \{(x,y) : x^2+y^2 \leq a^2\}$ and $\qquad ii)\ D(a) = \{(x,y) : 0 \leq x \leq a,\ 0 \leq y \leq a\}$.

   Hence prove that $\int\limits_0^{\infty} \exp^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

7. Change the order of integration to prove that

   $i)\ \int\limits_0^x \int\limits_0^u \exp^{m(x-t)} f(t)dtdu = \int\limits_0^x (x-t)\exp^{m(x-t)} f(t)dt,$

   $ii)\ \int\limits_0^x \int\limits_0^v \int\limits_0^u \exp^{m(x-t)} f(t)dtdudv = \int\limits_0^x \frac{(x-t)^2}{2} \exp^{m(x-t)} f(t)dt.$