The idempotent graph associated with commutative rings

Abstract

The study of algebraic structures through their associated graphs has developed into a rich and active area of research in recent decades. The investigation of these algebraic graphs leads to an interplay between the algebraic properties of respective algebraic structures and the graph-theoretic properties of graphs associated with them. Some well-known examples include Cayley graphs of groups and zero-divisor graphs of rings. To further investigate this connection, researchers have introduced various algebraic graphs using ring elements such as zero-divisors, units and idempotents. In this talk, we begin with zero-divisor graphs as a foundational example and then focus on idempotent graphs.

Let R be a commutative ring with unity. The *idempotent graph* $G_{\mathrm{Id}}(R)$ of a ring R is a simple undirected graph whose vertices are the set of all the elements of the ring R and two vertices x and y are adjacent if and only if $x + y$ is an idempotent element of R. In this talk, first, we discuss the structural aspects of the idempotent graphs. In this context, we determine the precise structure of the idempotent graph of a local ring. Further, we obtain a necessary and sufficient condition on the ring R such that $G_{\mathrm{Id}}(R)$ is planar. Moreover, we classify finite commutative rings according to when their idempotent graphs belong to certain graph classes, including cographs, split graphs, and threshold graphs.