Indian Institute of Technology Kanpur

Department of Mathematics

Excess decay without commutativity

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Venue: L13, Lecture Hall Complex

Abstract

Given any $1 < q < \infty$ and any $\phi : \mathbb{R}^n \to \mathbb{R}$, the q-excess of ϕ in the ball of radius r is defined as

$$E_q(\phi, r) := \left(\fint_{B_r} |\phi - (\phi)_r|^q \right)^{\frac{1}{q}}, \qquad \text{where } (\phi)_r := \fint_{B_r} \phi.$$

This measures the mean oscillation of ϕ in the ball of radius r in the L^p sense. Thanks to a celebrated theorem of Campanato, these quantities are a natural object to investigate the Hölder continuity of ϕ. When u is weakly p-harmonic, then the regularity properties for the derivatives of u is encoded in the excess decay estimates

$$E_q(\nabla u, \rho) \leq c\,(\rho/R)^{q\alpha}\, E_q(\nabla u, R).$$

These estimates are central to the regularity theory of p-Laplace type equations. Unfortunately, the proof of these estimates relied crucially on the fact that Euclidean derivatives *commute*, i.e. $\partial_i \partial_j - \partial_j \partial_i = 0$. In this talk, based on joint works with **Arka Mallick** (IISc), we shall discuss how one can derive excess decay estimates for the *subelliptic* p-Laplace equation in Heisenberg groups:

$$\operatorname{div}_{\mathbb{H}} \left(\mathfrak{a}(x)\, |\mathfrak{X} u|^{p(x)-2}\, \mathfrak{X} u \right) = 0 \qquad \text{in } \mathbb{H}^n.$$

Here $\mathbb{H}^n$ is the Heisenberg group, $\operatorname{div}_{\mathbb{H}}$ denotes the horizontal diveregnce, $\mathfrak{X} u = (X_1 u, \ldots, X_{2n} u)$ denotes the horizontal gradient. Despite the apparent similarity, the subelliptic setting is *very different* and quite challenging compared to the Euclidean case, as the horizontal vector fields do not commute and we have

$$X_i X_{n+i} - X_{n+i} X_i = T \quad \text{for all } 1 \leq i \leq n,$$

where T is the verical derivative, which is formally absent in the equation. The talk will begin with a history of ideas in the Euclidean case and should be accessible without any prior knowledge of regularity theory in the calculus of variations.

About the Speaker

Dr. Swarnendu Sil is an assistant professor at IISc Bangalore, specializing in geometric analysis, partial differential equations and calculus of variations. After transitioning from a background in engineering at Jadavpur University to a Ph. D. at EPFL, he held postdoctoral positions at both EPFL and ETH Zürich. More about him: https://math.iisc.ac.in// ssil/