



LECTURE SERIES ON CLUSTER ALGEBRAS

SPEAKER: PROF. THOMAS BRÜSTLE
UNIVERSITÉ DE SHERBROOKE AND BISHOP'S UNIVERSITY, CANADA



ORGANIZED BY

DEPARTMENT OF MATHEMATICS AND STATISTICS

TUESDAY (11TH NOV), WEDNESDAY (12TH NOV) AND FRIDAY (14TH NOV)

TIME: 3-4 PM, VENUE: FB 567

ABOUT THE SPEAKER

PROF. THOMAS BRÜSTLE IS A LEADING EXPERT IN REPRESENTATION THEORY WITH OVER 1800 CITATIONS. HE SPECIALISES IN REPRESENTATION THEORY OF QUIVERS, CLUSTER ALGEBRAS AND IN TOPOLOGICAL DATA ANALYSIS VIA PERSISTENCE THEORY. HE OBTAINED HIS PHD FROM UNIVERSITY OF ZÜRICH UNDER THE SUPERVISION OF PROF. PETER GABRIEL, WHO WAS THE FOUNDER OF REPRESENTATION THEORY OF QUIVERS. PROF. BRÜSTLE CURRENTLY HOLDS MAURICE AUSLANDER CHAIR AT BISHOP'S UNIVERSITY. HE IS AN ACTIVE MEMBER OF THE PRESTIGIOUS CENTRE DE RECHERCHES MATHÉMATIQUES (CRM) IN MONTRÉAL. IN RECENT YEARS, HE HAS SUCCESSFULLY LED EFFORTS TO BRING TOGETHER TOPOLOGICAL DATA ANALYSIS (TDA) AND REPRESENTATION THEORY COMMUNITIES THROUGH PUBLICATIONS, ORGANIZATION OF WORKSHOPS AS WELL AS THROUGH COLLABORATIONS WITH RESEARCH INSTITUTES AND INDUSTRY.

LECTURE 1:

Introduction to Cluster Algebras

Cluster algebras, introduced by Fomin and Zelevinsky, are combinatorially defined commutative algebras generated by recursively mutating collections of variables arranged in quivers. In this lecture, we give an informal introduction to the framework, beginning with quiver mutation and skew-symmetrizable matrices. We explain how phenomena from the theory of root systems and semisimple Lie algebras naturally appear in the background, and we discuss the finite type classification, which mirrors the classification of Dynkin diagrams. After formulating the definition of a cluster algebra and highlighting key results such as the Laurent phenomenon and positivity, we work through several concrete examples to illustrate how surprisingly rich algebraic structure can arise from simple combinatorial rules.

LECTURE 2:

Categorification and Cluster Categories

The second lecture explores the categorification of cluster algebras via cluster categories. We begin with the repetitive quiver and the derived category of representations of a quiver, then introduce the cluster category as a certain orbit category with Calabi–Yau properties. Within this framework, cluster variables correspond to indecomposable objects, and mutation is realized through tilting theory and cluster-tilting objects. The Caldero–Chapoton map provides an explicit bridge between categorical and algebraic viewpoints, allowing cluster combinatorics to be understood through homological algebra. Throughout, we emphasize the conceptual meaning of categorification and how it clarifies and unifies many structural features of cluster algebras.

LECTURE 3:

Cluster Algebras from Surfaces

In the final lecture, we discuss the rich interplay between cluster algebras and geometry of marked surfaces. Given an oriented surface with marked points, triangulations yield quivers and seeds whose mutations correspond to flips of arcs. This geometric model gives a transparent and visual way to understand cluster variables as curves on the surface. We describe how certain gentle algebras arise naturally from these constructions, and how the associated categories provide a categorification of the surface cluster algebra. This perspective connects cluster theory with low-dimensional topology, mapping class groups, and the geometry of Teichmüller spaces, offering a unifying and highly visual setting in which algebra, geometry, and combinatorics meet.