A noncoforming virtual element approximation for the Oseen eigenvalue problem

Dibyendu Adak

Virtual element methods (VEMs) are an accurate and computationally efficient generalization of the FEM with a solid mathematical foundation. The key advantage offered by the VEM is its great flexibility with mesh elements. The usual finite element formulations are restricted to a few simple elemental shapes like triangles, quadrilaterals, tetrahedra, and hexahedra. The VEM can handle elements with more general polygonal shapes, including random and smoothly distorted Voronoi elements and polygonal nonconvex elements. This flexibility is because the basis functions in VEM are virtual: we never compute them directly. Instead, we assume that the virtual element space always contains a polynomial subspace of a degree k, and we compute their L^2 or H^1 polynomial projections from their degrees of freedom (DoFs).

Using such projection operators, we decompose the bilinear forms of the variational formulations into two main components: a directly computable polynomial part and a remaining non-computable part and non-polynomial part that we approximate by using the degrees of freedom. This approach is challenged by nonlinear problems, as in this last case, the bilinear forms cannot be split as usual. We plan to address this issue by incorporating the nonlinearity into the polynomial component as detailed in Reference [1]. We can use this approach to address *nonlocal nonlinear problems*, i.e., problems whose analytical solution is not defined point-wise. The study of nonlocal problems has experienced considerable attention in the last decades, since these models have significant applications in studying physical and biological phenomena. For example, the analytical solution represents the density of bacteria population subject to spreading, bidomain models (Mitchell-Schaeffer), and monodomain models (Fitz Hugh-Nagumo).

In this talk, we analyze a nonconforming virtual element method to approximate the eigenfunctions and eigenvalues of the two dimensional Oseen eigenvalue problem. The spaces under consideration lead to a divergence-free method [3] that is capable to capture properly the divergence at discrete level and the eigenvalues and eigenfunctions. Under the compact theory for operators, we prove convergence and error estimates for the method [2]. By employing the theory of compact operators, we recover the double order of convergence of the spectrum. Finally, we present numerical tests to assess the performance of the proposed numerical scheme.

References

[1] D. Adak, E. Natarajan, and S. Kumar. Convergence analysis of virtual element methods for semilinear parabolic problems on polygonal meshes. *Numer. Methods Partial Differ. Equ.*, 35(1):222–245, 2019.

[2] I. Babuška and J. Osborn. Eigenvalue problems. In *Handbook of numerical analysis, Vol. II*, Handb. Numer. Anal., II, pages 641–787. North-Holland, Amsterdam, 1991.

[3] Jikun Zhao, Bei Zhang, Shipeng Mao, and Shaochun Chen. The divergence-free nonconforming virtual element for the Stokes problem. *SIAM J. Numer. Anal.*, 57(6):2730–2759, 2019.