Grade Table (for checker use only)

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Total:	90	

Team Members:

- ....
- ....
- ....

INSTRUCTIONS:

- **Write your team name on top of each page.**
- If you have any queries, contact an invigilator. Any sort of interaction with another team can lead to a penalty or disqualification.
- Submit any electronic devices that you possess, to one of the invigilators. You may collect them after the event. Any team caught using any electronic device will be immediately disqualified.
- Enough space has been provided in the question paper. Use it wisely. However, if you need extra sheets, contact an invigilator.

1. (10 points) Three distinct points with integer coordinates lie in the plane on a circle of radius $r > 0$. Show that these points are seperated by a distance of atleast $r^{1/3}$.

2. (10 points) For each positive integer n, determine the set of n distinct positive integers having the property that no subset of them adds up to a perfect square.

3. (10 points) At the vertices of a regular hexagon are written six non-negative integers whose sum is 2017^{2017}.
   Mansi is allowed to make moves of the following form: she may pick a vertex and replace the number written there by the absolute value of the difference between the numbers written at the two neighbouring vertices. Prove that Mansi can make a sequence of moves, after which the number 0 appears at all six vertices.

4. (10 points) You are presented with three large buckets, each containing an integral number of ounces of some non-evaporating fluid. At any time, you may double the contents of one bucket by pouring into it from a fuller one; in other words , you may pour from a bucket containing x ounces into one containing $y \leq x$ ounces until the latter contains $2y$ (and the former $x - y$). Prove that no matter what the initial contents, you can eventually empty one of the buckets.

5. (10 points) Suppose that a sequence $a_1, a_2, \ldots$ of positive real numbers satisfies

$$a_{k+1} \geq \frac{ka_k}{a_k^2 + (k-1)}$$

for every positive integer k. Prove that $a_1 + a_2 + \ldots + a_n \geq n$ for every $n \geq 2$.

6. (10 points) Find all functions $f\colon \mathbb{Q} \longrightarrow \mathbb{Q}$ satisfying

$$f(x) + f(t) = f(y) + f(z)$$

for all rational numbers $x < y < z < t$ that form an arithmetic progression.($\mathbb{Q}$ is the set of all rational numbers.)

7. (10 points) In a sports tournament of n players, each pair of players plays against each other exactly one match and there are no draws. Show that the players can be arranged in an order P_1, P_2,..., P_n such that P_i defeats P_{i+1} for all $1 \leq i \leq n$-1

8. (10 points) In triangle ABC, $\angle BAC = 94^\circ$, $\angle ACB = 39^\circ$. Prove that

$$BC^2 = AC^2 + AC \cdot AD$$

9. (10 points) **NOTE:** *This is a difficult question and is here only to demonstrate how powerful mathematics is. Attempt this when you have attempted all other questions.*

   In a game, one scores on a turn either a points or b points, a and b positive integers with $b < a$. Given that there are 35 non-attainable cumulative scores, and that one of them is 58, what are the values of a and b?
   (Cumulative scores have the form $ax + by$, x and y non-negative integers, obtained from scoring a on x turns and b on y turns.)