1) The expression \((1+q)(1+q^2)(1+q^4)(1+q^8)(1+q^{16})(1+q^{32})(1+q^{64})\), where \(q \neq 1\), equals
   A. \(\frac{1-q^{128}}{1-q}\);
   B. \(\frac{1-q^{64}}{1-q}\);
   C. \(\frac{1-q^{2^{1+2+\cdots+6}}}{1-q}\);
   D. None of the foregoing expressions

2) A boy walks from his home to school at 6 km per hour (kmph). He walks back at 2 kmph. His average speed, in kmph, is
   A. 3;
   B. 4;
   C. 5;
   D. \(\sqrt{12}\).

3) If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of G.P. is:
   A. \(\frac{7}{4}\);
   B. \(\frac{8}{5}\);
   C. \(\frac{4}{3}\);
   D. 1.

4) The value of \(\lim_{x \to \infty} (3^x + 7^x)^{\frac{1}{x}}\) is
   A. 7;
   B. 10;
   C. \(e^{7}\);
   D. \(\infty\)

5) Suppose that \(F(n + 1) = \frac{2F(n) + 1}{2}\) for \(n = 1, 2, 3, \ldots\), and \(F(1) = 2\). Then \(F(101)\) equals
   A. 50;
   B. 52;
   C. 54;
   D. None of the foregoing quantities.

6) The area in square units of the region described by \((x, y): y^2 \leq 2x\) and \(y \geq 4x - 1\) is:
   A. \(\frac{5}{64}\);
   B. \(\frac{15}{64}\);
   C. \(\frac{9}{32}\);
   D. \(\frac{7}{32}\)
7) The equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$
   A. Cannot have a common real root for any value of $a$;
   B. Have a common real root for exactly one value of $a$;
   C. Have a common real root for exactly two values of $a$;
   D. Have a common real root for exactly three values of $a$.

8) A man invests INR 10,000 for a year. Of this INR 4,000 is invested at the interest rate of 5% per year, INR 3,500 at 4% per year and the rest at $\alpha$% per year. His total interest for the year is INR 500. Then $\alpha$ equals
   A. 6.2;
   B. 6.3;
   C. 6.4;
   D. 6.5.

9) A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is
   A. \( \frac{5}{17} \);
   B. \( \frac{12}{17} \);
   C. \( \frac{17}{30} \);
   D. \( \frac{3}{5} \).

10) The equation of the circle circumscribing the triangle formed by the lines $y = 0$, $y = x$ and $2x + 3y = 10$ is
    A. $x^2 + y^2 + 5x - y = 0$;
    B. $x^2 + y^2 - 5x - y = 0$;
    C. $x^2 + y^2 - 5x + y = 0$;
    D. $x^2 + y^2 - x + 5y = 0$.

11) A salesman sold two pipes at INR 12 each. His profit on one was 20% and the loss on the other was 20%. Then on the whole, he
    A. Lost INR 1;
    B. Gained INR 1;
    C. Neither gained nor lost;
    D. Lost INR 2.

12) The integral
    \[ \int \frac{dx}{x^2(x^4 + 1)^{3/4}} \]
equals:
    A. $(x^4 + 1)^{1/4} + c$
    B. $-(x^4 + 1)^{1/4} + c$
    C. $-\left( \frac{x^4 + 1}{x^4} \right)^{\frac{1}{2}} + c$
    D. $\left( \frac{x^4 + 1}{x^4} \right)^{\frac{1}{2}} + c$
13) In an election, 10% of the voters on the voters’ list did not cast their votes and 60 voters cast their ballot papers blank. There were only two candidates. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. The number of voters on the lost was

A. 3600;
B. 6200;
C. 4575;
D. 6028.

14) For all angle \( A \) \[
\frac{\sin 2A \cos A}{(1+\cos 2A)(1+\cos A)}
\]
equals

A. \( \sin \frac{A}{2} \);
B. \( \cos \frac{A}{2} \);
C. \( \tan \frac{A}{2} \);
D. \( \sin A \)

15) There are 3 bags which are known to contain 2 white and 3 black balls; 4 white and 1 black balls and 3 white and 7 black balls respectively. A ball is drawn at random from one of the bags and found to be a black ball. Then the probability that it was drawn from the bag containing the most black balls is

A. \( \frac{7}{15} \);
B. \( \frac{5}{19} \);
C. \( \frac{3}{4} \);
D. None of these

16) If a, b, c, and d satisfy the equations

\[
\begin{align*}
    a + 7b + 3c + 5d &= 0, \\
    8a + 4b + 6c + 2d &= -16, \\
    2a + 6b + 4c + 8d &= 16, \\
    5a + 3b + 7c + d &= -16,
\end{align*}
\]

Then \( (a + d)(b + c) \) equals

A. 16;
B. -16;
C. 0;
D. None of the foregoing numbers.

17) IITK MTech admission test has 5 multiple choice questions with four choices with one correct answer in each. If you just randomly guess on each of the 5 questions, what is the probability that you get exactly 2 questions correct?

A. 0.625;
B. 0.25;
C. 0.0625;
D. 0.2636.
18) Let $y(x)$ be the solution of the differential equation

$$
(x \log x) \frac{dy}{dx} + y = 2x \log x, \,(x \geq 1). 
$$

Then $y(e)$ is equal to:
A. 0;
B. 2;
C. 2e;
D. e.

19) A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from the club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is
A. 380
B. 320
C. 260
D. 95

20) For a real number $x$, let $[x]$ denote the greatest integer less than or equal to $x$. Then the number of real solutions of $|2x - [x]| = 4$ is
A. 1;
B. 2;
C. 3;
D. 4.

21) The sum of the series $1 + 11 + 111 + \ldots$ to $n$ terms is
A. $\frac{1}{9} \left\{ \frac{10^n}{9} - 1 + n \right\}$;
B. $\frac{1}{9} \left\{ \frac{10^n}{9} - 1 - n \right\}$;
C. $\frac{10^n}{9} \left\{ \frac{1}{9} (10^n - 1) - n \right\}$;
D. $\frac{10^n}{9} \left\{ \frac{1}{9} (10^n - 1) + n \right\}$;

22) The sum of coefficients of integral powers of $x$ in the binomial expansion of $(1 - 2\sqrt{x})^{501}$ is
A. $\frac{1}{2} (3^{50})$
B. $\frac{1}{2} (3^{50} - 1)$
C. $\frac{1}{2} (2^{50} + 1)$
D. $\frac{1}{2} (3^{50} + 1)$

23) If $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_{10} 10$, then a possible value of $x$ is given by
A. 10;
B. 1/100
C. 1/1000
D. None of these
24) Consider the two arithmetic progressions 3, 7, 11, … , 407 and 2, 9, 16, … , 709. The number of common terms of these two progressions is
   A. 0;
   B. 7;
   C. 15;
   D. 14

25) Three coins are tossed. If one of them shows tail, then the probability that all three coins show tail, is
   A. $\frac{1}{7}$
   B. $\frac{1}{8}$
   C. $\frac{1}{7}$
   D. $\frac{1}{6}$

26) The equation $x^2y – 2xy + 2y = 0$ represents
   A. A straight line;
   B. A circle;
   C. A hyperbola;
   D. None of the foregoing curves.

27) The equation $x - \log_e(1 + e^x) = c$ has a solution
   A. For every $c \geq 1$;
   B. For every $c < 1$;
   C. For every $c < 0$;
   D. For every $c > -1$

28) Let $A$ be the fixed point $(0,4)$ and $B$ be a moving point $(2t,0)$. Let $M$ be the mid-point of $AB$ and let the perpendicular bisector of $AB$ meet the y-axis at $R$. The locus of the mid-point $P$ of $MR$ is
   A. $y + x^2 = 2$;
   B. $x^2 + (y - 2)^2 = 1/4$;
   C. $(y - 2)^2 - x^2 = 1/4$;
   D. None of the foregoing curves

29) A fair coin is tossed 99 times. Let $X$ be the number of times heads occurs. Then $P(X=r)$ is maximum when $r$ is
   (A) 49
   (B) 52
   (C) 51
   (D) None of these