ESC101 : Fundamental of computing

Tutorial sheet 4 **4 September, 2008**

Avoiding overflow by careful programming

Suppose we want to write a program to compute $\binom{n}{i}$. We may assume that $i <= n/2$, otherwise we may replace i by $n-i$. Furthermore, the values assigned to `n` and `i` are such that $\binom{n}{i}$ will have value well within the range of long. (This is not a hypothetical example. You may see the third programming challenge to be put on the website tomorrow to convince about its reality).

For the problem, the trivial way of computing $factorial(n), factorial(i), factorial(n-i)$ and then using multiplication and division won't work (even for $\binom{100}{2}$). Even the following approach will not work.

compute $x = n \times (n-1) \times \cdots \times (n-i+1)$.
compute $y = i \times (i-1) \times \cdots \times 2 \times 1$.
$z = x/y$;
system.out.println(z);

The reason is that at intermediate stages we are generating numbers which are too large to be stored in long (hence the result will be incorrect). So we should make sure that we NEVER generate any intermediate number which is larger than the final number to be output. One solution which meets this constraint is to use the following equation and apply the *cancellation* of factors whenever possible.

$$\binom{n}{i} = \frac{n}{i} \times \binom{n-1}{i-1}$$

Note that this equation follows from expressing z as x/y above. (So it should not appear to the students that it is based on any deep mathematical fact). If we have the value of $\binom{n-1}{i-1}$, how can we get the value of $\binom{n}{i}$ using the above equation without any overflow ?
First compute $\gcd(n, i)$, let it be g.
Replace n by n/g and i by i/g.
Divide $\binom{n-1}{i-1}$ by i. (note that it will be a complete division, that is, remainder will be 0)
For sake of completeness, I am providing the complete pseudocode below.

```
long n = 100;    long i = 4;
long x;         long y;
long nCr = n-i+1;
long count = 1;
while(count<i)
    {
                x = n-i+1+count;
                y = count+1;    // now compute gcd of x and y
                long big = x;
                long small = y;
                while(big%small!=0)
                {
                    long temp = big%small;
                    big = small;
                    small=temp;
                }
                long gcd = small;
                x = x/gcd;
                y = y/gcd;
                nCr = nCr/y;
                nCr = nCr*x;
                count = count+1;
    }
System.out.println("The value of nCi is "+nCr);
```

I am sure some of you or some student in the ESC101 class might come up with even simpler way to solve this problem. You may discuss that if you wish. However, please try to ensure that the students fully realize the objective behind this problem.