Proving correctness of Recursive methods

as part of Lecture 33

As an example, we consider the method PatternS(int n, int m, String S). We shall prove that this recursive method is indeed correct, that is, it prints all strings of the form S+P with n |'s and m ∗'s.

```
public static long PatternS(int n, int m, String S)
{ if(n==0 && m==0)
     System.out.println(S);
  else
  {
     if(n!=0) PatternS(n-1,m,S+'|');
     if(m!=0) PatternS(n,m-1,S+'*');
  }
}
```

The proof will require the knowledge of recursive formulation of the corresponding set **PatternS**(n,m,S) (discussed in details during the class) and the principle of mathematical induction.

Proof by Induction : First we have to formulate the inductive assertion appropriately. (Note that I have simplified the inductive assertion since some students were not feeling comfortable with inductive assertion with two variables)
Inductive Assertion :
$\mathcal{P}(k)$: For all pairs (n, m) with $n+m \leq k$ and any string S, the method PatternS(n, m, S) prints all strings of the form S+P where P consists of n |'s and m ∗'s.
Now the proof is similar to any other inductive proof.
Base Case : The base case is $\mathcal{P}(0)$. This corresponds to PatternS$(0, 0, S)$ since the only non-negaitve values of n, m with $n + m = 0$ are $n = m = 0$. There is only one string which is S in this case which should be printed. It can be seen from the code of the method that PatternS$(0, 0, S)$ prints S on the screen. Hence $\mathcal{P}(0)$ holds true.
Induction Step : We have to prove $\mathcal{P}(k)$ for some $k > 0$ given that $\mathcal{P}(k-1)$ holds. Consider any particular instance of n, m with $n + m = k$. Consider the execution of PatternS(n, m, S). It is easy to observe that we shall execute the code inside the `else` statement. Without loss of generality assume that both $n > 0$ and $m > 0$. (The other cases $n = 0, m = k$ and $n = k, m = 0$ can be handled similarly and are left as exercises for you.) It can be seen that PatternS(n,m,S) will execute PatternS(n-1,m,S) and PatternS(n,m-1,S) both. We can apply Induction Hypothesis $\mathcal{P}(k-1)$ for PatternS(n-1,m,S) as well as PatternS(n,m-1,S) to conclude that

- PatternS(n-1,m,S) will print all strings of the form S+'|'+P where P has n-1 |'s and m ∗'s.
- PatternS(n,m-1,S) will print all strings of the form S+'∗'+P where P has n |'s and m-1 ∗'s.

Now it follows from the recursive formulation of the set **PatternS**() that the set of strings printed by the two recursive calls mentioned above is indeed the set of all those strings which are of the form S+P where P has n |'s and m ∗'s. Hence $\mathcal{P}(k)$ holds. This completes the proof.

Just to make sure you have understood the above proof, try to write the proof of correctness for methods CombS(A,i,L,S) and PermuteS(A,i,L,S) given in the lecture notes.