

# Programming Challenge 4

## Physical balance

Your friend wants to set up a small scale industry which produces weights used in a physical balance. Physical balance is required to weigh a given quantity. See Figure 1 to understand how weights can be arranged on the pans to weigh a given quantity.

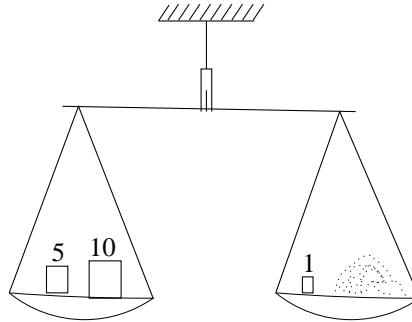


Figure 1: Physical balance : using weights of values 1,5,10 to weigh an object of weight 14

His customers are the shopkeepers located in his state. Each shopkeeper wants a set of weighing objects using which he can produce all integer weights in range  $[1, n]$ , where  $n$  may vary for each shopkeeper. For a small grocery shop,  $n$  could be upto 50 Kg, and for a whole sale shopkeeper  $n$  could be 1000 Kg. For a given  $n$ , a trivial solution is that the shopkeeper keeps  $n$  weighing objects each of weight one kilogram. But it is very cumbersome to use and manage so many weighing objects. So he wants the minimum number of weighing objects using which he can weigh any object whose weight is in the range  $[1, n]$ . Such a set of weighing objects is called a *Min-weighing-objects*( $n$ ) set. Such set need not be unique. For example,  $\{1, 3, 5\}$  as well as  $\{1, 2, 6\}$  can serve to be a *Min-weighing-objects*(9) set.

Your friend wants to expand his business. So he wants to ensure that his customers get full satisfaction. In particular, for a given  $n$ , he would like to present to the shopkeeper all possible *Min-weighing-objects*( $n$ ) sets. The shopkeeper can place an order for the set of his choice. Finally your friend delivers weights of that set to the shopkeeper. Your friend is unable to come up with any solution to this problem. But, he is aware that you are very brilliant, and that you are studying in IIT Kanpur. He approaches you with this problem. Solve this problem. More formally, he wants you to write a program which for a positive integer  $n$ , enumerates all *Min-weighing-objects*( $n$ ) sets.

Provide a proof of correctness for your program. At least you should provide an outline/sketch of the proof.