Hierarchical DWT-based optimal diversity power allocation for video transmission in OFDMA/MIMO wireless systems

Sohil Mahajan and Aditya K. Jagannatham*

Electrical Engineering Department,
Indian Institute of Technology Kanpur,
Kanpur, 208016, India
E-mail: sohil@iitk.ac.in
E-mail: adityaj@iitk.ac.in
*Corresponding author

Abstract: In this paper, we propose novel algorithms for optimal power allocation towards video transmission in orthogonal frequency division multiple access (OFDMA) and multiple-input multiple-output (MIMO)-based 4G networks. A unique feature of the schemes is that they employ diversity dependent subcarrier power distribution relying on a discrete wavelet transform (DWT)-based hierarchical video decomposition, with exclusively order statistics-based partial channel state information (CSI). This is formulated as a convex minimisation problem and closed form expressions are derived for diversity-based optimal power allocation. We also propose novel algorithms for diversity order-based optimal power allocation for video transmission in MIMO systems. Further, we consider a practical MIMO scenario and employ a limited feedback technique utilising the index of the quantised beamforming vector. A closed form expression for the optimal power allocation is presented based on an iterative convex optimisation framework. Simulation results demonstrate superior performance of the proposed optimal power allocation schemes.

Keywords: orthogonal frequency division multiple access; OFDMA; multiple-input multiple-output; MIMO; convex optimisation, optimal power allocation; OPA; wireless video transmission.

Reference to this paper should be made as follows: Mahajan, S. and Jagannatham, A.K. (2012) ‘Hierarchical DWT-based optimal diversity power allocation for video transmission in OFDMA/MIMO wireless systems’, *Int. J. Internet Protocol Technology*, Vol. 7, No. 1, pp.39–51.

Biographical notes: Sohil Mahajan received his Bachelors and Masters degree in Electrical Engineering from the Indian Institute of Technology Kanpur. From June 2011 till date he is employed as a Software Engineer at Ericsson Global, RnD., Gurgaon. His research interests are in the area of wireless communication, digital video processing for wireless systems, information theory and probability.

Aditya K. Jagannatham received his Bachelors degree from the Indian Institute of Technology, Bombay and MS and PhD from the University of California, San Diego, USA. From 2007 to 2009, he was employed as a Senior Wireless Systems Engineer at Qualcomm Inc., San Diego, California, where he worked on developing 3G UMTS/WCDMA/HSDPA mobile chipsets. His research interests are in the area of next-generation wireless communications and networking, sensor and ad-hoc networks, digital video processing for wireless systems, wireless 3G/4G cellular standards and CDMA/OFDM/MIMO wireless technologies. He received the Upendra Patel Achievement Award for his efforts towards developing HSPA+ WCDMA technologies at Qualcomm. Since 2009, he has been a faculty member at IIT Kanpur and is also associated with the BSNL-IITK Telecom Centre of Excellence (BITCOE).

This paper is a revised and expanded version of a paper entitled ‘Hierarchical DWT based optimal diversity power allocation for video transmission in 4G OFDMA wireless systems’ presented at Australasian Telecommunication Networks and Applications Conference 2011 (ATNAC 2011), Melbourne, Australia, 9–11 November 2011.

1 Introduction

With the advent of fourth generation (4G) wireless standards such as WiMAX and LTE, broadband video transmission with robust quality of service (QoS) has become a reality. Reliable video communication is essential for key applications such as surveillance, video conferencing, mobile gaming, etc. Orthogonal frequency division multiple access (OFDMA) and multiple-input multiple-output (MIMO) are the most attractive physical layer technologies for 4G broadband wireless networks due to their robustness to inter symbol interference arising from multipath interference combined with a low complexity IFFT/FFT-based implementation and high data throughput of such systems resulting from the spatial-multiplexing of multiple information streams in parallel without the necessity of additional bandwidth. Hence, OFDMA/MIMO is suitable for transmitting high data rates over wireless links, one of the key requirements for video transmission. However, video transmission in such scenarios is challenging, particularly due to the fading nature of wireless channels resulting in severe degradation of video quality.

To achieve high quality video transmission in an OFDMA system, it is critical to optimally distribute the limited power resources amongst the subcarriers. One such power allocation based on the water filling algorithm was examined in Kim et al. (2004). In Oh et al. (2004), the authors propose an iterative power allocation method to minimise power usage in OFDM for multiple access channels. These are optimal solutions for transmitting generic data, and are not specifically targeted at minimising the overall video distortion. Hence, one is motivated to develop optimal schemes, especially suited for video transmission.

Specific properties of the video stream can be exploited to minimise distortion in video transmission. Cross-layer resource allocation techniques, such as in Ha et al. (2008), have been proposed for video communications. The scheme in Zhao et al. (2009) uses joint power allocation and antenna selection for scalable video delivery over MIMO OFDMA systems. In our work, we consider a novel ordered subcarrier diversity-order dependent power allocation relying on a DWT-based hierarchical decomposition of the video stream. We motivate this paradigm for video transmission in wireless systems, since it naturally merges the hierarchical layering of video data with that of the graded reliability associated with the diversity order of the wireless channel.

We formulate the above problem as the minimisation of a constrained convex video distortion objective function and demonstrate that the optimal power allocation problem reduces to one of polynomial root computation. Further, the proposed scheme employs only partial CSI feedback, which significantly reduces the overhead on the reverse link, thus making it attractive for practical implementation. Simulation results for OFDMA-based wireless transmission employing several video sequences demonstrate that the proposed schemes outperform equal power allocation in terms of both PSNR and visual quality.

Moreover, MIMO video transmission is also challenging, particularly due to the time varying nature of wireless channels combined with the lack of channel state information (CSI) at the transmitter. In this context, employing the singular value decomposition (SVD) of the MIMO channel matrix **H**, a MIMO system can be decomposed as a set of parallel spatial channels corresponding to the singular modes. To maximise the throughput across the MIMO channel, one needs to optimally allocate the limited power resources across the different spatial channels, followed by transmit beamforming. Hence, to achieve high quality video transmission in a MIMO system, it is critical to optimally distribute the limited power resources amongst the singular modes of the MIMO channel. Water filling algorithm-based optimal schemes (Tse and Viswanath, 2005) have been proposed for power allocation in MIMO systems, which are not specifically optimised for video transmission. The scheme in Sabir et al. (2010) utilises a quality layers-based unequal power allocation scheme for JPEG transmission in MIMO systems based on a heuristic algorithm. Other empirical schemes for unequal power distribution between the channel singular modes have been studied for video transmission in works such as Tesanovic et al. (2008), but these schemes are adhoc and depend on a scaling parameter. Hence, there is a significant dearth of precise analytical schemes especially suited for MIMO video transmission.

Moreover, in a practical MIMO wireless scenario, obtaining perfect CSI at the transmitter is unrealistic. In 4G wireless systems, a limited number of bits are assigned for feedback on the reverse link. In such scenarios, it is key to design an optimal quantised feedback-based MIMO power allocation scheme to maximise the video transmission quality. Authors in Zhao et al. (2009) propose an algorithm for subcarrier power allocation and antenna selection for scalable video delivery in MIMO-OFDM systems based on an order feedback mechanism which is limited in scope as it results in a loss of coding gain. A novel feedback scheme for MIMO systems which yields a significant performance enhancement is codebook-based feedback of the quantised MIMO beamforming vectors (Roh and Rao, 2004). After each channel estimation epoch at the receiver, the indices of quantised vectors belonging to the codebook and closest to the transmit beamforming vectors in the mean-squared inner product (MSIP) norm are fed back to the transmitter. Such a scheme has the dual advantage of requiring only a few feedback bits on the reverse channel, while achieving a performance very close to that of perfect CSI at the transmitter. An added advantage is that such a fixed codebook of beamforming vectors can be constructed offline, thus greatly limiting the computational complexity of the scheme.

Hence, motivated by the above discussion, we propose a framework for optimal MIMO transmit power allocation employing codebook-based limited feedback in the context of video transmission. For this purpose, we consider hierarchical spatio-temporal layering of the video frames followed by MIMO SVD-based substream allocation. We develop an analytical framework for diversity-order dependent power allocation in MIMO systems based on the average video distortion minimisation criterion. The proposed framework is robust, since it also takes into consideration the distortion in the transmission due to multistream interference arising out of the limited CSI feedback. We formulate the above optimisation paradigm as a constrained cost optimisation. Following this, we demonstrate that the optimal power allocation can be computed by solving an iterative sequence of video distortion-based constrained convex objective minimisation problems. We provide a closed form expression for the optimal power allocation at each iterative step above based on the solution of a polynomial root computation. Simulation results for video transmission in quantised feedback-based MIMO wireless systems employing several video sequences demonstrate that the proposed schemes outperform equal power allocation in terms of both PSNR and visual quality.

The subsequent sections of this paper are organised as follows. We begin with a formulation of our diversity-based subcarrier allocation scheme for OFDMA wireless systems in the next section and present a framework to characterise the probability of bit-error for the proposed transmission scheme. Subsequently, in Section 3 we propose the diversity-based hierarchical allocation (DHA) scheme for video transmission. This scheme employs DWT filtering for spatio-temporal hierarchical decomposition of the video sequence. This framework is utilised to formulate an optimisation criterion based on a convex objective function for optimal power allocation (OPA) amongst the OFDMA subcarriers in Section 4. A closed form power allocation (CFPA), with performance close to OPA and significantly lower computational complexity is also proposed. Next we begin with a description of the MIMO system model and quantised beamforming vector codebook construction in the next section. The expression for the mode received SNR for this paradigm of quantised feedback-based transmit beamforming is presented subsequently. In Section 6, we describe a singular mode diversity order-based hierarchical video transmission (DHVT) scheme. The distortion can be further reduced through optimal power allocation-based video transmission (OPVT) and we derive an optimisation framework towards achieving this objective in Section 7. It is demonstrated therein that the optimal power allocation for the MIMO substreams can be computed as the solution of an iterative series of convex objective minimisation problems. Simulation results are given in Section 8 to illustrate the performance gains of the proposed DHA, OPA and CFPA schemes over equal power allocation (EPA) and of the proposed DHVT and OPVT schemes over the suboptimal single layer video transmission (SLVT) in OFDMA and MIMO systems respectively.

2 Order diversity in OFDMA

In this section, we propose a diversity-based subcarrier characterisation scheme for video transmission in OFDMA systems. Consider an OFDMA system of bandwidth W, for data transmission over a multipath Rayleigh fading channel with L independent fading multipath components. The time-domain baseband system model for the above system can be described for sampling instants $0 \le m \le N + L_{cp} - 1$ as,

$$y(m) = \sum_{l=0}^{L-1} h(l)x(m-l) + v(m),$$

where N is the number of subcarriers in the OFDMA system and L_{cp} is the cyclic prefix (CP) length.

After CP removal and FFT at the receiver, the resulting system can be expressed as,

$$Y_k = H_k X_k + V_k, \quad 0 \le k \le N-1 \tag{1}$$

where X_k and Y_k represent the data transmitted and received respectively through subcarrier k. It is known (van Nee and Prasad, 2000) that the subcarrier gains $[H_0, H_1, \ldots, H_{N-1}]^T$ for the above OFDMA system are obtained through an N-point DFT of the L-tap multipath wireless channel $\mathbf{h} \triangleq [h(0), h(1), \ldots, h(L-1)]^T$, where each path gain $h(i)$ is an independent Rayleigh fading complex channel coefficient. We denote by $[V_0, V_1, \ldots, V_{N-1}]^T$, the N-point DFT of the noise vector $\mathbf{v} = [v(0), v(1), \ldots, v(N-1)]^T$ where the noise $\mathbf{v} \sim \mathcal{N}(0, \sigma_v^2 \mathbf{I}_N)$ is complex additive white Gaussian with covariance matrix $E(\mathbf{v}\mathbf{v}^H) = \sigma_v^2 \mathbf{I}_N$.

In the context of the above model, we consider an OFDMA system with N subcarriers and K video users such that N_u subcarriers are allocated to each user. Note that $KN_u < N$, implying that there are other users in the system with applications not based on video transmission (such as voice, data, etc.) occupying the rest of the subcarriers. To minimise the video distortion, it is critical to appropriately distribute layers of the video stream amongst the N_u subcarriers allotted to a particular user and further to optimally allocate power to each of the subcarriers.

Consider the N_u independent Rayleigh fading subcarriers allotted to a particular user as per the property described in the Appendix. The CSI related to the instantaneous fading state of the wireless channel is unknown to the transmitter. Explicit feedback of the channel coefficients $H(i)$ requires a large number of feedback bits. We therefore motivate an order feedback of indices $[i]$, $i \in \{1, 2, \ldots, N_u\}$ such that $|H_{[i]}| > |H_{[j]}|$ for $i > j$. For this purpose, the N_u subcarrier gains for each user are ordered at the receiver as follows,

$$\left|H_{[N_u]}\right| \ge \left|H_{[N_u-1]}\right| \ge \cdots \ge \left|H_{[2]}\right| \ge \left|H_{[1]}\right|.$$

Subsequently, the receiver only feeds back this ordering information of the user subcarrier gains to the transmitter. Thus, since an elaborate feedback of the exact subcarrier gains is avoided, this scheme results in a significantly lower overhead on the reverse link. For instance, in the case of $N_u = 2$, only 1 bit is needed to be fed back by the receiver.

Hence, our order statistics-based partial feedback scheme results in a significant bit saving compared to a bit-expensive explicit feedback containing exact values of the channel coefficients. Below we present the closed form expression for the probability of bit-error for the above ordered subcarriers-based transmission scheme.

Theorem 1: The bit-error rate (BER) corresponding to the $[n]^{\text{th}}$ ordered subcarrier has a diversity order n. Further, the probability of bit-error ϕ_e for binary phase-shift keyed (BPSK) data transmission through this subcarrier can be expressed as,

$$\phi_e(P_n) = \sum_{i=n}^{\infty} \alpha_{ni} \left(\frac{\sigma_v^2}{P_n} \right)^i . \tag{2}$$

where the coefficient α_{ni} in the above expression is given as,

$$\alpha_{ni} \triangleq \sum_{k=0}^{n-1} \binom{n-1}{k} \binom{N_u}{n} n a_i 2^{i-1} \left(k + N_u - n + 1\right)^{i-1} (-1)^{k+1}$$

and $a_i = \frac{(2i)!(-1)^i}{2^{2i}(i!)^2}$, P_n is the allocated power to the $[n]^{\text{th}}$ ordered subcarrier and σ_v^2 is the variance of the additive white Gaussian noise.

Proof: Let the random variable U_n be defined as the gain of the $[n]^{\text{th}}$ ordered subcarrier, i.e., $U_n \triangleq |H_{[n]}|^2$. From the independence property of the subcarrier gains described in the previous section, the probability density $f_{U_n}(u_n)$ corresponding to the gain of the $[n]^{\text{th}}$ ordered subcarrier can be obtained as,

$$p_{U_n}(u_n) = n \binom{N_u}{n} \left(1 - e^{-u_n}\right)^{n-1} \left(e^{-u_n}\right)^{N_u - n + 1} .$$

A more elaborate discussion on order statistics can be found in David and Nagaraja (2003). The average probability of bit-error for BPSK transmission can be obtained in a straight forward manner as,

$$\phi_e(P_n) = \int_0^{\infty} Q\left(\sqrt{u_n \left(\frac{P_n}{\sigma_v^2} \right)} \right) p_{U_n}(u_n)\, du_n \tag{3}$$

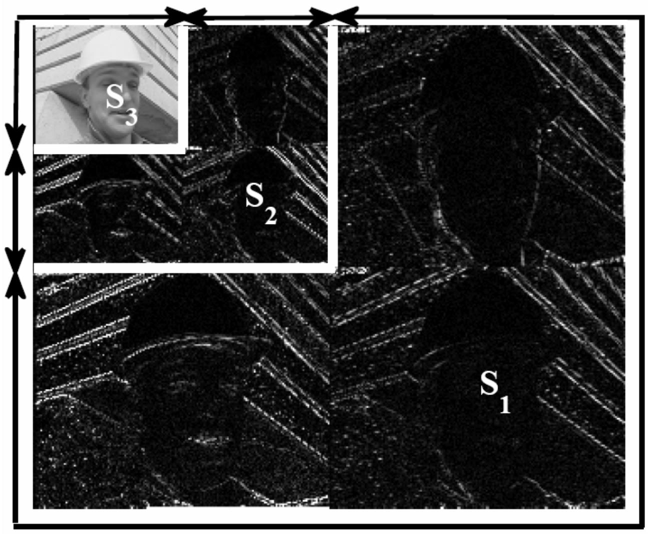
Substituting the probability density $p_{U_n}(u_n)$ from (3), followed by further simplification results in the BER expression above. ■

The above polynomial form of expression in $\left(\frac{1}{P_n} \right)$ can be readily seen to have a diversity order of n, since the lowest power of $\left(\frac{\sigma_v^2}{P_n} \right)$ in the above polynomial is n.

3 Diversity-based hierarchical allocation (DHA) for video

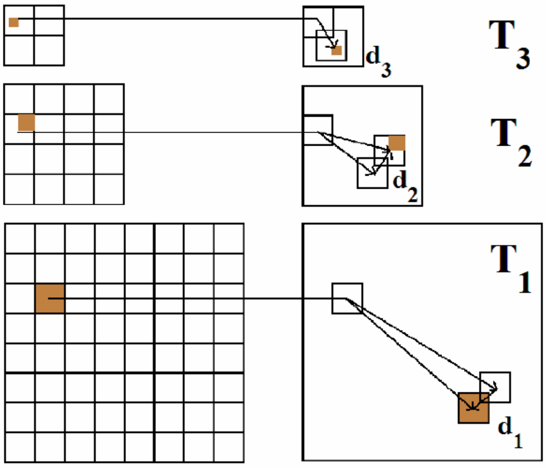
In this section, we propose a scheme for hierarchical decomposition of the given video bit-stream into N_l spatio-temporal layers. For spatial layering, a discrete wavelet transform (DWT) (Bovik, 2005)-based multi-resolution decomposition can be employed to decompose the intra-coded (I) frames into a spatial base layer S_{N_l} corresponding to the spatial low-frequency component and several enhancement layers S_i, $i \in 1, 2,, \ldots, N_l - 1$ corresponding to spatial high-frequency components. An example of such a 3-layer DWT based decomposition is shown in Figure 1. The base-layer content S_3 of the I frame is shown in the top left corner of the transformed frame, which is naturally the layer of highest significance for frame decoding as compared to the enhancement blocks of the frame.

Figure 1 Spatial decomposition of a frame into three layers



Similarly, in the temporal domain, HBMA (Wang et al., 2001)-based hierarchical motion estimation and compensation can be employed to decompose the motion vectors of target P, B frames into corresponding base and enhancement components. The base layer motion component T_{N_l} is computed from a direct comparison of base resolutions of the target and anchor frames. Subsequently, these motion vector estimates at the lowest spatial resolution are propagated down the pyramidal structure of higher resolution images, with the progressive addition of enhancement motion vector estimates T_j, $j \in 1, 2, \ldots, N_l - 1$. The resulting motion vector is a sum of the corresponding motion vectors from each level of the pyramid. An example of such motion estimation is shown in Figure 2 and described in detail in Wang et al. (2001). The motion vector in the base layer T_{N_l}, clearly has a higher impact on video integrity when compared to the enhancement components T_j, $j \neq N_l$ at higher resolutions in the pyramid.

Figure 2 Temporal decomposition of a frame into three layers (see online version for colours)



Thus, the video data can be naturally organised into hierarchical layers of varied significance. One can use this property of video advantageously for optimal ordered subcarrier diversity-based power allocation. The user video sequence can be hierarchically decomposed such that $N_l = N_u$. The data layers with highest priority (S_{N_l}, T_{N_l}) are transmitted on the $[N_u]^{\text{th}}$ ordered subcarrier, i.e., the subcarrier with the largest gain. Each spatial and temporal layer of progressively lower significance S_l, T_l is transmitted on the $[l]^{\text{th}}$ ordered subcarrier (i.e., the lower ordered subcarriers). Thus, DHA combines spatio-temporal video layering with diversity-based wireless communications to significantly minimise video distortion. The next section further minimises the transmission distortion by optimally allocating the limited transmission power.

4 Optimal and closed form power allocation

Employing the diversity-based hierarchal video transmission scheme motivated in the above sections, we further minimise the overall video distortion through optimal power allocation amongst the N_u ordered subcarriers for a given user. For a given spatio-temporal video sequence $\mathcal{V}(x, y, t)$, the distortion $D_l(\mathcal{V})$ arising out of erroneous reception of the bit-stream corresponding to layer $l \in 1, 2, \ldots, N_u$ can be derived as,

$$D_l(\mathcal{V}) = D_l^S(\mathcal{V}) + D_l^T(\mathcal{V}), \tag{4}$$

where $D_l^S(\mathcal{V})$, $D_l^T(\mathcal{V})$, are the spatial and temporal distortion coefficients corresponding to layer l. The component $D_l^S(\mathcal{V})$ is defined as $E(\|\psi^{-1}(\Delta S_l)\|^2)$, where ψ is the wavelet employed for the spatial-layering DWT and ΔS_l is the error corresponding to the spatial layer S_l. The temporal distortion coefficient $D_l^T(\mathcal{V})$ is obtained as, $D_l^T(\mathcal{V}) = E(\|\mathcal{V}(x,y) - \mathcal{V}(x+\Delta x_l, y+\Delta y_l\|^2)$, where $(\Delta x_l, \Delta y_l)$ is the MV component corresponding to layer l. Hence, employing the expression for the probability of error $\phi_e(P_l)$ given in (2), the mean overall distortion $D(\mathcal{V})$ for video transmission can be expressed as $D(\mathcal{V}) = \sum_{l=1}^{N_u} D_l(\mathcal{V})\phi_e(P_l)$. The optimisation problem for minimising overall video distortion $D(\mathcal{V})$ for a video stream $\mathcal{V}$ is described as:

$$\begin{aligned} \min. \quad & \sum_{l=1}^{N_u}\sum_{i=l}^{\infty} \alpha_{li} D_l(\mathcal{V}) \left(\frac{\sigma_v^2}{P_l}\right)^i \\ \text{s.t.} \quad & \sum_{l=1}^{N_u} P_l = P_T \\ & P_l \geq 0, \quad 1 \leq l \leq N_u \end{aligned} \tag{5}$$

It can be observed that the distortion function $D(\mathcal{V})$ is a polynomial in $\left(\frac{1}{P_l}\right)$ and hence convex. The parameters $D_l(\mathcal{V})$, which are fixed for a video sequence $\mathcal{V}$ can be obtained through an offline computation. Further, the optimal power allocation (OPA) vector $\bar{P}^\star \triangleq [P_1^\star, P_2^\star, \ldots, P_{N_u}^\star]^T$ for this problem can be computed using the primal-dual interior point method (Boyd and Vandenberghe, 2004). By approximating the distortion cost employing the most significant term corresponding to the layer diversity order in the overall distortion function $D(\mathcal{V})$ one can obtain a simplified objective function, with much lower computational complexity.

Thus, the optimisation paradigm simplifies to,

$$\begin{aligned} \min. \quad & \sum_{l=1}^{N_u} B_l(\mathcal{V}) \left(\frac{\sigma_v^2}{P_l}\right)^l \\ \text{s.t.} \quad & \sum_{l=1}^{N_u} P_l = P_T \\ & P_l \geq 0, \quad 1 \leq l \leq N_u \end{aligned} \tag{6}$$

where the constant $B_l(\mathcal{V}) \triangleq \alpha_{ll} D_l(\mathcal{V})$. The standard Lagrangian cost function $\mathrm{L}(\mathcal{V}, \lambda, \mu)$ can be formulated for the above optimisation problem as,

$$\sum_{l=1}^{N_u} B_l(\mathcal{V}) \left(\frac{\sigma_v^2}{P_l}\right)^l + \lambda \left(\sum_{j=1}^{N_u} P_j - P_T\right) - \sum_{k=1}^{N_u} \mu_k P_k \tag{7}$$

Assuming that $\mu_k = 0$, the KKT conditions for the above optimisation problem can be readily obtained as,

$$\tilde{\lambda}^\star = \left(\frac{l B_l(\mathcal{V})\left(\sigma_v^2\right)^l}{\left(\tilde{P}_l^\star\right)^{l+1}}\right) \geq 0, \ \tilde{\lambda}^\star \left(\sum_{j=1}^{N_u} \tilde{P}_j^\star - P_T\right) = 0$$

The optimal dual variable $\tilde{\lambda}^{\star}$ can be obtained as the solution of the polynomial equation,

$$\sum_{l=1}^{N_u}\left(\frac{lB_l(\mathcal{V})\left(\sigma_v^2\right)^l}{\tilde{\lambda}^{\star}}\right)^{\frac{1}{l+1}} = P_T. \quad (8)$$

Hence, the optimal subcarrier power $\tilde{P}_l^{\star}$ corresponding to the modified optimisation problem in (6) can be computed from the optimal dual variable $\tilde{\lambda}^{\star}$ as,

$$\tilde{P}_l^{\star} = \left(\frac{lB_l(\mathcal{V})\left(\sigma_v^2\right)^l}{\tilde{\lambda}^{\star}}\right)^{\frac{1}{l+1}}.$$

Thus, the closed form optimal power allocation (CFPA) vector $\tilde{P}^{\star}$ can be obtained as the solution of a polynomial root finding problem (8). The allocation computed from the above simplified optimisation problem performs close to the OPA vector $\bar{P}^{\star}$ computed from (5) as demonstrated in the simulation results. Both the above schemes yield a significant performance enhancement compared to the sub-optimal equal power allocation (EPA) scheme corresponding to the power allocation vector $\hat{P} = \left(\frac{P_T}{N_u}\right)[1,1,\ldots 1]^T$ without diversity consideration.

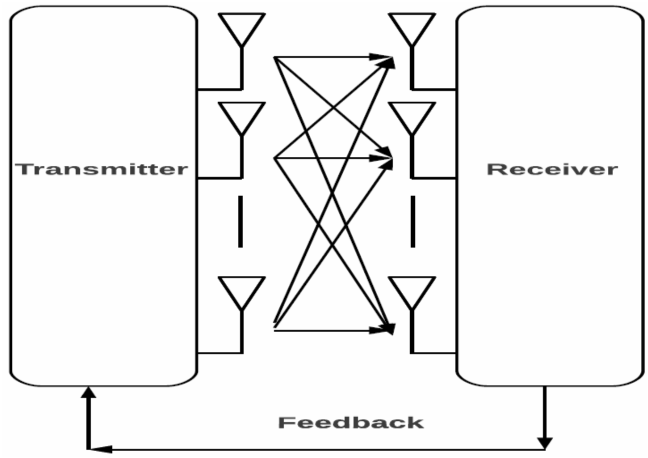
5 MIMO system model

A schematic of a MIMO wireless system with multiple transmit and receive antennas is shown in Figure 3. We consider a MIMO system with t transmit antennas and r receive antennas, modeled as,

$$\mathbf{y}(k) = \mathbf{H}\mathbf{x}(k) + \eta(k),$$

where $\mathbf{x}(k) \in \mathbb{C}^{t\times 1}$ and $\mathbf{y}(k) \in \mathbb{C}^{r\times 1}$ are the transmitted and received signal vectors respectively. The element h_{ij}, $1 \le i \le r$, $1 \le j \le t$ of the channel matrix $\mathbf{H} \in \mathbb{C}^{r\times t}$ is the flat-fading channel coefficient between the j^{th} transmit antenna and i^{th} receive antenna. These complex coefficients h_{ij} are IID Rayleigh distributed. The vector $\eta \sim \mathbb{C}\mathcal{N}\left(0, \sigma_\eta^2 \mathbf{I}_r\right)$ is complex additive spatio-temporally white Gaussian noise with covariance matrix $E\left(\eta\eta^H\right) = \sigma_\eta^2 \mathbf{I}_r$. The SVD of the channel matrix $\mathbf{H}$ defined above is given as $\mathbf{H} = \mathbf{U}\sum\mathbf{W}^H$ where $\mathbf{U} \in \mathbb{C}^{r\times r}$ and $\mathbf{W} \in \mathbb{C}^{t\times t}$ are unitary matrices and $\sum \in \mathbb{R}^{r\times t}$ is a diagonal matrix, whose elements are the non-negative ordered singular values $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_m \ge 0$ of $\mathbf{H}$, where $m \triangleq \min(r, t)$.

Figure 3 A schematic of MIMO system with feedback



It has been demonstrated in literature (Scaglione et al., 2002) that the capacity of the MIMO system can be maximised by performing transmit beamforming along the columns of the right singular matrix, i.e., $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m]$ combined with optimal power allocation amongst the singular modes. The resulting MIMO system model for transmit beamforming, corresponding to beamforming the symbols $\tilde{\mathbf{x}}(k) = [\tilde{x}_1(k), \tilde{x}_2(k), \ldots, \tilde{x}_m(k)]^T$ along the right singular vectors can be described as,

$$\mathbf{y}(k) = \mathbf{H}\mathbf{W}\tilde{\mathbf{x}}(k) + \eta(k), \quad (9)$$

where $E\{\tilde{x}_k^H \tilde{x}_k\} = P_k$, the power allocated to the k^{th} singular mode. For symbol detection, the receiver employs receive beamforming with the left singular matrix $\mathbf{U}$ as $\tilde{\mathbf{y}}(k) = \mathbf{U}^H \mathbf{y}(k)$ to decompose the received vectors $\mathbf{y}(k)$ into m parallel substreams corresponding to the transmit streams $\tilde{x}_j, 1 \le j \le m$.

However, in a practical MIMO wireless system, the transmitter does not possess knowledge of the CSI $\mathbf{W}$, $\sum$ since the channel is estimated at the receiver. Hence, the channel estimate has to be relayed from the receiver to transmitter, termed as *feedback* in the context of wireless communications. Further, exact feedback of the coefficients of each of the transmit beamforming vectors $\mathbf{w}_j$, incurs a high bit-rate overhead on the reverse link. Thus, quantised beamforming vector-based limited feedback schemes, where the receiver quantises each vector $\mathbf{w}_j$, $1 \le j \le m$ as,

$$\hat{\mathbf{w}}_j = \mathcal{Q}\left(\mathbf{w}_j\right) \in \mathcal{C}, \quad 1 \le j \le m,$$

have gained immense popularity. The quantised vectors $\hat{\mathbf{w}}_j$ belong to the codebook $\mathcal{C}$ and the receiver feeds back the index of the quantised beamforming vector, thereby greatly limiting the number of feedback bits required. In the section below we describe the algorithm for the transmit beamforming vector codebook construction. In the discussion that follows, the inner product between two vectors $\mathbf{v}_i$, $\mathbf{v}_j$, denoted by $\langle \mathbf{v}_i, \mathbf{v}_j \rangle$ is defined as,

$$\langle \mathbf{v}_i, \mathbf{v}_j \rangle \triangleq \mathbf{v}_i^H \mathbf{v}_j.$$

5.1 Beamforming vector codebook

We employ the mean-squared inner product (MSIP)-based nearest-neighbour scheme elaborated in Roh and Rao (2004) for codebook construction and quantisation of the transmit beamforming vectors $\mathbf{w}_j$. Let B denote the number of feedback bits per singular mode on the reverse link. Employing the scheme in the work above, we construct a vector codebook $\mathcal{C}$ of $N = 2^B$ unit-norm quantised vectors. The vector quantiser $\mathcal{Q}(\cdot)$ is designed such that it maximises the mean squared inner-product (MSIP) $E|\langle \mathbf{v}, \mathcal{Q}(\mathbf{v})\rangle|^2$. This approach has been naturally shown to result in maximising the gains resulting from transmit beamforming employing the quantised beamforming vectors. This codebook construction scheme is based on the Lloyd's algorithm for optimal quantiser design and iteratively computes the Voronoi regions R_i and the codebook vectors $\mathbf{c}_i \in \mathcal{C}$, $1 \le i \le N$.

- *Nearest neighbourhood computation:* Let the codebook $\mathcal{C}^{(k)}$ at the k^{th} iteration comprise of the N quantiser beamforming vectors $\{\mathbf{c}_1^{(k)}, \mathbf{c}_2^{(k)}, \ldots, \mathbf{c}_N^{(k)}\}$. The N Voronoi regions $R_i^{(k)}$ corresponding to the vectors $\mathbf{c}_i^{(k)}$, $1 \le i \le N$ at the k^{th} iteration are computed as,

$$\left\{ \mathbf{v} \in \mathbb{C}^{t \times 1} : \| \mathbf{v} \| = 1 \text{ and } \left| \left\langle \mathbf{v}, \mathbf{c}_i^{(k)} \right\rangle \right| \ge \left| \left\langle \mathbf{v}, \mathbf{c}_j^{(k)} \right\rangle \forall j \ne i \right| \right\}.$$

- *Centroid computation:* For the given set of Voronoi regions $R_i^{(k)}$, $1 \le k \le N$ at the end of the $k^{(k)}$ iteration above, we compute the optimum quantisation vectors in the $(k + 1)^{\text{th}}$ iteration as follows. Consider the matrix $\mathbf{G}_i^{(k)}$ defined as $\mathbf{G}_i^{(k)} = E\left(\mathbf{v}\mathbf{v}^H\right)$, $\mathbf{v} \in R_i$. The quantisation vectors $\mathbf{c}_i^{(k+1)}$ can be computed as the solution to,

$$\mathbf{G}_i^{(k)} \mathbf{c}_i^{(k+1)} = \lambda_{\max} \mathbf{c}_i^{(k+1)}, \left\| \mathbf{c}_i^{(k+1)} \right\| = 1,$$

where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{G}_i^{(k)}$. In other words, $\mathbf{c}_i^{(k+1)}$ is the principal unit-norm eigenvector of $\mathbf{G}_i^{(k)}$. This process is iterated until $E|\langle \mathbf{v}, \mathcal{Q}(\mathbf{v})\rangle|^2$ converges. In practice, this algorithm for codebook computation can be implemented very efficiently by choosing a large ensemble of vectors $\tilde{\mathbf{v}}_i \in \tilde{\mathbf{V}}$, $1 \le i \le J$, where J is a large number and the vectors $\tilde{\mathbf{v}}_i$ are chosen randomly according to the distribution of the transmit beamforming vectors $\mathbf{w}_j$, $1 \le j \le m$. The final codebook $\mathcal{C}$ obtained as a result of the above iterative process is denoted by $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$.

5.2 Codebook-based transmit beamforming

The codebook $\mathcal{C}$ constructed through the scheme described above is utilised to quantise each beamforming vector $\mathbf{w}_i$ at the receiver as $\hat{\mathbf{w}}_j = \mathbf{c}_j = \mathcal{Q}(\mathbf{w}_i)$ where,

$$j = \mathcal{I}\left\{\mathcal{Q}(\mathbf{w}_i)\right\} = \arg \max_{1 \le l \le N} \left| \left\langle \mathbf{w}_i, \mathbf{c}_l \right\rangle \right|.$$

The quantity $\mathcal{I}\{\mathcal{Q}(\mathbf{v})\}$ represents the index of the quantised vector in the codebook $\mathcal{C}$ and is fed back to the transmitter on the reverse link. The MIMO quantisation codebook $\mathcal{C}$ is known to the transmitter since it is computed offline and depends only on the ensemble statistical information and not on the instantaneous time-varying CSI. Hence, the transmitter employs the quantised vectors $\{\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2, \ldots, \hat{\mathbf{w}}_m\}$ for beamforming, where $\hat{\mathbf{w}}_i = \mathcal{Q}(\mathbf{w}_i)$ as described in (9). The resulting model for symbol detection corresponding to the i^{th} substream $1 \le i \le m$ can be expressed as,

$$\tilde{y}_i(k) = \sigma_i \left\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \right\rangle \tilde{x}_i(k) + \sigma_i \left(\sum_{\substack{n=1 \\ n \ne i}}^{m} \left\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \right\rangle \tilde{x}_n(k) \right) + \eta(k) \tag{10}$$

where the inter-stream interference term $\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \rangle$ above arises due to the transmit beamformer quantisation error corresponding to the limited feedback scheme. Hence, SNR_i, the received SNR corresponding to the i^{th} parallel transmit substream, is given as,

$$\text{SNR}_i = \frac{\mathrm{E}\left\{\left|\left\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \right\rangle\right|^2\right\} \sigma_i^2 P_i}{\sigma_\eta^2 + \sigma_i^2 \sum_{\substack{n=1 \\ n \ne i}}^{m} \mathrm{E}\left\{\left|\left\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \right\rangle\right|^2\right\} P_n}, \quad 1 \le i \le m.$$

From the results in Roh and Rao (2006), the quantities $\mathrm{E}\left\{\left|\left\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \right\rangle\right|^2\right\}$ and $\mathrm{E}\left\{\left|\left\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \right\rangle\right|^2\right\}$ can be approximated as,

$$\mathrm{E}\left\{\left|\left\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \right\rangle\right|^2\right\} = 1 - \delta\left(\frac{t+1}{t}\right) \approx 1 - \delta,$$

$$\mathrm{E}\left\{\left|\left\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \right\rangle\right|^2\right\} = \left(\frac{\delta}{N-1}\right)\left(\frac{t-1}{t}\right) \approx \frac{\delta}{N-1},$$

where $\delta \triangleq 2^{-\left(\frac{B}{t-1}\right)}$. Substituting these approximations in the expression for SNR_i derived above, the resulting expression for the SNR of the k^{th} MIMO substream can be simplified as,

$$\mathrm{SNR}_i = \frac{\beta_i\left(1-2^{-\left(\frac{B}{t-1}\right)}\right)P_i}{\sigma_\eta^2+\beta_i\sum_{\substack{n=1\\n\neq i}}^{m}\left(\frac{2^{-\left(\frac{B}{t-1}\right)}}{N-1}\right)P_n},\ 1\le i\le m \tag{11}$$

where $\beta_i \triangleq E(\sigma_i^2)$. This expression is employed in Section 4 below to formulate the framework for optimal MIMO singular mode power allocation in the context of video transmission. In the next section we propose a singular mode diversity order-based video substream allocation for distortion minimisation in MIMO wireless systems.

6 DER diversity-based MIMO video transmission

In the context of the quantised feedback MIMO system model described above, we consider a MIMO user with t_u transmit antennas, r_u receive antennas and m_u = min (r_u, t_u) parallel substreams for video transmission. The SVD of the MIMO channel results in a natural ordering of singular modes of the MIMO channel matrix corresponding to the singular values $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_{m_u}$. The key result below, which has been presented in Garcia-Ordonez et al. (2005), provides a handle on the diversity order and probability of bit-error for QPSK transmission for the i^{th} MIMO mode corresponding to σ_i.

Theorem 2: The bit-error rate (BER) corresponding to the k^{th} MIMO substream with channel gain σ_k has a diversity order $G_d(k) = (m_u - k + 1)(n_u - k + 1)$, where m_u = min $\{r_u, t_u\}$ and n_u = max (r_u, t_u). Further, the probability of bit-error ϕ_e for QPSK modulated data transmission through this mode can be expressed as,

$$\phi_e\left(\mathrm{SNR}_k\right) = \left(G_a(k)\mathrm{SNR}_k\right)^{-G_d(k)} \tag{12}$$

where the array gain $G_a(k)$ is given by the expression,

$$G_a(k) = \frac{3}{n_u}\left(\frac{\left(\sqrt{2}-1\right)a_k 2^{d_k}\Gamma\left(d_k+3/2\right)}{\sqrt{\pi}\left(d_k+1\right)}\right)^{-\left(\frac{1}{(d_k+1)}\right)}.$$

The quantities d_k, a_k and $K_{n_u m_u}$ are given as,

$$d_k = G_d(k) - 1$$

$$a_k = K_{n_u m_u}^{-1}\,|A(k)|\,|B(k)|$$

$$K_{n_u m_u} = \prod_{i=1}^{m_u}\left(m_u - i\right)!\left(n_u - i\right)!$$

Let the quantity $b(i)$ be defined as $b(i) \triangleq n_u - m_u + i$. The matrix $A(k)$ can be defined as follows. For $k = 1$, $A(1) = 1$. For other values of k, i.e., $k \ge 2$ and $i, j = 1, 2, \ldots, (k-1)$, $A(k)_{ij}$ the $(i, j)^{\text{th}}$ entry of $A(k)$ is given as,

$$A(k)_{ij} = \left(b(i+j) + 2\left(m_u - k\right)\right)!$$

The matrix $B(k)$ is defined as follows. The matrix $B(m_u) = 1$, corresponding to $k = m_u$. For other values of k, the $(i, j)^{\text{th}}$ entry of $B(k)$ with $1 \le i, j \le m_u - k$ can be expressed as,

$$B(k)_{ij} = \frac{2}{\left(b(i+j)^2 - 1\right)b(i+j)}.$$

The above result plays a key role in the context of video transmission through MIMO systems as follows. Consider a MIMO system with $r = t = m_u = n_u$. Hence, the strongest mode corresponding to the singular value σ_1 has a diversity order m_u, n_u, while the weakest mode, corresponding to singular value σ_{m_u} has diversity order 1. Thus, it can be readily seen that the BER corresponding to the strongest mode decreases as $\mathrm{O}(\mathrm{SNR}^{-m_u n_u})$, while that corresponding to the weakest mode decreases as $\mathrm{O}(\mathrm{SNR}^{-1})$. Hence, the strongest mode has a significantly lower BER and higher reliability owing to its higher diversity order. The diversity order is progressively lower for the weaker modes, eventually becoming 1 for the weakest mode. This salient property relating to MIMO substream diversity order can be advantageously employed for minimisation of video distortion corresponding to video transmission over the MIMO wireless system as described below.

Any given video sequence can be readily decomposed into L hierarchical spatio-temporal layers, comprising of a base spatial layer S_1, base temporal layer T_1 and $L - 1$ spatial, temporal enhancement layers $S_2, \ldots, S_L, T_2, \ldots, T_L$, as described in Section 3 above. In this context, a natural scheme for MIMO video transmission is diversity-based hierarchical video transmission (DHVT), wherein the i^{th} spatiotemporal layers S_i, T_i are transmitted over the i^{th} MIMO singular mode corresponding to gain σ_i and transmit beamforming vector $\mathbf{w}_i$. Thus, by transmitting the base layers S_1, T_1 through the strongest MIMO mode and choosing progressively weaker modes for the transmission of the enhancement layers, DHVT significantly reduces video distortion. In the next section, we describe the optimal power allocation-based video transmission (OPVT) scheme for MIMO singular mode power distribution to further reduce the video distortion of DHVT.

7 Optimal power allocation-based video transmission (OPVT)

Consider a MIMO wireless system with total transmit power P_T. The video distortion of the DHVT scheme described above can be further reduced through optimal power allocation amongst the m_u MIMO substreams. Let $D_l(\mathcal{V})$ denote the distortion coefficients of the video sequence $\mathcal{V}(x, y, t)$ corresponding to the spatio-temporal layer l, where $1 \le l \le m_u$ as computed above. The mean overall distortion $D(\mathcal{V})$ for MIMO transmission of the video sequence $\mathcal{V}(x, y, t)$ can be expressed as

$D(\mathcal{V}) = \sum_{l=1}^{m_u} D_l(\mathcal{V}) \phi_e(P_l)$. Employing the DHVT-based hierarchical video layer singular mode allocation, the expression for the interference constrained l^{th} mode SNR_l given in (11) and the relation for l^{th} MIMO mode probability of bit-error from (12), the constrained cost function for overall video distortion $D(\mathcal{V})$ minimisation through optimal power allocation can be described as, min

$$\min. \sum_{l=1}^{m_u} D_l(\mathcal{V}) \left(\frac{\beta_l G_a(l) \left(1 - 2^{-\left(\frac{B}{t-1}\right)}\right) P_l}{\sigma_\eta^2 + \beta_l \sum_{\substack{j=1 \\ j \neq l}}^{m_u} \left(\frac{2^{-\left(\frac{B}{t-1}\right)}}{N-1} \right) P_j} \right)^{-G_d(l)}$$

$$\text{s.t.} \quad \sum_{l=1}^{m_u} P_l = P_T \tag{13}$$

$$P_l \geq 0, \quad 1 \leq l \leq m_u$$

The above optimisation can be solved through the following iterative procedure. Consider the optimal power allocation vector after the i^{th} iterative step given as $\tilde{P}^{(i)} = [\tilde{P}_1^{(i)}, \tilde{P}_2^{(i)}, \ldots, \tilde{P}_{m_u}^{(i)}]$. The optimal power vector $\tilde{P}^{(i+1)}$ corresponding to the $(i + 1)^{\text{th}}$ iteration can be obtained by solving the simplified convex optimisation problem,

$$\min. \sum_{l=1}^{m_u} B_l^{(i)}(\mathcal{V}) \left(\frac{1}{P_l} \right)^{G_d(l)}$$

$$\text{s.t.} \quad \sum_{l=1}^{m_u} P_l = P_T \tag{14}$$

$$P_l \geq 0, \quad 1 \leq l \leq m_u,$$

where the coefficients $B_l^{(i)}$, $1 \leq l \leq m_u$ are defined as,

$$B_l^{(i)}(\mathcal{V}) \triangleq \frac{D_l(\mathcal{V}) \left(\beta_l G_a(l) \left(1 - 2^{-\left(\frac{B}{t-1}\right)} \right) \right)^{-G_d(l)}}{\left(\sigma_\eta^2 + \beta_l \sum_{\substack{j=1 \\ j \neq l}}^{m_u} \left(\frac{2^{-\left(\frac{B}{t-1}\right)}}{N-1} \right) \tilde{P}_j^{(i)} \right)^{-G_d(l)}}$$

It can be observed that the above constrained objective minimisation paradigm is a standard form convex optimisation problem. Hence, the optimal power allocation vector $\tilde{P}^{(i+1)}$ can be readily computed from the above optimisation problem using the primal-dual interior point method (Boyd and Vandenberghe, 2004). This iterative procedure can be initialised with the uniform power vector, $P^{(0)} \triangleq \left(\frac{P_T}{m_u} \right) [1, 1, \ldots, 1]^T$ and has been observed to converge very rapidly to the optimal power allocation solution.

Below, we describe a fast Lagrangian procedure to compute the allocation vector $\tilde{P}^{(i+1)}$. The Lagrangian cost function $L(\mathcal{V}, \lambda, \bar{\mu})$ can be formulated for the convex optimisation problem described in (14) above as,

$$\sum_{l=1}^{m_u} B_l^{(i)}(\mathcal{V}) \left(\frac{1}{P_l} \right)^{G_d(l)} + \lambda \left(\sum_{j=1}^{m_u} P_j - P_T \right) - \sum_{k=1}^{m_u} \mu_k P_k.$$

From the KKT conditions for the above optimisation problem, it can be shown that $\tilde{P}_l^{(i+1)}$. the optimal power allocation for the l^{th} mode can be expressed as a function of the dual variable $\tilde{\lambda}^{(i+1)} > 0$ as,

$$P_l^{(i+1)} = \left(\frac{G_d(l) B_l^{(i)}(\mathcal{V})}{\tilde{\lambda}^{(i+1)}} \right)^{\frac{1}{G_d(l)+1}}. \tag{15}$$

Finally, the optimal dual variable $\tilde{\lambda}^{(i+1)}$ can be obtained as the root of the polynomial equation,

$$\sum_{l=1}^{m_u} \left(\frac{G_d(l) B_l^{(i)}(\mathcal{V})}{\tilde{\lambda}} \right)^{\frac{1}{G_d(l)+1}} = P_T. \tag{16}$$

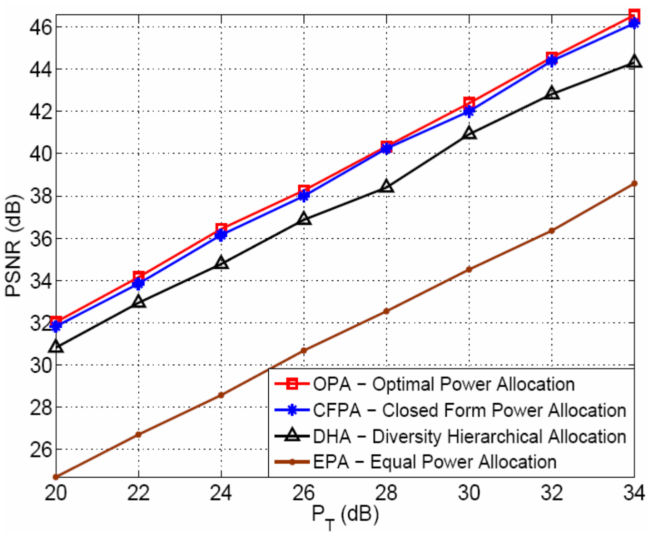
Hence, the optimal substream power $\tilde{P}_l^{(i+1)}$ corresponding to the l^{th} iterative step convex optimisation problem in (14) can be obtained by substituting this value of $\tilde{\lambda}^{(i)}$ in (15). Thus, the computation of the optimal power vector $\tilde{P}$ for each iterative step reduces essentially to finding the solution of the polynomial root finding problem in (16). This can be computed very efficiently through a Newton-step iterative procedure. It has been observed that the optimal power allocation obtained above for the hierarchical layered video transmission results in a significant reduction in video distortion compared to the suboptimal single layer video transmission (SLVT) with equal power allocation across the singular modes corresponding to the power allocation vector $\hat{P} = \left(\frac{P_T}{m_u} \right) [1, 1, \ldots, 1]^T$ without MIMO mode diversity consideration.

8 Simulation results

In our OFDMA simulation setup, we employ the standard video test sequences from Video sequences, http://media.xiph.org/video/derf/ for wireless video transmission. We consider an OFDMA system with $K = 4$ users. Each user is allocated $N_u = 2$ subcarriers, with BPSK modulated data transmitted on each subcarrier. We employ the level-1 Haar DWT for spatial decomposition of the intra-coded frame into $N_l = N_u = 2$ hierarchical layers. The lowest spatial frequency content, filtered in both the spatial dimensions forms the spatial base layer S_2 transmitted through the second ordered subcarrier, while the DWT filtered detail coefficients corresponding to S_1 are transmitted through the first ordered subcarrier. Similarly, 2-level HBMA (Wang et al., 2001) algorithm is employed for hierarchical motion estimation as discussed in Section 3. We employ a block size of 8 × 8 pixels and the search range is set to four pixels in each level of HBMA. After motion compensation of target frames, 2-layer Haar DWT is employed for spatial decomposition of corresponding residual frames. The distortion parameters $D_l(\mathcal{V})$, $0 \le l \le N_l = N_u$ corresponding to the hierarchical layers of the video sequences are computed offline as described in Section 4.

Users one to four transmit the Foreman, Hall, Mobile and Coastguard video sequences respectively. We compare the performance of the optimal diversity schemes DHA, OPA and CFPA for video transmission with the sub-optimal equal power allocation (EPA). Figures 4 and 5 demonstrate the PSNR of the decoded video streams corresponding to the received Foreman and Coastguard video sequences employing the different transmission schemes described above. It can be observed that the diversity-based hierarchical allocation (DHA) (described in Section 3) where ordered subcarriers are allocated to the hierarchically decomposed layers of the video sequence (while allocating equal power to the ordered subcarriers) results in a 7 dB performance enhancement over the sub-optimal EPA scheme. Thus, diversity order-based hierarchical video allocation results in significant performance gains.

Figure 4 PSNR plots for decoded Foreman video sequence (see online version for colours)



This PSNR performance can be further improved by 2 dB (i.e., 9 dB compared to EPA) through the optimal power allocation schemes OPA and CFPA described in Section 4. Further, the performance of the OPA power allocation vector $\bar{P}^\star$ is very close to that of the optimal CFPA vector $\tilde{P}^\star$ computed as given in (9), thus demonstrating that the modified optimisation problem in (6) for video distortion is an accurate approximation of the exact problem in (5). The superior performance of OPA over EPA is also visually illustrated by a comparison of the decoded received frames for the Foreman and Coastguard video sequences in Figures 6 and 7 respectively. Thus, the DHA, OPA and CFPA schemes are ideally suited for video distortion minimisation over OFDMA-based wireless channels.

Figure 5 PSNR plots for decoded Coastguard video sequence (see online version for colours)

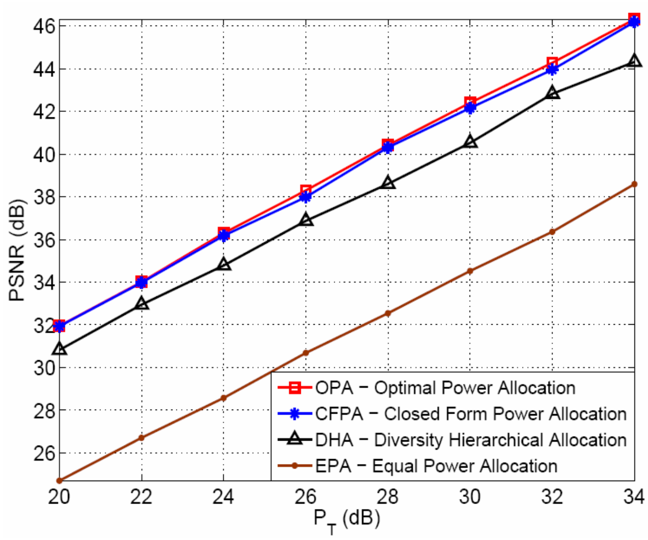


Figure 6 Frame quality comparison of decoded Foreman video sequence from the transmission employing (a) OPA and (b) EPA

(a) (b)

Figure 7 Frame quality comparison of decoded Coastguard video sequence from the transmission employing (a) OPA and (b) EPA

(a) (b)

To illustrate the performance of the proposed MIMO video power allocation schemes, we consider a $t_u = 2$ transmit antenna and $r_u = 2$ receive antenna, i.e., a 2×2 MIMO wireless system. Each user transmits the video over the MIMO wireless system comprising of $m_u = 2$ parallel substreams. The physical layer information symbols are QPSK modulated for wireless transmission. The reverse link channel feedback is limited to B = 8 bits per MIMO mode, i.e., for each of the right singular vectors $\mathbf{w}_1$, $\mathbf{w}_2$. The MIMO transmit beamforming codebook for this system therefore contains $N = 2^B = 256$ quantised vectors and is constructed employing the MSIP minimisation procedure described in Section 5. After each channel estimation epoch, the receiver feeds back the indices of the quantised vectors in the codebook, closest in MSIP to each of the instantaneous transmit beamforming vectors $\mathbf{w}_1$, $\mathbf{w}_2$. The transmitter beamforms the data along the corresponding quantised vectors $\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2$ as described by the system model in (10). We consider the standard Foreman, Hall, Coastguard and Mobile video sequences for MIMO-based video transmission. For the hierarchical decomposition of the video sequences, we employ the level-1 Haar DWT-based spatial decomposition of the intra-coded video frame into two hierarchical layers comprising the base spatial layer S_0 and enhancement layer S_1. Similarly, 2-level HBMA (Wang et al., 2001) is employed for DPCM-based hierarchical motion estimation and decomposition into temporal layers T_0, T_1. In the DHVT scheme discussed in Section 2, the base layers are transmitted across the strongest MIMO mode corresponding to gain σ_l, while the enhancement layers are transmitted over progressively weaker modes, thus significantly reducing distortion. The transmit power is distributed equally over both the spatial MIMO modes. The video distortion can be further minimised through the OPVT scheme elaborated in (13) for distribution of the limited transmit power P_T across the modes. The procedure for the optimal power vector computation is described in Section 7. It has been observed that the iterative process converges converges very rapidly to the optimal solution, requiring usually not more than five iterative cycles.

We compare the performance of the optimal video transmission schemes OPVT, DHVT with the suboptimal SLVT scheme. Figures 8 and 9 demonstrate the PSNR of the decoded Coastguard and Foreman video streams corresponding to the different video transmission schemes described above. It can be observed that the diversity-based hierarchical transmission scheme DHVT, which employs hierarchical decomposition followed by singular mode diversity order-based video layer allocation for MIMO transmission (while allocating equal power to all the substreams) results in a significant performance enhancement of around 6 to 7 dB in PSNR over the sub-optimal SLVT scheme. This is due to the fact that SLVT is diversity agnostic and does not employ video decomposition, which is key to achieving performance gains in the context of MIMO video transmission. This performance of DHVT is further improved by about 1 to 2 dB in PSNR through the optimal power allocation-based video transmission scheme OPVT as described in Section 7. Also, the optimal power vector obtained from the constrained convex optimisation problem solver CVX agrees well with the theoretical results presented in (15), thus lending support to the derivation of the optimal power allocation vector in Section 7. Further, in Figure 10 we plot the decoded PSNR for several values of B, the number of feedback bits per singular mode. It can be observed therein that the PSNR achieved for the OPVT scheme for $B = 4$, i.e., a total feedback of $mB = 8$ bits on the reverse link, is close to that of perfect CSI feedback (i.e., $B = \infty$). Hence, the codebook-based quantised beamforming vector feedback scheme is ideally suited for practical wireless systems since it achieves a performance close to the ideal system with perfect CSI without incurring large bit overheads on the reverse link.

Figure 8 PSNR plots for decoded Coastguard video sequence (see online version for colours)

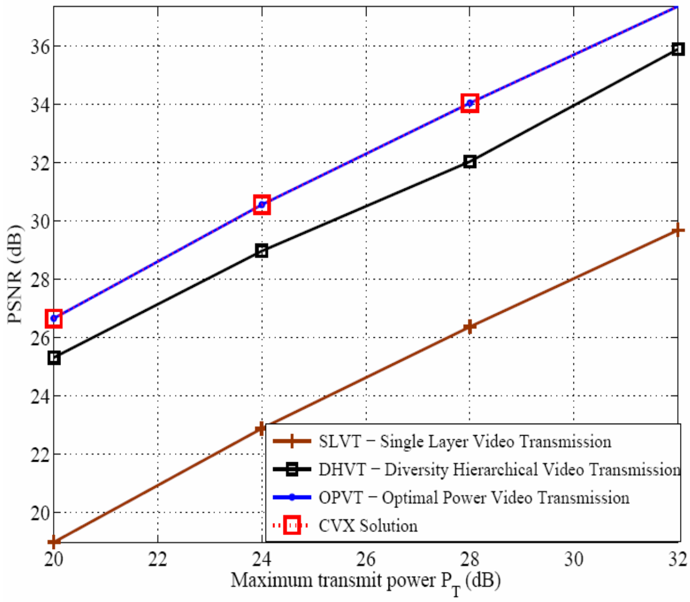


Figure 9 PSNR plots for decoded Foreman video sequence (see online version for colours)

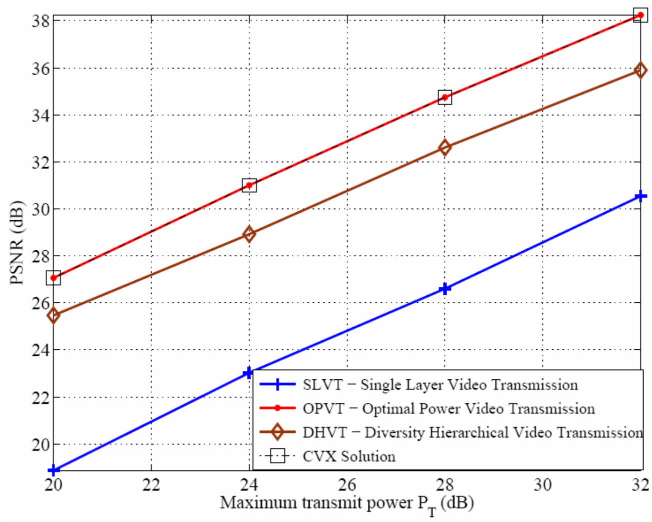
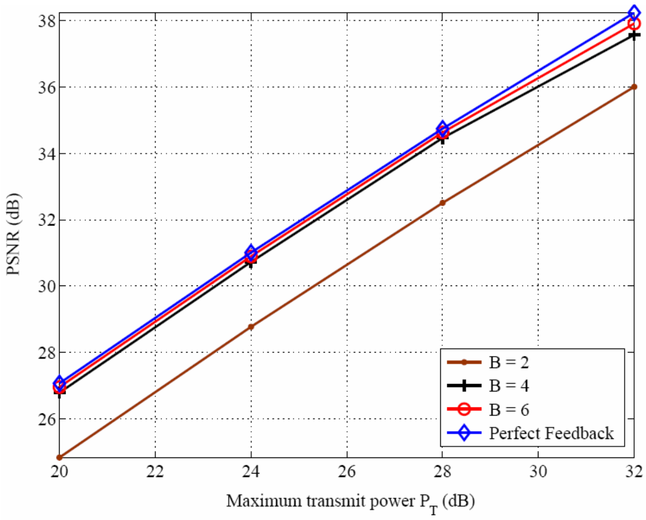


Figure 10 PSNR values for decoded video sequence for different feedback bits (*B* = 2, 4, 6) per MIMO channel model compared with perfect feedback (see online version for colours)

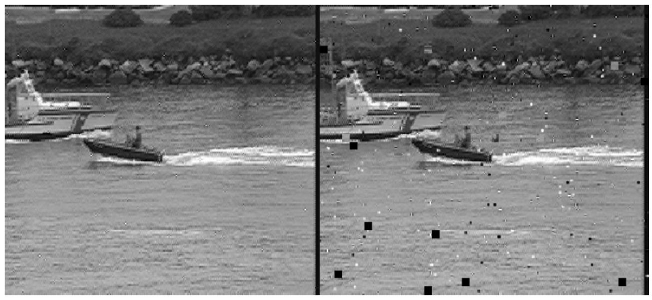


The superior performance of OPVT over SLVT is visually illustrated by a comparison of the decoded received frames for the Foreman and Coastguard video sequences in Figures 11 and 12 respectively. The OPVT frames have significantly lower artefacts introduced by the bit-errors over the fading MIMO wireless channel. Thus, the DHVT and OPVT schemes are ideally suited for video transmission over MIMO wireless channels as they minimise the video distortion through intelligent use of the diversity properties of the MIMO transmission modes. Further, it only involves codebook-based feedback of quantised transmit beamforming vectors, thereby greatly reducing the overheads on the MIMO reverse wireless link.

Figure 11 Frame quality comparison of decoded Foreman video sequence from the transmission employing (a) OPVT and (b) SLVT

(a) (b)

Figure 12 Frame quality comparison o1ecoded Coastguard video sequence from the transmission employing (a) OPVT and (b) SLVT



(a) (b)

9 Conclusions

In this work we proposed several order statistics-based optimal power allocation schemes viz. DHA, OPA and CFPA for minimum distortion video transmission in OFDMA wireless systems. Since they are based only on order feedback, explicit feedback of CSI is not required, thus saving bandwidth and avoiding overheads on the reverse link. Simulation results show that the proposed optimal power allocation schemes are significantly superior to the equal power allocation (EPA) scheme for video transmission in an OFDMA system. We also proposed MIMO SVD-based video transmission schemes viz. DHVT, OPVT for minimisation of video distortion in 4G MIMO wireless systems. Further, the proposed schemes employ codebook-based quantised feedback, thus resulting in significantly lower overheads on the reverse link, while simultaneously not compromising on the beamforming gain achievable in the MIMO system.

We described the paradigm of DHVT for MIMO mode diversity order-based hierarchical video allocation, which results in a significant reduction in video distortion. Subsequently, we formulated the optimisation problem for OPVT taking into account the distortion caused by quantised beamforming due to limited feedback. The Lagrangian-based closed form solution for iterative computation of the MIMO optimal power vector has been derived as the solution of a simplistic polynomial root computation. Simulation results show that the proposed OPVT and DHVT schemes are significantly superior to the diversity order agnostic equal power allocation-based SLVT scheme for video transmission in MIMO wireless systems.

References

Bovik, A.C. (2005) *Handbook of Image and Video Processing*, Academic Press, Inc., Orlando, FL, USA.

Boyd, S. and Vandenberghe, L. (2004) *Convex Optimization*, Cambridge University Press, Cambridge, UK.

David, H.A. and Nagaraja, H.N. (2003) *Order Statistics*, John Wiley & Sons, Hoboken, New Jersey, USA.

Garcia-Ordonez, L., Palomar, D.P., Pages-Zamora, A. and Fonollosa, J.R. (2005) 'Analytical BER performance in spatial multiplexing MIMO systems', in *Signal Processing Advances in Wireless Communications, 2005 IEEE 6th Workshop on*, June, pp.460–464.

Ha, H., Yim, C. and Kim, Y.Y. (2008) 'Cross-layer multiuser resource allocation for video communication over OFDM networks', *Comput. Commun.*, Vol. 31, No. 15, pp.3553–3563.

Kim, K., Kim, H. and Han, Y. (2004) 'Subcarrier and power allocation in OFDMA systems', *Vehicular Technology Conference*, September, Vol. 2, p.1058.

Oh, J., Kim, J. and Cioffi, J.M. (2004) 'Optimum power allocation and control for OFDM in multiple access channels', *Vehicular Technology Conference*, September, Vol. 2, pp.774–778.

Roh, J.C. and Rao, B.D. (2004) 'Channel feedback quantization methods for MISO and MIMO systems', in *Personal, Indoor and Mobile Radio Communications, PIMRC 2004, 15th IEEE International Symposium on*, September, Vol. 2, pp.805–809.

Roh, J.C. and Rao, B.D. (2006) 'Transmit beamforming in multiple-antenna systems with finite rate feedback: a VQ-based approach', *Information Theory, IEEE Transactions on*, March, Vol. 52, No. 3, pp.1101–1112.

Sabir, M.F., Bovik, A.C. and Heath, R.W. (2010) 'Unequal power allocation for JPEG transmission over MIMO systems', *Image Processing, IEEE Transactions on*, February, Vol. 19, No. 2, pp.410–421.

Scaglione, A., Stoica, P., Barbarossa, S., Giannakis, G.B. and Sampath, H. (2002) 'Optimal designs for space-time linear precoders and decoders', in *IEEE Trans. Signal Processing*, May, Vol. 50, No. 5, pp.1051–1064.

Tesanovic, M., Bull, D.R. and Doufexi, A. (2008) 'Enhancing video quality for multiple-description MIMO transmission through unequal power allocation between eigen-modes', in *Image Proc., ICIP 2008, 15th IEEE International Conference on*, October, pp.2024–2027.

Tse, D. and Viswanath, P. (2005) *Fundamentals of Wireless Communication*, Cambridge University Press, Cambridge, UK.

van Nee, R. and Prasad, R. (2000) *OFDM for Wireless Multimedia Communications*, Artech House, Inc., Norwood, MA, USA.

'Video sequences', available at http://media.xiph.org/video/derf/ (accessed in May 2011).

Wang, Y., Zhang, Y. and Ostermann, J. (2001) *Video Processing and Communications*, Prentice Hall PTR, Upper Saddle River, New Jersey, USA.

Zhao, S., You, M. and Gui, L. (2009) 'Scalable video delivery over MIMO OFDM wireless systems using joint power allocation and antenna selection', in *Multimedia Sig. Proc, 2009. MMSP '09. IEEE International Workshop on*, October, pp.1–6.

Appendix

The result below describes a key property of the gains of the OFDMA subcarriers. The subcarriers occupy a bandwidth W, with inter-subcarrier spacing $\left(\frac{W}{N}\right)$.

Theorem 3: For a WSSUS Rayleigh fading channel, the correlation between subcarriers separated by $\left(\frac{W}{L}\right)$ is zero. Further, the correlation between subcarrier gains $H(m)$, $H(m + r)$ separated $r = \alpha \frac{N}{2L}$ by subcarriers decreases as,

$$R_H(r) \triangleq \left| E\left(H(m)H*(m+r) \right) \right| \leq \frac{1}{\alpha} R_H(0)$$

Proof: Proof is similar, as stated in Tse and Viswanath (2005). ■

From the above result, it is clear that the correlation between the subcarrier gains decreases rapidly as $O\left(\frac{1}{\alpha}\right)$, becoming insignificant (<0.5) for $r > \frac{N}{L}$. In other words, the gains of the subcarriers separated by more than the coherence bandwidth $\left(W_c = \frac{W}{L}\right)$ are essentially uncorrelated, and hence independent due to their Rayleigh fading nature. Thus, in an OFDMA system, each of the data streams transmitted through subcarriers with inter-subcarrier separation greater than $\left(\frac{W}{L}\right)$ can be assumed to experience independent Rayleigh fading.