

Indian Institute of Technology Kanpur
Ph.D. (Economics) Admission Test – Sample

Time: 2 Hours

Full Marks: 100 Points

Candidates with Economics Background

1. Zinnia's utility function is $U(x_1, x_2) = 2(\ln x_1) + x_2$. Given her current income and the current relative prices, she consumes 10 units of x_1 and 15 units of x_2 . If her income doubles, while prices stay constant, how many units of x_1 will she consume after the change in income? (10 points)

2. An industry consists of two firms, each of which has variable costs of \$10 per unit but no fixed costs. The industry demand curve is given by $P = 130 - 2Q$
 - a. Solve for the Cournot equilibrium. In doing so, derive the reaction function of each firm (call Q_1 and Q_2 the output of each firm). Solve for each firm's output, the market price, and each firm's profits. (8 points)
 - b. Now suppose that firm 1 is a Stackelberg leader while firm 2 is a follower. Assume, as usual, that the follower behaves like a Cournot duopolist (that is, assumes that the leader's output is fixed). What will each firm produce, what will the price be, and what profits will each firm earn? (8 points)

3. Integrate (10 points)

$$\int_0^{\frac{1}{2}} \frac{4}{1 + 4t^2} dt$$

4. Determine whether the following matrix is invertible or not? If YES, derive the inverse. (10 points)

$$\underline{\mathbf{A}} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

5. Consider the linear regression model: $y_i = \beta_1 + \beta_2 x_i + e_i$

You are given the following observations.

Y	5	2	3	2	-2
X	3	2	1	-1	0

- a. Find least squares estimates for β_1 and β_2 . (6 points)
 - b. Calculate the estimated variances of the estimators of β_1 and β_2 . (6 points)
 - c. Is there a statistically significant relationship between X and Y? Test. (4 points)
6. A company manufactures a new kind of light bulb based on the latest LED technology. The (population) mean life of a bulb is 7.0 years with a population standard deviation of 1.4 years. The mean life of a bulb is normally distributed.
- a. What proportion of bulbs will last more than 9.5 years? (5 points)
 - b. What proportion of bulbs will last less than 4.0 years? (5 points)
 - c. What proportion of bulbs will last between 8.0 and 9.0 years? (5 points)
 - d. Calculate the 90th percentile for a bulb. (5 points)
7. Multiple choice questions: Mention the correct answer option. ($1 \times 8 = 8$ points)
- A. Technological improvements are incorporated in
 - (a) cyclical macroeconomic models.
 - (b) growth models.
 - (c) long run macroeconomic models.
 - B. Output in the long run macroeconomic models
 - (a) are entirely 'demand-determined'.
 - (b) are both demand and supply determined.
 - (c) are entirely supply determined.
 - C. Stabilization policies
 - (a) stabilize the long term growth of the economy.
 - (b) are meant to counter the cyclical changes in the economy.
 - (c) None of the above.
 - D. Personal Disposable Income
 - (a) does not contain Transfer Payments.
 - (b) does not contain Dividend Income.
 - (c) does not contain Retained Earnings.
 - E. The balanced budget multiplier in the IS-LM Model

- (a) is one.
 - (b) is less than one.
 - (c) can be greater than one.
- F. In liquidity trap
- (a) fiscal policy is the best option.
 - (b) monetary policy is the best option.
 - (c) the LM curve is vertical.
- G. The rate of interest will adjust
- (a) whenever the goods market is in disequilibrium.
 - (b) whenever the money market is in disequilibrium.
 - (c) None of the above.
- H. If there is excess supply of money in the economy
- (a) there will be excess supply in the non-money market.
 - (b) the prices of securities will fall.
 - (c) the interest rate will decline.
8. Assume that the following structural equations describe behavior in the economy's goods market and money market, respectively, during the current period (i.e. given exogenous effects).

Goods Market	Money Market
$Y = C + I + G$	$(M/P)^d = (M/P)^s$
$C = C_a + \theta (Y - T)$	$(M/P)^d = \lambda Y - \beta r$
$I = I_o - \alpha r$	$(M/P)^s = (M_o/P_o)$
$T = T_o + \tau Y$	
$G = G_o$	

Assume known parameter values of θ , τ , and λ that are the marginal propensity to consume (C), to tax (T), and to demand money $((M/P)^d)$, respectively, that are induced from a change in income (Y). Also assume known parameter values of α , and β that are the marginal responsiveness of investment (I), and money demand, respectively, that are induced from a change in the interest rate (r).

Now, suppose that the government spending (G) increases because of war. Use graphical analysis as tools to briefly analyze the influences on the:

- interest rate
- real income
- real consumption
- real investment spending. (2.5 × 4 = 10 points)

- a. Find least squares estimates for β_1 and β_2 . (5 points)
 - b. Calculate the estimated variances of the least squares estimators of β_1 and β_2 . (5 points)
6. A bulb company manufactures a new kind of light bulb based on the latest LED technology. The (population) mean life of a bulb is 7.0 years with a population standard deviation of 1.4 years. The mean life of a bulb is normally distributed.
- a. What proportion of bulbs will last more than 9.5 years? (5 points)
 - b. What proportion of bulbs will last less than 4.0 years? (5 points)
 - c. What proportion of bulbs will last between 8.0 and 9.0 years? (5 points)
 - d. Calculate the 90th percentile for a bulb. (5 points)
7. A Survey indicated that chocolate ice cream is India's favorite ice-cream flavor. For each of the following, use diagrams to indicate the possible effects on demand and/or supply and equilibrium price and quantity of chocolate ice cream. ($2.5 \times 4 = 10$ points)
- a. A severe drought in Gujarat causes dairy farmers to reduce the number of milk-producing cattle in their herds by a third. These dairy farmers supply cream that is used to manufacture chocolate ice cream.
 - b. A news report by the Indian Medical Association reveals that chocolate does, in fact, have significant health benefits.
 - c. The discovery of cheaper synthetic vanilla flavoring lowers the price of vanilla ice cream.
 - d. New technology for mixing and freezing ice cream lowers manufacturers' costs of producing chocolate ice cream.
8. A U.S. Pharmaceuticals firm sells its patent-protected drug Levemir in the U.S. market. The domestic demand function is given by $Q_{US} = 120 - 2P_{US}$, where price is measured in U.S. dollars and quantity is measured in vials. The firm's marginal cost is $MC = \$10$. Find out firms' optimal price and number of vials sold in the markets. (10 points)
9. Define the following terms: (i) GDP, (ii) inflation, (iii) money supply, (iv) price index, (v) VAT. ($2 \times 5 = 10$ points)