Indian Institute of Technology Kanpur
Proposal for a New Course

1. **Course Number:** MTH XXX
2. **Course Title:** Numerical Methods for Hyperbolic Problems
3. **Per Week Lectures:** 2 (L), **Tutorial:** 0 (T), **Laboratory:** 0 (P), **Additional Hours:** 0
   Credits: $2 \times L + T + P + A$: 9 credits
4. **Duration of Course:** Full Semester
5. **Proposing Department:** Department of Mathematics and Statistics
6. **Proposing Instructor:** Abhijit Biswas
7. **Overview of the Course:** This course provides an introduction to the numerical analysis of first-order hyperbolic PDEs. We begin with linear hyperbolic equations and develop numerical methods based on finite difference approximations. We study the fundamental concepts of consistency, stability, and convergence of these methods. We then turn to scalar nonlinear conservation laws. Starting with the method of characteristics, we analyze shock formation, derive the Rankine–Hugoniot conditions, study rarefaction waves, and discuss the theory of weak and entropy solutions. The course then focuses on finite volume methods for scalar conservation laws. We study first-order schemes, including the Godunov method and monotone schemes, and examine the concepts of consistency, conservation, and monotonicity together with their stability and convergence properties. We then move to high-resolution finite volume methods, including Lax–Wendroff, TVD, MUSCL, and WENO schemes. Time integration techniques will be discussed, with emphasis on Strong Stability Preserving (SSP) properties. Finally, we extend these ideas to linear and nonlinear systems of hyperbolic conservation laws. Applications include linear systems such as the linear acoustics equations and nonlinear systems such as the shallow water equations and the Euler equations.
8. **Objectives:** Students will understand the mathematical structure of first-order hyperbolic conservation laws, including shock formation, weak solutions, and entropy conditions. They will gain the ability to analyze numerical methods such as finite difference and finite volume schemes for first-order hyperbolic PDEs. Furthermore, students will implement and apply high-resolution methods to solve linear and nonlinear hyperbolic systems. Throughout the course, students will gain hands-on experience by implementing these numerical methods in MATLAB or Python, as well as in the Clawpack software that is used to simulate complex physical phenomena such as water waves and fluid flow.
9. **Pre-requisites:** Numerical Analysis, Partial Differential Equations, and students should be comfortable with programming simple numerical algorithms.
10. **Short summary for inclusion in the Courses of Study Booklet:** This course introduces the analytical and numerical treatment of hyperbolic conservation laws. Topics include linear transport equations, scalar conservation laws, finite difference and finite volume methods, stability and convergence analysis, and high-resolution schemes such as TVD, ENO, and WENO. Applications to systems including linear acoustics, shallow water equations, and the Euler equations are discussed.

Recommended books:

1. Randall J. LeVeque: Finite Volume Methods for Hyperbolic Problems, Cambridge Texts in Applied Mathematics, 2002.
2. J.S. Hestheven, Numerical methods for conservation laws: From Analysis to Algorithms, SIAM publisher.
3. E. F. Toro: Reimann Solvers and Numerical Methods for Fluid Dynamics, Springer, 1999.

Tentative Lecture Plan (26 Lectures)

1. **Linear Transport Equations** (2 Lectures)
   Method of characteristics, finite difference schemes, upwind method, stability analysis.
2. **Scalar Nonlinear Conservation Laws** (4 Lectures)
   Characteristics, shock formation, weak solutions, entropy conditions, Riemann problems.
3. **Finite Volume Methods for Scalar Conservation Laws** (5 Lectures)
   Finite volume framework, Godunov method, monotone schemes, stability properties, convergence analysis.
4. **Second-Order and High-Resolution Finite Volume Methods** (4 Lectures)
   Order of accuracy, reconstruction techniques, limiters, TVD property, semi-discrete schemes, time stepping.
5. **Very High-Order Methods (ENO and WENO)** (3 Lectures)
   High-order reconstructions, ENO and WENO procedures, numerical fluxes.
6. **Linear Hyperbolic Systems** (2 Lectures)
   Examples (linear acoustics), hyperbolicity, characteristic decomposition, Riemann problems.
7. **Nonlinear Hyperbolic Systems** (3 Lecture)
   Structural properties, entropy conditions, Riemann problems.
8. **Finite Volume Methods for Systems** (3 Lectures)
   Godunov-type methods, Roe solver, HLL-type solvers.

Dated: February 13, 2026 **Proposer:** Abhijit Biswas

Dated: **DUGC Convener:**

The course is approved / not approved

Chairman, SUGC

Dated: