# Indian Institute of Technology Kanpur
Proposal for a New Course

## Course Information

- **Course Number:** MTH 7XX (PhD course)
- **Course Title:** Numerical Analysis
- **Per Week Lectures:** 3 (L), Tutorial: 0 (T), Lab: 0 (P)
  **Credits:** 9 credits
- **Duration of Course:** Full Semester
- **Proposing Department:** Department of Mathematics and Statistics
- **Proposing Instructor:** Mrinmay Biswas .

  Other faculty members interested in teaching the proposed course: Prof. Akash Anand, Prof. B. V. Rathish Kumar, Prof. Saktipada Ghorai, Prof. Abhijit Biswas.

## Course Objectives

This course is designed to provide an exposure to PhD students in basic areas of advanced numerical analysis techniques and computation. Main objectives of this course are as follows:

- **Build foundational understanding**: To develop a strong conceptual foundation in numerical methods for solving linear and nonlinear problems, enabling students from diverse backgrounds to engage with computational research.
- **Analyze errors and stability**: To understand sources of numerical error, conditioning, and stability, and to evaluate how these affect the reliability and accuracy of computational solutions.
- **Develop skills in numerical linear algebra**: To equip students with direct and iterative techniques (e.g., LU/Cholesky factorization, Krylov subspace methods) for solving large-scale linear systems and eigenvalue problems arising in scientific applications.
- **Solve nonlinear systems effectively**: To introduce robust numerical techniques such as Newton and quasi-Newton methods for solving multidimensional nonlinear equations and analyzing their convergence behavior.
- **Apply numerical methods to ODEs**: To enable students to construct and analyze numerical schemes for ordinary differential equations (ODEs), including stability, convergence, and adaptive methods.
- **Understand computational approaches for PDEs**: To provide knowledge of finite difference and variational methods for elliptic, parabolic, and hyperbolic partial differential equations, including stability and convergence analysis.
- **Introduce modern numerical techniques**: To familiarize students with advanced approaches such as finite element methods, projection methods, and discontinuous Galerkin methods used in high-level research and simulations.
- **Promote implementation and research readiness**: To develop the ability to implement numerical algorithms and apply them to real-world problems in science and engineering, preparing students for independent computational research.

## Course content:

Total lectures = 39, 1 lecture = 50 minutes

### Numerical Linear Algebra (13 Lectures)

- **Conditioning and sensitivity (1 lecture)**: Sources of error and propagation of error, condition number, examples.
- **Gaussian Elimination, LU Decomposition, Cholesky Decomposition (3 lectures)**: Forward elimination, backward substitution, pivoting, Doolittle, Crout, operation counts, applications, Symmetric positive definite systems, examples.
- **Iterative Methods (3 lectures)**: Jacobi, Gauss–Seidel methods, Successive Over-Relaxation (SOR), convergence analysis, Conjugate Gradient method for SPD systems.
- **Krylov Subspaces (4 lectures)**: Introduction to QR factorization, Arnoldi/Lanczos processes, basic concepts, GMRES, BiCG, MINRES methods, Convergence and applications.
- **Eigen value problems and applications (2 lectures)**: Methods to compute eigen values, Singular Value Decomposition.

### Numerical Solution of nonlinear system (2 Lectures)

- **Newton's Method**: Multidimensional Newton, convergence analysis, Modified Newton, quasi-Newton, secant methods.

### Numerical Solution of ODE (10 Lectures)

- **One step methods for IVP (6 lectures)**: Introduction to IVP, Euler method, Improved Euler, Taylor Series Methods, Classical Runge–Kutta Methods, stability, consistency, Adaptive RK, RK-Fehlberg.
- **Linear Multistep Methods (2 lectures)**: Adams-Bashforth, Adams-Moulton, BDF methods, Predictor-corrector methods.
- **BVP – Finite Difference (2 lectures)**: Second-order linear ODE, numerical solution, convergence analysis.

### Numerical Solution of PDE (14 Lectures)

- **Finite Difference for Elliptic PDEs (1 lecture)**: Laplace/Poisson, grid, implementation.
- **Finite Difference for Hyperbolic PDEs (3 lectures)**: Transport and Wave equation, Von Neumann stability analysis, convergence of numerical solutions.
- **Finite Difference for Parabolic PDEs (1 lecture)**: Heat equation, stability, CFL condition.
- **Projection Methods (6 lectures)**: Weak formulation, variational approach, Error analysis, Examples: Galerkin (Spectral, FEM, Higher-order elements, adaptive refinement), Collocation, implementation, FEM for Laplace/Poisson, Heat, Wave equation.
- **Discontinuous Galerkin (3 lectures)**: Basics, numerical fluxes, stability, Implementation, examples, Applications, error analysis.

## Pre-requisites

Instructor consent.

## Short summary for inclusion in the Courses of Study Booklet:

Including a course in Numerical Analysis in a PhD curriculum is especially valuable as a foundational bridge. It strengths students' ability to solve complex, real-world problems using computational methods. This course introduces essential concepts like basic algorithms design, approximation methods, and error estimation in a structured way, which are essential for research involving simulations, data analysis, and modeling. This foundation enhances both the rigor and efficiency of their work, especially in fields like engineering, physics, and data science, where analytical solutions are often impractical. Such a course can level the playing field, enabling students from diverse academic backgrounds to confidently engage with research that involves simulations, data processing, or computational modeling.

## Recommended books:

**Reference for Numerical Linear Algebra:**

- Kendall E. Atkinson, *An Introduction to Numerical Analysis*, Wiley, 2008.
- M Heath, *Scientific Computing - An introductory Survey*, SIAM, 2018.

**Reference for Numerical Finite Difference Methods:**

- R. J. LeVeque, *Finite difference methods for ordinary and partial differential equations: steady-state and time-dependent problems*, vol. 98, SIAM, 2007.
- Arnold, Douglas N. *Lecture notes on Numerical Analysis of Partial Differential Equations, 2012.* 2017-2018 version is available at https://www-users.cse.umn.edu/ arnold/8445-8446.17-18/notes.pdf

**Reference for Finite Element Methods:**

- Susanne C. Brenner, L. Ridgway Scott, *The Mathematical Theory of Finite Element Methods*, Springer, 2008.
- Philippe G. Ciarlet, *The Finite Element Method for Elliptic Problems*, SIAM, 2002.

**Reference for Numerical DG Methods:**

- B. Cockburn, G. E. Karniadakis and C.W. Shu, *Discontinuous Galerkin methods. Theory, computation and applications*, Lecture Notes in Computational Science and Engineering, 11. Springer-Verlag, Berlin, 2000.
- B. Rivière, *Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and Implementation.* SIAM Frontiers in Applied Mathematics, 2008.

## Additional Remarks and Approval

8. **Any other remarks:**

**Dated:** _____01/05/26_____

**Proposer:** _____Mrinmay Biswas_____

**Dated:** _____

**DUGC/DPGC Convener:**

**The course is approved / not approved**

**Chairman, SUGC/SPGC**

**Dated:** __________