Indian Institute of Technology Kanpur

Proposal for a New Course

Course Information

- **Course Number:** MTH 7XX (PhD course)
- **Course Title:** Topology and Geometry
- **Per Week Lectures:** 2 (L), Tutorial: 0 (T), Lab: 0 (P)
  Credits: 9 credits
- **Duration of Course:** Full Semester
- **Proposing Department:** Department of Mathematics and Statistics
- **Proposing Instructor:** Debasis Sen.

  Other faculty members interested in teaching the proposed course: Prof. Ashis Mandal, Prof. Abhijit Pal, Prof. Ajay Singh Thakur, Prof. Bidyut Sanki.

Course Objectives

This course aims to provide an exposure to PhD students in basic areas of topology and geometry. It reviews fundamental concepts such as topological spaces, compactness, connectedness, and quotient constructions, and develops tools for studying algebraic invariants like the fundamental group and covering spaces. The course further explores smooth and Riemannian manifolds, including tangent structures, vector fields, geodesics, and curvature, while also introducing simplicial complexes and homology theory. By the end of the course, students will be equipped with tools of topology and geometry and will be prepared for research in topology, geometry, and related areas.

Course content:

Total lectures = 26, 1 lecture = 1.15 hours

Review of Basic Topics (3 Lectures)

Topological spaces, Compact spaces, Connected spaces. Tychonoff Theorem, Lebesgue Lemma, Heine–Borel Theorem, Quotient topology, Examples: X/A, Group action quotients, Projective spaces, Grassmannians.

Fundamental Groups and Covering Spaces (10 Lectures)

Fundamental groups, Contractible and simply connected spaces, Covering spaces, Lifting criterion, Universal cover, $\pi_1(S^1)$, Properly discontinuous actions, Fundamental group of X/G, Seifert–van Kampen Theorem, Effect of attaching a cell on the fundamental group, Fundamental group of surfaces, Statement of classification of surfaces.

Smooth Manifolds (4 Lectures)

Differentiable manifolds, Tangent spaces and tangent bundle, Differentiable maps, Vector fields, Lie derivative, Exponential map.

Riemannian Manifolds (6 Lectures)

Riemannian metric, Covariant derivative, Riemannian connection, Geodesics, Curvature, Hopf–Rinow Theorem.

Simplicial Complexes and Homology (3 Lectures)

Simplicial complexes, Simplicial homology.

Pre-requisites:

Instructor consent.

Short summary for inclusion in the Courses of Study Booklet:

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Recommended books:

- Algebraic Topology — A. Hatcher
- Algebraic Topology — J. Munkres
- An Introduction to Differentiable Manifolds and Riemannian Geometry — W. Boothby
- Riemannian Geometry — M. do Carmo
- Morse Theory — J. Milnor

Additional Remarks and Approval

8. **Any other remarks:**

Dated: _____28/04/26_____ **Proposer:** _____Debasis Sen_____

Dated: _____ **DUGC/DPGC Convener:**

The course is approved / not approved

Chairman, SUGC/SPGC **Dated:** _____