Indian Institute of Technology Kanpur

Proposal for a New Course

Course Information

- **Course Number:** MTH 7XX (PhD course)
- **Course Title:** Advanced ODE and PDE
- **Per Week Lectures:** 3 (L), Tutorial: 0 (T), Lab: 0 (P)
  Credits: 9 credits
- **Duration of Course:** Full Semester
- **Proposing Department:** Department of Mathematics and Statistics
- **Proposing Instructor:** Indranil Chowdhury .

  Other faculty members interested in teaching the proposed course: Prof. T. Muthukumar, Prof. Kaushik Bal, Prof. Prosenjit Roy, Prof. Vedansh Arya.

Course Objectives

This course is designed to provide an exposure to PhD students in basic areas of advanced ODEs and PDEs. Main objectives of this course are as follows:

- **Establish rigorous foundations in ODE theory** To develop a solid understanding of first-order ODEs, including existence and uniqueness results (e.g., Picard–Lindelöf and Cauchy–Peano theorems), and their theoretical significance in modeling real-world phenomena.
- **Understand qualitative behavior of dynamical systems**: To analyze continuous dependence on initial data and parameters, and to study stability of solutions using tools such as phase portraits, eigenvalue analysis, and Lyapunov methods.
- **Develop insight into boundary value problems**: To introduce linear second-order boundary value problems, including conditions for existence, uniqueness, and the role of linear independence concepts.
- **Introduce analytical techniques for PDEs**: To provide a foundation in first-order PDEs and methods of characteristics, along with classical solutions of key equations such as the wave and heat equations.
- **Build familiarity with functional analytic tools**: To equip students with essential concepts such as distributions, weak derivatives, and Sobolev Spaces, forming the basis for modern PDE theory.
- **Develop understanding of weak solutions and variational methods**: To introduce variational formulations and key results such as the Lax–Milgram lemma for proving existence and uniqueness of weak solutions to elliptic PDEs.
- **Explore fundamental PDE models and their properties**: To study classical equations like Laplace, Poisson, and heat equations, including their fundamental solutions and qualitative behavior.
- **Prepare students for advanced mathematical and computational research**: To build the theoretical and analytical skills required for further study in applied mathematics, numerical analysis, and scientific computing.

Course content:

Total lectures = 39, 1 lecture = 50 minutes

ODE(13 Lectures)

- **Introduction to 1st order ODEs, Existence-Uniqueness Theory (4 lectures)**: Initial value problems, examples, Existence and uniqueness results, Picard–Lindelöf theorem – Contraction mapping approach, Statement and idea of the proof, Cauchy Peano Theorem – (Local and Global) Existence results and proofs.
- **Continuous dependence (2 lectures)**: Dependence on initial data, and on forcing term, examples, stability interpretation.
- **Stability analysis of ODE (4 lectures)**: Stable and unstable solutions, Exponential matrix, Equilibrium solutions, Eigenvalue analysis, Phase diagram, Lyapunov stability.
- **Boundary value problems (3 lectures)**: Linear second-order ODE, Overview of Wronskian, Linear dependency and independency (statements), Existence and Uniqueness results.

PDE (26 Lectures)

- **1st Order PDE (3 lectures)** : Concept of Linear, Semi linear and nonlinear PDEs, Derivation of Method of characteristics for quasi-linear PDE, Characteristic curves and solutions, Existence and Uniqueness results.
- **Wave equation (2 lectures)**: 1D wave equation and Derivation of D'Alembert solution, Duhamel's principle, Domain of dependence, Range of influence.
- **Introduction to distributions and Weak Derivatives (2 lectures)**: Motivation, Test function space and topology (via convergence of sequence), Definition of distributions, Weak derivatives, examples.
- **Sobolev spaces (2 lectures)**: Definition of $W^{k,p}$, Basic properties,
- **Density theorem (3 lectures)**: Approximation by smooth functions, Proof ideas, Applications.
- **Extension theorem (2 lectures)**: Extension operators, Construction methods, Applications.
- **Trace theorem (1 lecture)**: Boundary values of Sobolev functions.
- **Sobolev embedding (4 lectures)**: Gagliardo Nirenberg Sobolev embedding, Morrey's embedding, Poincare Inequality, Rellich Kondrachov compact embedding.
- **Lax–Milgram lemma (2 lectures)**: Variational formulation, Existence and Uniqueness of Weak solutions of elliptic equations.
- **Laplace equation (3 lectures)**: Harmonic functions and properties, Fundamental solution, Poisson Equation.
- **Heat Equation (2 lectures)**: Homogeneous equation and Fundamental solution, Non-homogeneous Equations and Duhamel's principle.

Pre-requisites

Instructor consent.

Short summary for inclusion in the Courses of Study Booklet:

This course offers a foundational yet rigorous introduction to Ordinary Differential Equations and Partial Differential Equations tailored for first-year PhD students from diverse academic backgrounds. It develops core concepts such as existence and uniqueness of solutions, stability analysis, and boundary value problems in ODEs, before progressing to first-order PDEs, wave and heat equations. The course also introduces modern tools including distributions, weak derivatives, and Sobolev Spaces. Emphasis is placed on building both intuition and mathematical rigor, enabling students to transition effectively into advanced research in applied mathematics, science, and engineering.

Recommended books:

Reference for ODE:

- Earl A. Coddington, Norman Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill, 2017.
- S. L. Ross, *Introduction to Ordinary Differential Equations*, Wiley, 1980.
- Lawrence Perko, *Differential Equations and Dynamical Systems*, Springer, 2001.

Reference for PDE:

- Yehuda Pinchover, Jacob Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press, 2005.
- Robert C. McOwen, *Partial Differential Equations*, Prentice Hall, 2003.
- Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010.
- Haim Brézis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2011.

Additional Remarks and Approval

8. **Any other remarks:**

Dated: _____01/05/26_____

Proposer: _____Indranil Chowdhury_____

Dated: _____

DUGC/DPGC Convener:

The course is approved / not approved

Chairman, SUGC/SPGC

Dated: __________