- This section contains Fifteen (15) questions
- Each question has FOUR options, ONE / MULTIPLE of these four options is/are the correct answer(s) or Question with Short Answer.
- Answer to each question will be evaluated according to the following marking scheme:
    - **Full Marks:** +4 If ONLY the correct option is chosen or correct answer is given
    - **Zero Marks:** 0 If none of the options is chosen (i.e. the question is unanswered);
    - **Negative Marks:** −1 In all other cases.

Question 1

At time $t = 0$, a solid ball of mass M and radius R *slides without rolling* up a fixed plane, inclined at an angle $\theta = 45°$ to the horizontal, with speed v_0 as shown in figure below. Assume that the coefficient of kinetic friction between the ball and the plane is $\mu_k = 1$. From $t = 0$ up to the instant when the ball starts to roll without slipping up the plane, the work done by the force of kinetic friction is W. If $\frac{|W|}{Mv_0^2} = \frac{1}{n}$, what is the value of n?
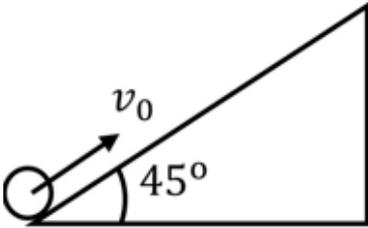
Answer: 9

Question 2

A system of two particles A and B of masses M and $2M$ respectively, can move on horizontal frictionless surface between two fixed walls, as shown in Figure (a). All collisions are elastic. At any time, if the speeds of A and B are u and v, respectively, then define $x = u\sqrt{M}$ and $y = v\sqrt{2M}$, with positive and negative values of x and y denoting motion towards the right and left, respectively. Initially, A and B start midway between the walls, and move in opposite directions, with A moving to the left, and their first collision with the walls is simultaneous. In the qualitative $x - y$ diagram of Figure (b), points P, Q and R represent the system before this first collision, just after this first collision and just before the second collision of A with a wall, respectively. If α and β are angles made by QP and QR with the positive x-axis respectively, what is the magnitude $\left|\frac{\tan \alpha}{\tan \beta}\right|$?
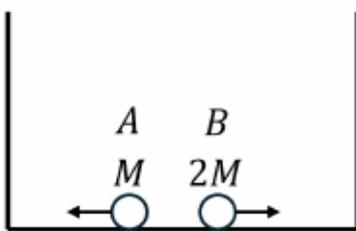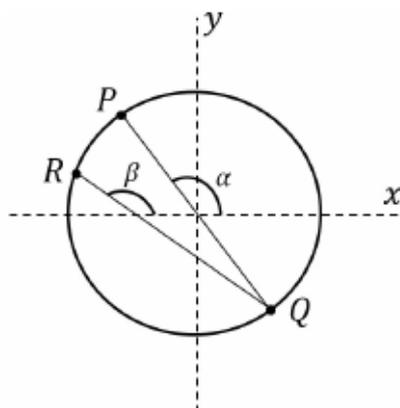

Figure (a)   Figure (b)

Answer: 2

Question 3

In a vertical cylindrical fluid column of height H, the pressure P varies with height z, with $z = 0$ denoting the horizontal bottom surface. In terms of the fluid density ρ, it is given that $P = P_0 + \frac{gH}{2\alpha\rho_0}\left(\rho^2 - \rho_0^2\right)$. Here, P_0 and ρ_0 are the pressure and density at $z = 0$, respectively. The acceleration due to gravity is g, and α is a positive constant. At a height $z = H/2$, the pressure is P_1 and the density is ρ_1. For $\alpha = \frac{1}{2}$, $\frac{(P_0 - P_1)}{H\rho_1 g} = \frac{n}{12}$. What is the value of n?

Answer: 7

Question 4

Consider a fixed inclined plane with inclination angle θ and four distinct objects, each of same mass M: A is a cube of edge length $2R$ with all surfaces frictionless. B is a cube of edge length $2R$ with its lower surface having coefficient of kinetic friction $\mu_k = 1$ with the plane. C is a solid sphere and D is a thin spherical shell, both of diameters $2R$. Four situations with two objects moving on the plane are depicted in Cases 1 - 4 as shown in figure below. In each case, a massless spring is inserted between the two objects, with the ends of the spring being frictionless and the spring parallel to the plane. Assume that C and D roll without slipping. The angle θ is chosen such that the common acceleration down the plane is a, and N is the magnitude of the normal reaction on the objects due to the spring. Which of the following options is(are) correct?


Case 1.   Case 2.   Case 3.   Case 4.
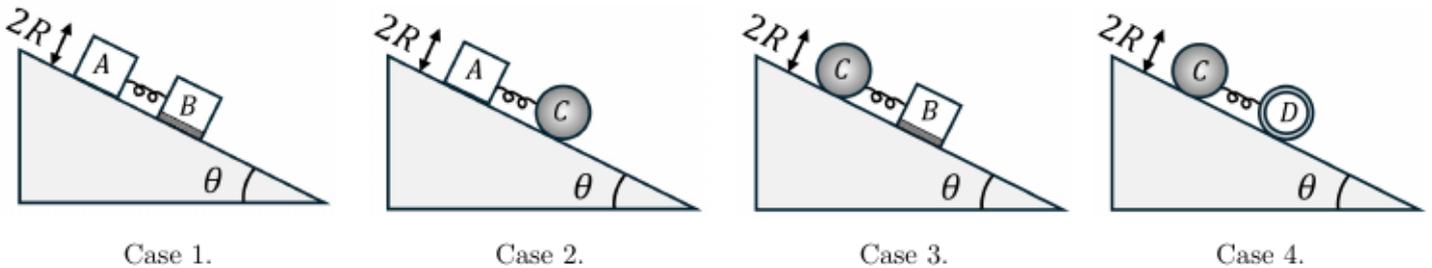
(A) For case 1, $N = \frac{Mg\sin\theta}{2}$
(B) For case 2, $a = \frac{5}{6}g\sin\theta$
(C) For case 3, $a = \frac{5g}{12}\left(2\sin\theta - \cos\theta\right)$
(D) For case 4, $N = \frac{2}{23}Mg\sin\theta$

Answer: (B), (C), (D)

Question 5

Two thin uniform horizontal discs A and B of masses M and $2M$ respectively, and radii R and $2R$ respectively, lie on a frictionless horizontal plane, and are infinitesimally separated by a small distance δ along the x axis. A and B are rotating in opposite directions about vertical axes passing through their centres of mass, with constant angular speeds 2ω and ω, respectively, as shown in figure below. They are instantaneously joined by making $\delta = 0$, after which they move as a single body. Assume that while joining, any resulting force can be neglected. If K_i and K_f are the initial and final kinetic energies of the system respectively, $\frac{K_f}{K_i} = \frac{1}{n}$. What is the value of n?
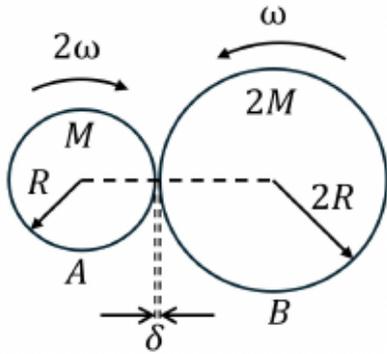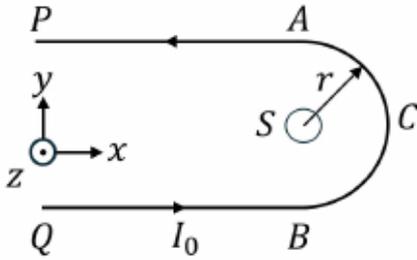

Answer: 7

Question 6

A non-conducting solid sphere of radius R carries a charge density $\rho(r) = \rho_0 \frac{r}{R}$ where ρ_0 is a positive constant and r denotes the radial distance from its centre. The sphere is placed between two infinite uniformly charged thin non-conducting sheets A and B, both in the $y - z$ plane, as shown in figure below. Sheet A carries uniform positive surface charge density σ, while B carries uniform negative surface charge density $-\sigma$, and they just touch the sphere at diametrically opposite points on the x-axis. The centre of the sphere is at the origin, and the zero of the electrostatic potential V is taken at $x \to \infty$. At the centre of the sphere, $V = \frac{1}{\epsilon_0}(P\rho_0 + Q\sigma)$ with ϵ_0 the permittivity of vacuum, and P, Q are functions of R. If $\frac{P}{Q} = \frac{R}{n}$, what is the value of n? (SI units are used).
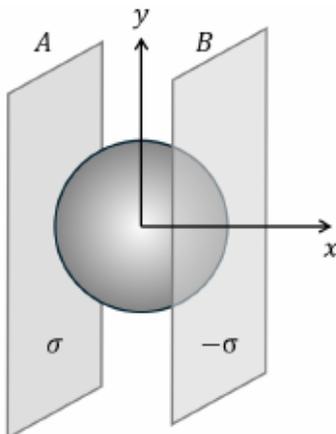
Answer: 3

Question 7

A thin long conductor lying on the $x - y$ plane consists of two parallel regions PA and QB along with a semi-circular region ACB of radius r, as shown in figure below. The point S lies at the centre of AB. The conductor is carrying a constant current I_0 (whose direction is shown by the arrows). A circular conducting wire loop of resistance R and area a is kept such that its centre at S is fixed. Assume that the radius of the loop is small compared to r, so that the magnetic field inside it can be thought of as uniform and entirely due to the one at S. The vector normal to the surface of the loop $\hat{n}$ is made to vary with time as $\hat{n} = \hat{j}\sin(\omega t) + \hat{k}\cos(\omega t)$ with constant ω. If $\vec{\tau}$ is the torque on the loop at $t = \frac{\pi}{3\omega}$, then with $\alpha = \frac{\mu_0^2 a^2 I_0^2 (2+\pi)^2 \omega}{64\pi^2 r^2 R}$, where μ_0 is the permeability of vacuum, which of the following options is correct? (SI units are used).

(A) $\vec{\tau} = 3\alpha\hat{i}$ (B) $\vec{\tau} = -3\alpha\hat{i}$ (C) $\vec{\tau} = \alpha\hat{j}$ (D) $\vec{\tau} = -\alpha\hat{j}$

Answer: A

Question 8

Given an initial amount of radioactive element, the fraction of the amount decays in first 5 hours is found to be four times the fraction it decays in next 5 hours. The fractions are calculated with respect to the initial quantity. The half- life of the element is (in hours)

(A) 2.5 (B) 5.0 (C) 7.5 (D) 10

Answer: A

Question 9

An electron from the n th orbit of a hydrogen atom jumps to the first orbit and gains a speed of $\frac{8hR}{9m}$ in the process, where h is the Planck's constant, R is the Rydberg constant, and m is the electron mass. What is the value of n?

Answer: 3

Question 10

Two Carnot engines run in such a manner that any heat released in one cycle of the first engine is absorbed in one cycle of the second engine. Both the engines run with the same efficiency. The net efficiency of them working together is 9/25. If heat absorbed in one cycle by the first engine is 500 J then how much work does the first engine perform in each cycle, in Joules?
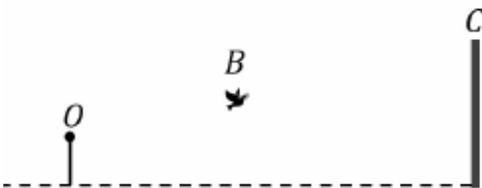
Answer: 100

Question 11

A square metallic loop of total resistance 1 KΩ, mass 1 g, and each side having length 1 m, starts entering a region of magnetic field of magnitude 1T and perpendicular to its plane, moving with a constant speed 10 m/s. Once the loop fully enters the region of the magnetic field, what will be the speed in m/s?

Answer: 9

Question 12

A bird B is flying horizontally towards a vertical cliff C at a constant speed of v_B m/s while emitting sound of frequency 1000 Hz (see figure below). A stationary observer O is positioned in such a way that the bird moves away from it. If the beat frequency perceived by the observer due to the superposition of the direct and reflected sounds from the cliff is 100 Hz, then what is the value of v_B?

Assume that the speed of sound in air is 340 m/s.

Answer: 16.96 (Range 16.9 - 17.1)

Question 13

A thermally insulated cylindrical container contains n moles of an ideal monoatomic gas at temperature T, which is kept in equilibrium using a freely movable piston of mass m, in such a way that in equilibrium the piston is at a height L from the base of the cylinder. There is also a spring of spring constant k and natural length L connected from the piston to the base of the cylinder. If the piston is slightly pushed, then it performs vertical simple harmonic motion. The angular frequency of the motion is ω_1 if the piston is thermally insulated, and ω_2 if the piston is thermally conducting with the outside being at temperature T. If $\omega_1^2 - \omega_2^2 = N\left(\frac{nRT}{mL^2}\right)$, then what is the value of N?
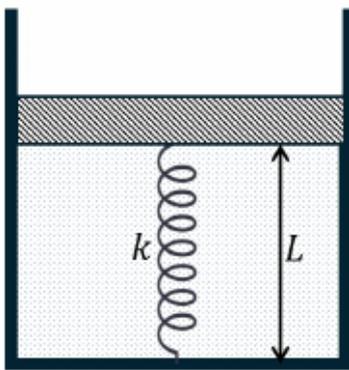
Answer: 0.67 (Range 0.66 - 0.68)

Question 14

A fish in still water looking up towards the sky, sees a cone of light due to total internal reflection. If another fluid layer of refractive index n is put on the water surface, such that $n_{air} < n_f < n_{water}$, then

(A) The cone angle would increase.
(B) The cone angle would decrease.
(C) The cone angle would remain the same.
(D) There will be no total internal reflection.

Answer: C

Question 15

A solenoid of radius R and length L is made of a light flexible material and a steady current is passed through it. Considering only electromagnetic forces, which of the following statements is true?

(A) Both R and L will decrease.
(B) Both R and L will increase.
(C) R will decrease, and L will increase.
(D) R will increase, and L will decrease.

Answer: D

Question 16

Light refracts from a denser medium to a rarer medium. θ_B is the Brewster angle and θ_{TIR} is the angle for total internal reflection. Which of the following is true?

(A) $\theta_B > \theta_{TIR}$.
(B) $\theta_B < \theta_{TIR}$.
(C) $\theta_B = \theta_{TIR}$.
(D) No relation can be established between θ_B and θ_{TIR}, and this depends on the specific refractive index ratio.

Answer: B

Question 17

A particle moves in a potential $V(x) = -x^2 + 2x^4$. Which of the above statements is true?

(A) The particle experiences no force at 3 points (two stable and one unstable).
(B) The particle experiences no force at 3 points (one stable and two unstable).
(C) The particle experiences no force at 3 points (all unstable).
(D) There is only one point of stable equilibrium $x = 0$.

Answer: B

Question 18

An experiment is carried out in a balanced Michelson interferometer. When a coherent blue light of wavelength 400 nm is used, one mirror needs to be moved by a distance d_b to obtain the next bright fringe. The same experiment is repeated with red light of wavelength 600 nm, wherein the distance moved to obtain the next bright fringe is d_r. What is he ratio of d_b/d_r?

(A) 1/3. (B) 2/3. (C) 1 (D) 4/3.

Answer: B

Question 19

Three polarizers P_1, P_2 and P_3 are placed parallel to one another, in that order. The transmission angle between P_1 and P_2 is θ, which is also the transmission angle between P_2 and P_3. If unpolarized light of intensity I_0 is incident on P_1, what will be the intensity of the transmitted light through P_3?

(A) $\frac{I_0}{2}\cos^4\theta$. (B) $2I_0\cos^4\theta$. (C) $I_0\sin^2(2\theta)$. (D) $\frac{I_0}{2}\cos^2(2\theta)$.

Answer: A

Question 20

A sinusoidal signal of amplitude 10 V and frequency 50 Hz is given to a full wave rectifier. For the output signal, what will be the peak amplitude and frequency?

(A) 10 V, 50 Hz. (B) 5 V, 50 Hz. (C) 10 V, 100 Hz. (D) 5 V, 100 Hz.
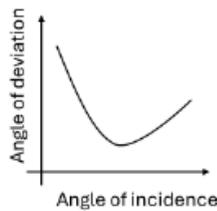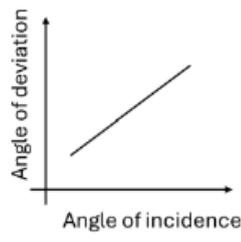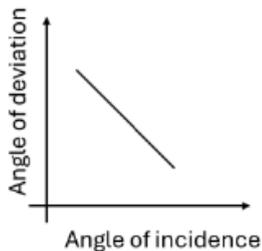
Answer: C

Question 21

An object is placed on the optic axis of a concave mirror. If the distance between the object and the mirror is variable, which of the following options is(are) true?

(A) A Real image could be formed.
(B) A virtual image could be formed.
(C) There is no magnification of the images.
(D) The real image is magnified but not the virtual image.

Answer: A, B

Question 22

Which of the following plots represent the correct relation between the angle of incidence and angle of deviation for a triangular prism?

Plot 1.   Plot 2.   Plot 3.   Plot 4.
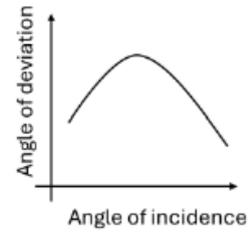
(A) Plot 1. (B) Plot 2. (C) Plot 3. (D) Plot 4.

Answer: C