- This section contains Fifteen (15) questions
- Each question has FOUR options, ONE / MULTIPLE of these four options is/are the correct answer(s) or Question with Short Answer.
- Answer to each question will be evaluated according to the following marking scheme:
  - **Full Marks:** +4 If ONLY the correct option is chosen or correct answer is given
  - **Zero Marks:** 0 If none of the options is chosen (i.e. the question is unanswered);
  - **Negative Marks:** −1 In all other cases.

1. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

(A) 5 (B) 7 (C) 9 (D) 11.

Answer: (C)

2. Let T be the set of all non-zero real numbers such that the quadratic equation $cx^2 - x + c = 0$ has two distinct real roots α and β satisfying the inequality $|\alpha - \beta| < 1$. Then which of the following intervals is(are) a subset(s) of T ?

(A) $(-\frac{1}{2}, -\frac{1}{\sqrt{5}})$ (B) $(-\frac{1}{\sqrt{5}}, 0)$ (C) $(0, \frac{1}{\sqrt{5}})$ (D) $(\frac{1}{\sqrt{5}}, \frac{1}{2})$

Answer: (A), (D)

3. Let $f : \mathbb{R} \to [-2, 2]$ be a twice differentiable function satisfying $(f(0))^2 + (f'(0))^2 = 85$. Then, which of the following statement(s) is (are) TRUE ?

(A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s).

(B) There exists $a \in (-4, 0)$ such that $|f'(a)| \leq 1$.

(C) $\lim_{x\to\infty} f(x) = 1$.

(D) There exists $b \in (-4, 4)$ such that $f(b) + f''(b) = 0$ and $f'(b) \neq 0$.

Answer: (A), (B), (D)

4. If $f(x) = \begin{vmatrix} cos2x & cos2x & sin2x \\ -cosx & cosx & -sinx \\ sinx & sinx & cosx \end{vmatrix}$, then

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$.

(B) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$.

(C) $f(x)$ attains its maximum at $x = 0$.

(D) $f(x)$ attains its minimum at $x = 0$.

Answer: (B), (C)

5. Three randomly chosen non-negative integers a, b and c are found to satisfy the equation $a+b+c=10$. Then the probability that c is even, is

(A) $\frac{36}{55}$, (B) $\frac{6}{11}$ (C) $\frac{1}{2}$, (D) $\frac{5}{11}$.

Answer: (B)

6. Let $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ be two vectors. Consider a vector $\vec{c}=\alpha\vec{a}+\beta\vec{b}$. If the projection of $\vec{c}$ on the vector $\vec{a}+\vec{b}$ is $3\sqrt{2}$, then the minimum value of $(\vec{c}-(\vec{a}\times\vec{b})).\vec{c}$ is

(A) 12 (B) 15 (C) 16 (D) 18.

Answer: (D)

7. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(1)=2$ and it satisfies the relation $f(x+y)=f(x)f(y)$ for all natural numbers x and y. Then the value of the natural number n such that $\sum_{k=1}^{r} f(n+k)=16(2^r-1)$ is

(A) 3 (B) 4 (C) 5 (D) 6.

Answer: (A)

8. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is

(A) of area zero.

(B) right angled isosceles.

(C) equilateral.

(D) obtuse angled isosceles.

Answer: (C)

9. Let $f : [1, \infty] \to [2, \infty]$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{8}{3}$

Answer: (D)

10. Let $f : [\frac{1}{2}, 1] \to \mathbb{R}$ be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f(\frac{1}{2}) = 1$. Then, the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval

(A) $(2e-1, 2e)$ (B) $(e-1, 2e-1)$ (C) $(\frac{e-1}{2}, e-1)$ (D) $(0, \frac{e-1}{2})$.

Answer: (D)

11. The graph of the function $f(x) = cosx\ cos(x+2) - cos^2(x+1)$ is

(A) a straight line passing through $(0, -sin^2 1)$ with slope 2.
(B) a straight line passing through $(0, 0)$.
(C) a parabola with vertex $(1, -sin^2 1)$
(D) a straight line passing through the point $(\frac{\pi}{2}, -sin^2 1)$ and parallel to the X-axis.

Answer: (D)

12. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal to that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(A) equation of the ellipse is $x^2 + 2y^2 = 2$
(B) the foci of the ellipse are $(\pm 1, 0)$
(C) equation of the ellipse is $x^2 + 2y^2 = 4$
(D) the foci of the ellipse are $(\pm\sqrt{2}, 0)$

Answer: (A), (B)

13. Considering only the principal values of inverse trigonometric functions, the set

$$A = \{x \geq 0 : tan^{-1}(2x) + tan^{-1}(3x) = \frac{\pi}{4}\}$$

(A) is an empty set

(B) is a singleton

(C) contains more than two elements

(D) contains two elements

Answer: (B)

14. A plane passes through $(1, -2, 1)$ and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, then the distance of the plane from the point $(1, 2, 2)$ is

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$.

Answer: (D)

15. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the Y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

Answer: (C)

16. The least value of a for which $4ax^2 + \frac{1}{x} \geq 2$, for all $x > 0$, is

(A) $\frac{1}{27}$ (B) $\frac{1}{3}$ (C) $\frac{8}{27}$ (D) $\frac{2}{27}$

Answer: (C)

17. How many 3×3 matrices A with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $A^T A$ is 5

(A) 126 (B) 162 (C) 198 (D) 135

Answer: (C)

18. Let $f : \mathbb{R} \to \mathbb{R}$ be such that for all $x \in \mathbb{R}$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P.. Then the minimum value of $f(x)$ is :

(A) 0 (B) 4 (C) 3 (D) 2

Answer: (C)

19. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{49}$

Answer: (A)

20. Let $f : [0, 2] \to \mathbb{R}$ be a continuous function on $[0, 2]$ and it is differentiable on $(0, 2)$ with $f(0) = 1$. Let $F(x) = \int_0^{x^2} f(\sqrt{t})dt$ for all $x \in [0, 2]$. If $F'(x) = f(x)$ for all $x \in (0, 2)$, then the value of $F(2)$ equals

(A) $e^2 - 1$ (B) $e^4 - 1$ (C)$e - 1$ (D) e^4

Answer: (B)

21. Let $A = (a_{ij})$ be a 3×3 matrix such that $a_{ij} = |i - j| + 1$ for all $1 \le i, j \le 3$. Let $Adj(A)$ denote the adjoint of A. Then $det(Adj(Adj(A))) =$

(A) 2^{10} (B) 2^{12} (C) 2^{16} (D) 2^{24}

Answer: (B)