Lecture # 13: Particle Image Velocimetry Technique

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Particle-based Flow Diagnostic Techniques

- Seeded the flow with small particles (~ μm in size)

- **Assumption**: the particle tracers move with the same velocity as local flow velocity!

\[
\text{Flow velocity } V_f = \text{Particle velocity } V_p
\]

Measurement of particle velocity
Particle-based techniques: Particle Image Velocimetry (PIV)

- To seed fluid flows with small tracer particles (~\(\mu\)m), and assume the tracer particles moving with the same velocity as the low fluid flows.
- To measure the displacements (\(\Delta L\)) of the tracer particles between known time interval (\(\Delta t\)). The local velocity of fluid flow is calculated by \(U = \frac{\Delta L}{\Delta t}\).

\[ U = \frac{\Delta L}{\Delta t} \]

\[ \Delta L \]

\[ t = t_0 + \Delta t \]

\[ t = t_0 \]

A. \(t = t_0\)

B. \(t = t_0 + 10 \ \mu s\)

C. Derived Velocity field
**PIV System Setup**

*Particle tracers:* to track the fluid movement.

*Illumination system:* to illuminate the flow field in the interest region.

*Camera:* to capture the images of the particle tracers.

*Synchronizer:* the control the timing of the laser illumination and camera acquisition.

*Host computer:* to store the particle images and conduct image processing.
Tracer Particles for PIV

- Tracer particles should be **neutrally buoyant and small enough** to follow the flow perfectly.
- Tracer particles should be **big enough** to scatter the illumination lights efficiently.
- The scattering efficiency of trace particles also strongly depends on the ratio of the **refractive index** of the particles to that of the fluid.

*For example: the refractive index of water is considerably larger than that of air.*

The scattering of particles in air is at least one order of magnitude more efficient than particles of the same size in water.
A primary source of measurement error is the influence of gravitational forces when the density of the tracer particles is different to the density of work fluid.

\[ U_g = d_p \frac{2(\rho_p - \rho)}{18\mu} g \]

The velocity lag of a particle in a continuously acceleration fluid will be:

\[ \vec{U}_s = \vec{U}_p - \vec{U} = d_p \frac{2(\rho_p - \rho)}{18\mu} g \]

\[ \vec{U}_p(t) = \vec{U}(1 - \exp(-\frac{t}{\tau_s})); \]

\[ \tau_s = d_p \frac{2\rho_p}{18\mu} \]

Fig. 2.1. Time response of oil particles with different diameters in a decelerating air flow.
Tracer Particles for PIV

- Tracers for PIV measurements in liquids (water):
  - Polymer particles (d=10~100 μm, density = 1.03 ~ 1.05 kg/cm³)
  - Silver-covered hollow glass beams (d =1 ~10 μm, density = 1.03 ~ 1.05 kg/cm³)
  - Fluorescent particle for micro flow (d=200~1000 nm, density = 1.03 ~ 1.05 kg/cm³).
  - Quantum dots (d= 2 ~ 10 nm)

- Tracers for PIV measurements in gaseous flows:
  - Smoke …
  - Droplets, mist, vapor…
  - Condensations ….
  - Hollow silica particles (0.5 ~ 2 μm in diameter and 0.2 g/cm³ in density for PIV measurements in combustion applications.
  - Nanoparticles of combustion products
The illumination system of PIV is always composed of light source and optics.

Lasers: such as Argon-ion laser and Nd:YAG Laser, are widely used as light source in PIV systems due to their ability to emit monochromatic light with high energy density which can easily be bundled into thin light sheet for illuminating and recording the tracer particles without chromatic aberrations.

Optics: always consisted by a set of cylindrical lenses and mirrors to shape the light source beam into a planar sheet to illuminate the flow field.
Double-pulsed Nd:Yag Laser for PIV

Fig. 2.17. Double oscillator laser system with critical resonators

Figure 4.5: Gemini with Extended Bate Plate for UV Operation
Optics for PIV

Side view

Light sheet

-50 mm 200 mm 500 mm

Top view

Thickness

-50 mm 200 mm 500 mm
Cameras

• The widely used cameras for PIV:
  • Photographic film-based cameras or Charged-Coupled Device (CCD) cameras.

• Advantages of CCD cameras:
  • It is fully digitized
  • Various digital techniques can be implemented for PIV image processing.
  • Conventional auto- or cross- correlation techniques combined with special framing techniques can be used to measure higher velocities.

• Disadvantages of CCD cameras:
  • Low temporal resolution (defined by the video framing rate):
  • Low spatial resolution:
Interlaced Cameras

- The fastest response time of human being for images is about ~ 15Hz.
- Video format:
  - PAL (Phase Alternating Line) format with frame rate of $f=25\text{Hz}$ (sometimes in $50\text{Hz}$). Used by U.K., Germany, Spain, Portugal, Italy, China, India, most of Africa, and the Middle East.
  - NTSC format: established by National Television Standards Committee (NTSC) with frame rate of $f=30\text{Hz}$. Used by U.S., Canada, Mexico, some parts of Central and South America, Japan, Taiwan, and Korea.

Old field
(1,3,5...639)

Even field
(2,4,6...640)

480 pixels by 640 pixels

Interlaced camera

1st field: Odd field
2nd field: Even field

One complete frame using interlaced scanning
Progressive scan camera

- All image systems produce a clear image of the background
- Jagged edges from motion with interlaced scan
- Motion blur caused by the lack of resolution in the 2CIF sample
- Only progressive scan makes it possible to identify the driver

Note: In these examples, the cameras have been using the same lens. The car has been driving at 20 km/h (15 mph) using cruise control.
**Synchronizer**

- **Function of Synchronizer:**
  - To control the timing of the laser illumination and camera acquisition

![Diagram of Synchronizer](image)

To laser

To camera

From computer

Timing of pulsed laser

Timing of CCD camera

1st frame exposure

2nd frame exposure

Δt

33.33ms (30Hz)

Time
Host computer

- To send timing control parameter to synchronizer.
- To store the particle images and conduct image processing.
Single-frame technique

Particle streak velocimetry

\[ L = V \Delta t \]

Single-frame technique

Single Frame/Single Pulse

Single Frame/Double Pulse

Single Frame/Multiple Pulse

Streak line

Particle

single-pulse

Multiple-pulse
Multi-frame technique

Double Frame/Single Exposure

\[ t = t_0 \]

\[ \Delta L \]

\[ U = \frac{\Delta L}{\Delta t} \]

Multi-Frame/Single Exposure

\[ t = t_0 \]

\[ \Delta L \]

\[ t = t_0 + \Delta t \]

Multi-Frame/Double-Exposure

\[ t = t_0 \]

\[ \Delta L \]

\[ t = t_0 + \Delta t \]

Fig. 4.2. Multiple frame techniques

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Image Processing for PIV

- To extract velocity information from particle images.

A typical PIV raw image pair:

\[ t = t_0 \quad \text{and} \quad t = t_0 + 4\text{ms} \]

Image processing

\[
\begin{array}{c}
\text{Velocity } U/U_w \\
1.100 \\
1.090 \\
1.080 \\
1.070 \\
1.060 \\
1.050 \\
1.040 \\
1.030 \\
1.020 \\
1.010 \\
1.000 \\
0.990 \\
0.980 \\
0.970 \\
0.960 \\
0.950 \\
0.940 \\
0.930 \\
0.920 \\
0.910 \\
0.900 \\
0.890 \\
0.880 \\
0.870 \\
0.860 \\
0.850 \\
0.840 \\
0.830 \\
0.820 \\
0.810 \\
0.800 \\
0.790 \\
0.780 \\
0.770 \\
0.760 \\
0.750 \\
0.740 \\
0.730 \\
0.720 \\
0.710 \\
0.700 \\
0.690 \\
0.680 \\
0.670 \\
0.660 \\
0.650 \\
0.640 \\
0.630 \\
0.620 \\
0.610 \\
0.600 \\
0.590 \\
0.580 \\
0.570 \\
0.560 \\
0.550 \\
0.540 \\
0.530 \\
0.520 \\
0.510 \\
0.500 \\
0.490 \\
0.480 \\
0.470 \\
0.460 \\
0.450 \\
0.440 \\
0.430 \\
0.420 \\
0.410 \\
0.400 \\
0.390 \\
0.380 \\
0.370 \\
0.360 \\
0.350 \\
0.340 \\
0.330 \\
0.320 \\
0.310 \\
0.300 \\
0.290 \\
0.280 \\
0.270 \\
0.260 \\
0.250 \\
0.240 \\
0.230 \\
0.220 \\
0.210 \\
0.200 \\
0.190 \\
0.180 \\
0.170 \\
0.160 \\
0.150 \\
0.140 \\
0.130 \\
0.120 \\
0.110 \\
0.100 \\
0.090 \\
0.080 \\
0.070 \\
0.060 \\
0.050 \\
0.040 \\
0.030 \\
0.020 \\
0.010 \\
0.000 \\
\end{array}
\]
Particle Tracking Velocimetry (PTV)

1. Find position of the particles at each images
2. Find corresponding particle image pair in the different image frame
3. Find the displacements between the particle pairs.
4. Velocity of particle equates the displacement divided by the time interval between the frames.

Low particle-image density case

Figure 2. The particle-tracking algorithm applied to the sequence of images (i-1) to (i+2). (a) Detected particles in frames (i) (light blue) and (i+1) (dark red) with overlapped centroids and velocity vectors. (b) Detected centroids of particles in all four frames with overlaid velocity vectors. Consecutive frames are colored from light to dark.
Particle Tracking Velocimetry (PTV)-2

1. Find position of the particles at each image
2. Find corresponding particle image pair in the different image frame
3. Find the displacements between the particle pairs.
4. Velocity of particle equates the displacement divided by the time interval between the frames.

Four-frame-particle tracking algorithm

(a) Original image (8 bit grayscale)

Overlap of four consecutive frames
Correlation-based PIV methods

Corresponding flow velocity field

\[ t = t_0 \]

\[ t = t_0 + \Delta t \]

high particle-image density
Correlation-based PIV methods

Correlation coefficient function

\[
R(p,q) = \frac{\int (f(x,y) - \bar{f})(g(x,y) - \bar{g}) \, dv}{\sqrt{\int (f(x,y) - \bar{f})^2 \, dv} \sqrt{\int (g(x,y) - \bar{g})^2 \, dv}}
\]
Cross Correlation Operation

Signal A:

Signal B:

\[ R(u) = \frac{\int [f(x) * g(x+u)]dx}{\sqrt{\int [f(x)^2]dx * \int [g(x+u)^2]dx}} \]

\[ \phi(u,v) = \sum_{x,y} f(x,y) \delta(x - u, y - v) \]
Correlation coefficient distribution

$$R(p,q) = \frac{\int (f(x,y) - \overline{f})(g(x,y) - \overline{g})dv}{\sqrt{\int (f(x,y) - \overline{f})^2 dv \int (g(x,y) - \overline{g})^2 dv}}$$

Peak location
Comparison between PIV and PTV

- **Particle Tracking Velocimetry:**
  - Tracking individual particle
  - Limited to low particle image density case
  - Velocity vector at random points where tracer particles exist.
  - Spatial resolution of PTV results is usually limited by the number of the tracer particles

- **Correlation-based PIV:**
  - Tracking a group of particles
  - Applicable to high particle image density case
  - Spatial resolution of PIV results is usually limited by the size of the interrogation window size
  - Velocity vector can be at regular grid points.
Estimation of differential quantities

\[
\frac{dU}{dX} = \begin{bmatrix}
\frac{\partial U}{\partial X} & \frac{\partial U}{\partial Y} & \frac{\partial U}{\partial Z} \\
\frac{\partial V}{\partial X} & \frac{\partial V}{\partial Y} & \frac{\partial V}{\partial Z} \\
\frac{\partial W}{\partial X} & \frac{\partial W}{\partial Y} & \frac{\partial W}{\partial Z}
\end{bmatrix}
\] (6.7)

This deformation tensor can be decomposed into a symmetric part and an antisymmetric part:

\[
\frac{dU}{dX} = \begin{bmatrix}
\frac{\partial U}{\partial X} & \frac{1}{2} \left( \frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial X} + \frac{\partial U}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) & \frac{\partial V}{\partial Y} & \frac{1}{2} \left( \frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Z} + \frac{\partial V}{\partial X} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial Z} + \frac{\partial V}{\partial Y} \right) & \frac{\partial W}{\partial Z}
\end{bmatrix}
\] (6.8)

\[
= \begin{bmatrix}
0 & \frac{1}{2} \left( \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial X} - \frac{\partial U}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) & 0 & \frac{1}{2} \left( \frac{\partial W}{\partial Y} - \frac{\partial V}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Z} - \frac{\partial V}{\partial X} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial Z} - \frac{\partial V}{\partial Y} \right) & 0
\end{bmatrix}
\] (6.9)

A substitution of the strain and vorticity components yields:

\[
\frac{dU}{dX} = \begin{bmatrix}
\epsilon_{XX} & \frac{1}{2} \epsilon_{XY} & \frac{1}{2} \epsilon_{XZ} \\
\frac{1}{2} \epsilon_{XY} & \epsilon_{YY} & \frac{1}{2} \epsilon_{YZ} \\
\frac{1}{2} \epsilon_{XZ} & \frac{1}{2} \epsilon_{YZ} & \epsilon_{ZZ}
\end{bmatrix}
+ \begin{bmatrix}
0 & \frac{1}{2} \omega_{Z} & -\frac{1}{2} \omega_{X} \\
-\frac{1}{2} \omega_{Z} & 0 & \frac{1}{2} \omega_{Y} \\
\frac{1}{2} \omega_{X} & -\frac{1}{2} \omega_{Y} & 0
\end{bmatrix}
\] (6.10)
## Estimation of differential quantities

<table>
<thead>
<tr>
<th>Operator</th>
<th>Implementation</th>
<th>Accuracy</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward difference</td>
<td>( \frac{f_{i+1} - f_i}{\Delta X} \approx O(\Delta X) )</td>
<td>( \approx 1.41 \frac{\varepsilon U}{\Delta X} )</td>
<td></td>
</tr>
<tr>
<td>Backward difference</td>
<td>( \frac{f_i - f_{i-1}}{\Delta X} \approx O(\Delta X) )</td>
<td>( \approx 1.41 \frac{\varepsilon U}{\Delta X} )</td>
<td></td>
</tr>
<tr>
<td>Center difference</td>
<td>( \frac{f_{i+1} - f_{i-1}}{2\Delta X} \approx O(\Delta X^2) )</td>
<td>( \approx 0.7 \frac{\varepsilon U}{\Delta X} )</td>
<td></td>
</tr>
<tr>
<td>Richardson</td>
<td>( \frac{f_{i-2} - 8f_{i-1} + 8f_{i+1} - f_{i+2}}{12\Delta X} \approx O(\Delta X^3) )</td>
<td>( \approx 0.95 \frac{\varepsilon U}{\Delta X} )</td>
<td></td>
</tr>
<tr>
<td>Least squares</td>
<td>( \frac{2f_{i+2} + f_{i+1} - f_{i-1} - 2f_{i-2}}{10\Delta X} \approx O(\Delta X^2) )</td>
<td>( \approx 1.0 \frac{\varepsilon U}{\Delta X} )</td>
<td></td>
</tr>
</tbody>
</table>
Estimation of Vorticity distribution

\[ \omega_z = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \]

Fig. 6.7. Vorticity field estimates obtained from twice oversampled PIV data, e.g. the interrogation window overlap is 50%. The vortex pair is known to be laminar and thus should have smooth vorticity contours.
Estimation of Vorticity distribution

**Stokes Theorem:**

\[ \Gamma = \oint_C \mathbf{V} \, d\mathbf{l} = - \iint_S \mathbf{\nabla} \times \mathbf{A} \cdot d\mathbf{A} \]

\[ \Rightarrow \omega_z = \frac{\Gamma_{x-y}}{dA} \]

\[ (\omega_z)_{i,j} \equiv \frac{\Gamma_{i,j}}{4\Delta X \Delta Y} \]

with

\[ \Gamma_{i,j} = \frac{1}{2} \Delta X (U_{i-1,j-1} + 2U_{i,j-1} + U_{i+1,j-1}) \]

\[ + \frac{1}{2} \Delta Y (V_{i+1,j-1} + 2V_{i+1,j} + V_{i+1,j+1}) \]

\[ - \frac{1}{2} \Delta X (U_{i+1,j+1} + 2U_{i,j+1} + U_{i-1,j+1}) \]

\[ - \frac{1}{2} \Delta Y (V_{i-1,j+1} + 2V_{i-1,j} + V_{i-1,j-1}) \]

**Fig. 6.10.** Vorticity field estimates obtained from PIV velocity fields by the circulation method: (left) the velocity field is twice oversampled, (right) four times oversampled. The contours of this laminar vortex pair are known to be smooth such that the nonuniformities are due to measurement noise.
Vorticity distribution Examples

Spanwise Vorticity (Z-direction)

\( \text{Re} = 6,700 \)
\( U_{in} = 0.33 \text{ m/s} \)

\( X \) mm
\( Y \) mm

shadow region

spanwise vorticity (1/s)

-3.2 -2.7 -2.2 -1.7 -1.2 -0.7 -0.2 0.3 0.8 1.3 1.8

water free surface

GA(W)-1 airfoil

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Ensemble-averaged quantities

- Mean velocity components in x, y directions:
  \[ U = \frac{1}{N} \sum_{i=1}^{N} u_i \]
  \[ V = \frac{1}{N} \sum_{i=1}^{N} v_i \]

- Turbulent velocity fluctuations:
  \[ u' = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (u_i - U)^2} \]
  \[ v' = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (v_i - V)^2} \]

- Turbulent Kinetic energy distribution:
  \[ TKE = \frac{1}{2} \rho \left( u'^2 + v'^2 \right) \]

- Reynolds stress distribution:
  \[ \tau = -\rho u'v' = -\rho \frac{1}{N} \sum_{i=1}^{N} (u_i - U)(v_i - U) \]
Ensemble-averaged quantities

- T.K.E
- Normalized Reynolds Stress
- Vorticity
- U m/s
- 10 m/s

Images depict airflow and shadow regions around GA(W)-1 airfoil.
Pressure field estimation

\[
\begin{align*}
    u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
    u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{align*}
\]

Figure 6. Instantaneous pressure field around a circular cylinder. (a) Stationary cylinder, (b) low-frequency oscillation ($S_t = 0.2, V_t = 2$) and (c) high-frequency oscillation ($S_t = 1, V_t = 2$).
Integral Force estimation

\[ \frac{\partial}{\partial t} \int_{C.V.} \rho \vec{V} \, dV + \int_{C.S.} (\rho \vec{V} \cdot \vec{V}) \cdot dA = \int_{C.S.} \vec{P} \cdot dA + \int_{C.V.} \rho f \, dV + \vec{F} \]
AerE 545 class notes #34

Microscopic Particle Image Velocimetry technique
Part - 2

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**System Setup of a conventional PIV system**

**Particle tracers:** to track the fluid movement.

**Illumination system:** to illuminate the flow field in the interest region.

**Camera:** to capture the images of the particle tracers.

**Synchronizer:** the control the timing of the laser illumination and camera acquisition.

**Host computer:** to store the particle images and conduct image processing.
System Setup of a Typical Micro-PIV system

Nd:YAG Laser → Beam Expander → Microfluidic device

Microscope Lens (NA=1.4, 60x) → Epi-fluorescent Prism / Filter Cube

l = 532 nm → Lens → l = 560 nm

1030 x 1300 x 12 bit Interline Transfer Cooled CCD Camera

Microscope
Test Section
Syringe Pump
Timing Boxes
Lasers
Camera
Laser Controls
Mirrors
Differences between conventional PIV and μ-PIV

- **Particle tracers:**
  - **Conventional PIV:**
    - Tracer particle size is about ~1 μm for gaseous flows and ~ 10 μm for liquid flows
    - To detect scattering light signal
  - **Micro-PIV:**
    - Coated fluorescent particles.
    - Particle size is much smaller 200 nm ~1000 nm for liquid flows
    - To detect fluorescent light signal

<table>
<thead>
<tr>
<th>Color</th>
<th>Excitation Maxima (nm)</th>
<th>Emission Maxima (nm)</th>
<th>Stokes Shift (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>469 (blue)</td>
<td>509 (green)</td>
<td>40</td>
</tr>
<tr>
<td>Red</td>
<td>541 (green) 365 (UV)</td>
<td>611 (red) 446 (blue)</td>
<td>70 81</td>
</tr>
<tr>
<td>Blue</td>
<td>388 (UV-violet) 412 (violet)</td>
<td>446 (blue) 473 (blue)</td>
<td>58 61</td>
</tr>
</tbody>
</table>
Diameter of Particle Images for Micro-PIV

Diffraction-limited spot size:

\[ d_s = 2.44 \left( M + 1 \right) \frac{\lambda}{2NA} \]

Effective size of the tracer particles:

\[ d_e = \left[ d_s^2 + M^2 d_p^2 \right]^{1/2} \]

Table 1. Effective particle image diameters when projected back into the flow, \( d_e/M \) (μm).

<table>
<thead>
<tr>
<th>Particle Size ( d_p ) (μm)</th>
<th>Microscope Objective Lens Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M = 60 ) ( NA = 1.4 )</td>
</tr>
<tr>
<td>0.01 μm</td>
<td>0.50</td>
</tr>
<tr>
<td>0.10 μm</td>
<td>0.50</td>
</tr>
<tr>
<td>0.20 μm</td>
<td>0.53</td>
</tr>
<tr>
<td>0.30 μm</td>
<td>0.58</td>
</tr>
<tr>
<td>0.50 μm</td>
<td>0.70</td>
</tr>
<tr>
<td>0.70 μm</td>
<td>0.86</td>
</tr>
<tr>
<td>1.00 μm</td>
<td>1.10</td>
</tr>
<tr>
<td>3.00 μm</td>
<td>3.00</td>
</tr>
</tbody>
</table>
Out-of-Plane Spatial Resolution for Micro-PIV

• **Conventional “Macro”-PIV measurements:**
  - The light sheet illuminates only particles contained within the depth of focus of the recording lens, providing high quality in-focus particle images to be recorded with low levels of background noise being emitted from the out-of-focus particles.
  - The out-of-plane spatial resolution of the velocity measurements is defined clearly by the thickness of the illuminating light sheet.

• **Micro-PIV measurements:**
  - Due to the small length scales associated with μ-PIV, it is difficult if not impossible to form a light sheet that is only a few microns thick, and even more difficult to align a light sheet with the object plane of an objective lens.
  - It is common practice in μ-PIV to illuminate the test section with a volume of light, and rely on the depth of field of the lens to define the out-of-plane thickness of the measurement plane.
Out-of-Plane Spatial Resolution for Micro-PIV

A typical Micro-PIV image

\[ \varepsilon = \frac{d_p^4(0)}{d_p^4(z_{cor})} \]

\[ z_{cor} = \frac{1}{2} \left[ \frac{1 - \sqrt{\varepsilon}}{\sqrt{\varepsilon}} \left( \frac{n^2}{NA^2} - 1 \right) \left( d_p^2 + \frac{1.49 (M + 1)^2 \lambda^2}{M^2 NA^2} \right) \right]^{1/2} \]

Table 2. Thickness of the measurement plane for typical experimental parameters, 2z_{cor} (\mu m).

<table>
<thead>
<tr>
<th>Particle Size</th>
<th>M = 60</th>
<th>M = 40</th>
<th>M = 40</th>
<th>M = 20</th>
<th>M = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_p</td>
<td>NA = 1.4</td>
<td>NA = 0.75</td>
<td>NA = 0.6</td>
<td>NA = 0.5</td>
<td>NA = 0.25</td>
</tr>
<tr>
<td>0.01 µm</td>
<td>0.62</td>
<td>2.47</td>
<td>4.67</td>
<td>7.46</td>
<td>34.95</td>
</tr>
<tr>
<td>0.10 µm</td>
<td>0.63</td>
<td>2.49</td>
<td>4.69</td>
<td>7.48</td>
<td>34.97</td>
</tr>
<tr>
<td>0.20 µm</td>
<td>0.66</td>
<td>2.53</td>
<td>4.74</td>
<td>7.53</td>
<td>35.02</td>
</tr>
<tr>
<td>0.30 µm</td>
<td>0.72</td>
<td>2.60</td>
<td>4.82</td>
<td>7.62</td>
<td>35.12</td>
</tr>
<tr>
<td>0.50 µm</td>
<td>0.87</td>
<td>2.80</td>
<td>5.08</td>
<td>7.90</td>
<td>35.43</td>
</tr>
<tr>
<td>0.70 µm</td>
<td>1.06</td>
<td>3.09</td>
<td>5.45</td>
<td>8.30</td>
<td>35.88</td>
</tr>
<tr>
<td>1.00 µm</td>
<td>1.39</td>
<td>3.62</td>
<td>6.15</td>
<td>9.09</td>
<td>36.83</td>
</tr>
<tr>
<td>3.00 µm</td>
<td>3.77</td>
<td>8.31</td>
<td>12.88</td>
<td>17.28</td>
<td>49.36</td>
</tr>
</tbody>
</table>
Visibility of Particles tracers

\[
V = \frac{I(0,0)}{I_B} = \frac{4M^2 \beta^2 (s_o - a)(s_o - a + L)}{\pi CL s_o^2 \left( M^2 d_p^2 + 1.49(M + 1)^2 \lambda^2 / NA^2 \right)}
\]

- For a given set of recording optics, particle visibility can be increased:
  - By decreasing particle concentration, \( C \),
  - By decreasing test section thickness, \( L \).

- For a fixed particle concentration, the visibility can be increased by
  - By decreasing the particle diameter,
  - By increasing the numerical aperture of the recording lens.

- Visibility depends only weakly on magnification, and object distance, so.

Table 3. Maximum percent volume fraction of particles, \( V_p \), while maintaining an in-focus visibility, \( V = 1.5 \), for imaging the center of an \( L = 100 \mu m \) deep device.

<table>
<thead>
<tr>
<th>Particle Size ( d_p )</th>
<th>Microscope Objective Lens Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M = 60 )</td>
<td>( M = 40 )</td>
</tr>
<tr>
<td>( s_o = 0.38 \text{ mm} )</td>
<td>( s_o = 0.89 \text{ mm} )</td>
</tr>
<tr>
<td>0.01 ( \mu m )</td>
<td>6.50E-6</td>
</tr>
<tr>
<td>0.10 ( \mu m )</td>
<td>6.25E-3</td>
</tr>
<tr>
<td>0.20 ( \mu m )</td>
<td>4.50E-2</td>
</tr>
<tr>
<td>0.30 ( \mu m )</td>
<td>1.29E-1</td>
</tr>
<tr>
<td>0.50 ( \mu m )</td>
<td>4.04E-1</td>
</tr>
<tr>
<td>0.70 ( \mu m )</td>
<td>7.47E-1</td>
</tr>
<tr>
<td>1.00 ( \mu m )</td>
<td>1.29E-0</td>
</tr>
<tr>
<td>3.00 ( \mu m )</td>
<td>4.68E-0</td>
</tr>
</tbody>
</table>
Effect of Brownian Motion on micro-PIV measurements

Particle response time:

\[ \tau_p = (1 + 2.76 \, Kn_p) \, d_p^2 \, \rho_p/(18 \mu_f) \]

For 300 nm diameter polystyrene latex spheres immersed in water, the particle response time is \(~10^{-9}\) s.

- Brownian motion is the random thermal motion of a particle suspended in a fluid. The motion results from collisions between fluid molecules and suspended particles.
- The velocity spectrum of a particle due to Brownian motion consists of frequencies too high to be resolved fully and is commonly modeled as Gaussian white noise.
- A quantity more readily characterized is the particle’s average displacement after many velocity fluctuations.
- For time intervals \(\Delta t\) much larger than the particle inertial response time, the dynamics of Brownian motion are independent of inertial parameters such as particle and fluid density.
- The mean square distance of diffusion is proportional to \(D\Delta t\), where \(D\) is the diffusion coefficient of the particle.
- For a spherical particle subject to Stokes drag law, the diffusion coefficient \(D\) was first given by Einstein (1905) as:

\[ D = \frac{\kappa T}{3 \pi \mu d_p} \]

where \(d_p\) is the particle diameter, \(\kappa\) is Boltzmann's constant, \(T\) is the absolute temperature of the fluid, and \(\mu\) is the dynamic viscosity of the fluid.
Effect of Brownian Motion on micro-PIV measurements

\[ \Delta x = u \Delta t \]
\[ \Delta y = v \Delta t \]

- The errors estimated by above Equations show that the relative Brownian intensity error decreases as the time of measurement increases. Larger time intervals produce flow displacements proportional to \( \Delta t \) while the root mean square of the Brownian particle displacements grow as \( \Delta t^{1/2} \).

- In practice, Brownian motion is an important consideration when tracing 50 to 500 nm particles in flow field experiments with flow velocities of less than about 1 mm/s.

For a velocity on the order of 0.5 mm/s and a 500 nm seed particle, the lower limit for the time spacing is approximately 100 \( \mu s \) for a 20% error due to Brownian motion.

- This error can be reduced by both averaging over several particles in a single interrogation spot and by ensemble averaging over several realizations.

- The diffusive uncertainty decreases as \( 1/\sqrt{N} \), where \( N \) is the total number of particles in the average.
**Saffman effect**

- Particles are found to intend to stay away from the regimes with high velocity gradient.
- Poiseuille (1836) is generally acknowledged to be the first modern scientist who recorded evidence of particle migration with his observations that blood cells flowing through capillaries tended to stay away from the walls of the capillaries.
- Taylor (1955) scanned the cross-section of a tube carrying a suspension and noticed areas of reduced cell concentration not only near the walls, but also near the center of the channel.
- Segré and Silberberg (1962) systematically performed experiments that confirmed both observations—migration away from the walls and migration away from the center of the channel and determined that migration rate was proportional to the square of the mean velocity in the channel as well as the fourth power of the particle radius.
- Saffman (1965) analytically considered the case of a rigid sphere translating in a linear unbounded shear field.

\[
\frac{V_m}{V_s} = 0.343 a \sqrt{\frac{G}{\nu}}.
\]

- *G*: velocity gradient
- *Vs*: is slip velocity
- *Vm*: migration velocity
Micro-PIV setup
Correlation averaging method

Conventional method:

- Instantaneous PIV image pair
- Instantaneous Correlation coefficient distribution
- Find peak and sub-pixel interpolation
- Instantaneous Velocity vectors
- Averaging processing
  - Averaged Velocity vectors

Correlation averaging method

- Instantaneous PIV image pair
- Instantaneous Correlation coefficient distribution
- Averaging processing
- Averaged correlation coefficient distribution
- Find peak and sub-pixel interpolation
  - Averaged Velocity vectors

Figure 5. Effect of ensemble correlation: (a) results with conventional correlation for one of the PIV recording pairs; (b) results with ensemble correlation for 101 PIV recording pairs (Wereley, et al., 2001).

Figure 6. Comparison of the evaluation function of (a) a single PIV recording pair with (b) the average of 101 evaluation functions (Wereley, et al., 2001).
Near Wall Flow Measurement Using Micro-PIV

Figure 12b. Near wall view of boxed region from Figure 12a (Meinhart, et al., 1999).

FIG 3. Velocity profiles for flow over a hydrophilic (square) and hydrophobic (triangle) microchannel surface. The velocity profiles are normalized by the free-stream velocity.
Total Internal Reflection

\[
\frac{\sin \varphi_1}{\sin \varphi_2} = \frac{n_2}{n_1}
\]

\(n_1 > n_2 \Rightarrow \varphi_2 > \varphi_1\)

\(\varphi_{2_{\text{max}}} = \pi / 2 \Rightarrow \varphi_{1_{\text{cri}}} = \sin^{-1}\left(\frac{n_2}{n_1}\right)\)

- **TIR occurs when** \(n_2 < n_1\) **and** \(\theta_i > \theta_c\)

---

Evanescent Field Intensity Coordinate System

---
Figure 5. Evanescent field intensity decays exponentially with distance. Altering the beam incident angle or wavelength will change the field penetration depth. Shown is a beam with $\lambda=403$ nm incident on the waveguide at an angle of 61 degrees.\textsuperscript{12}
Nano-PIV

C. M. Zettner, and M. Yoda, Experiments in Fluids (2002)
A Study of Insulin Occlusion Using Insulin Pump

- Objective lens
- Dichroic mirror
- Emitter filter
- Mirror
- Digital delay generator
- CCD camera (SensiCam)

Flexible tubing (i.d. 356μm)

Insulin pump

Objective lens

Beaker

Nd:Yag Laser

Optics

Dichroic mirror

Emitter filter

Host computer

An insulin pump administers insulin through a catheter in the abdominal fat to help control a person's blood sugar levels.

Insulin pump
Micro-PIV Measurement Results
Micro-PIV Measurement Results

![Image of Micro-PIV Measurement Result]

- **X (mm)**: 0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2
- **Y (mm)**: 0.0, 0.1, 0.2, 0.3, 0.4
- **Velocity (mm/s)**: 0.10, 0.50, 0.90, 1.30, 1.70, 2.10, 2.50, 2.90, 3.30

**Channel Wall**
The Lecture Contains:

- Particle image velocimetry
- Apparatus and Instrumentation
- Experimental Setup
  - Test Cell
- Particle Image Velocimetry
  - Seeding Arrangement for PIV
  - Particle Dynamics
  - Generating a Light Sheet
  - Synchronizer
Particle image velocimetry

The method relies on the fact that small particles introduced in a fluid stream would move with the local fluid velocity. These particles, ideally, are neutrally buoyant with respect to the fluid medium and would not respond to buoyancy forces. This is particularly true for particles of very small diameters where surface forces (that scale with the square of the particle diameter) are much larger than body forces (that scale as diameter cube).

The basic measurements in particle image velocimetry (PIV) relate particle displacement $\vec{\delta}$ over a time period $\Delta t$ in such a way that velocity is measured as the ratio of displacement and the time interval. The former being a vector, velocity components in the plane of illumination are jointly determined.

Since particle sizes are very small, a small interrogation area selected by the camera for determination for velocity would have several particles. These, in turn, are indistinguishable. The displacement measured is a statistical quantity, applicable for the collection of particles as a whole. The local velocity thus obtained is a group velocity of these particles. Statistical methods preferred are usually based on cross-correlation between a pair of images that are separated by a time interval of $\Delta t$. Clearly, smaller the time interval, better is the time estimate of velocity. Intervals as small as a few hundred nanoseconds are possible with pulsed lasers; conventional light sources severely fail in this regard. It should also be clear that the cameras used for imaging should record two images of the particle positions separated on the time axis by such a small interval.

The important components of a PIV system would then be (a) a pulsed light source, (b) an imaging system synchronized with the laser, (c) seeding arrangement for creating particles, and (d) software for calculating the cross-correlation function between the image pairs.

The details of a simple PIV system are presented in the following sections.
Apparatus and Instrumentation

A setup for conducting experiments where wake properties of a square cylinder can be studied has been constructed in the laboratory. The setup resembles a low speed wind tunnel, though smaller in the overall size. It is a vertical test cell made of Plexiglas with two optical windows, one for laser sheet and the other for recording images by the CCD camera. The working fluid is air and the direction of overall fluid motion is in the vertically upward direction. Particle Image Velocimetry (PIV) and Hotwire Anemometry (HWA) have been primarily used for velocity measurements. Flow visualization study has been carried out at low seeding density in the PIV setup. The cylinder is oscillated with the help of an electromagnetic actuator. This module describes details of the experimental hardware, including instruments and auxiliary equipment used in the present study. The validation results for proper PIV technique implementation, flow parallelism and turbulence intensity of the test cell and the effect of end plates have been discussed.
**Experimental Setup**

A schematic drawing of the experimental setup is shown in Figure 3.8. It comprises the following components: flow circuit, traversing mechanism for hotwire measurements, laser (pulsed), CCD camera, seeding arrangement for PIV measurements, and data acquisition system. The free-stream velocity approaching the cylinder has been determined using a pitot-static tube connected to a micro-manometer. The micro-manometer has a resolution of 0.001 mm of $H_2O$ it translates to an error in Reynolds number of about $\pm 2$. The details of the test cell are discussed here and the PIV and HWA techniques are presented in the following section.

![Fig 3.8: Schematic of the experimental setup](image)
Test cell

Experiments have been performed in a vertical open-loop airflow system. The crosssection of the active portion of the test cell (to be called the test section) is $9.5 \times 4.8 \text{ cm}^2$ with an overall length of 2 m. The active length of the test section where wake measurements have been carried out is 0.3 m. A contraction ratio of 10:1 ahead of the test section has been used. Prisms of square cross-section (3-4 mm edge) have been used for experiments as square cylinders. They are made either of Plexiglas or brass and carefully machined for sharp edges. Each cylinder is mounted horizontally with its axis perpendicular to the flow direction. It is supported along the two side walls for fixed cylinder experiments and mounted on actuators for oscillating cylinder experiments.

Fig 3.9: Picture of the PIV setup used for the present experiments

Two different $L/D$ ratios (also called aspect ratios) of 16 and 28 have been utilized in the experiments. The two aspect ratios were realized depending on the alignment of the cylinder axis with respect to test section. The corresponding blockage ratios are 0.03 and 0.06 respectively. With reference to Figure 3.8, the $x$- axis is vertical and aligned with the mean flow direction. The $z$- axis coincides with the cylinder axis and the $y$- axis is perpendicular to both $x$ and $z$. 
Flow in the test section was set up by a small fan driven by a single phase motor. The suction side of the fan was used to draw the flow from the test cell. The power supply to the blower was from an online uninterruptible power supply unit to ensure practically constant input voltage to the motor. For better control of the voltage setting, particularly at low fan RPM and hence at low flow rates, the output of the UPS was stepped down via two variacs connected in series. In turn, this had the effect of minimizing the velocity fluctuations in the approach flow. The free stream turbulence level in the approach flow was quite small and it was found to be less than the background noise of the anemometer (\(< 0.05\)%). Flow parallelism in the approach ow was better than 98% over 95% of the width of the test cell. The validation of the test cell is discussed in a later section of the present module.

![PIV components: (a) CCD Camera (b) Nd-YAG laser (c) Synchronizer](image_url)
The flow to the test cell goes through three parts, namely the settling chamber, a honeycomb section and a contraction cone. Fine screens are mounted in the settling chamber for reducing the turbulence level of flow entering the test section. The contraction ratio of the contraction cone in area units is 10:1. The contraction cone reduces the spatial irregularities in the velocity distribution and helps in the decay of turbulence intensity by proper stretching of the vortices. The function of the honeycomb is to straighten the flow by damping the transverse components of velocity, and to reduce the turbulence level by suppressing the turbulence scales that are larger than the size of a honeycomb cell. The screens are used to suppress the small disturbances generated at the outlet tips of the honeycomb. Proper mesh size gradation has been utilized by examining the diameter of the elements of the honeycomb, and hence the length scale of the vortices generated. Specially, two screens, one with a coarse grid (10 per cm²) and the other with a fine grid (100 per cm²) have been used in the test cell.

**Fig 3.11: Imaging system for PIV**
Contd...

The distance maintained between the mesh and honeycomb has been selected by trial and error, to ensure that the smoothest possible flow approaches the square cylinder. Stable velocities in the range of 0.5—3 m/s could realized in the test section. These values correspond to a Reynolds number range of 100 - 700 for the cylinder sizes referred earlier. A seeding arrangement is fitted prior to the honeycomb for PIV measurements.
Particle Image Velocimetry

Traditionally, quantitative measurements of fluid velocity have been carried out using a pitot-static tube and hotwire anemometry. Both these techniques require insertion of a physical probe into the flow domain. This process is intrusive and can alter the flow field itself. In addition, measurements are averages over a small representative volume. The probe has to be physically displaced to various locations to scan the entire region of interest. The development of cost-effective lasers led to the development of Laser Doppler velocimeter (LDV) that uses a laser probe to enable non-intrusive velocity measurements. Velocity information by LDV however, is obtained point-wise similar to that of the pitot-static tube and the hotwire probe. Particle image velocimetry (PIV) is the state-of-the-art technique for velocity measurement in experimental fluid mechanics. Original contributions towards its development were made by Adrian (1991), Gharib (1991), Melling (1997), and Westerweel (1997). The most important advantage of PIV is that it is a non-intrusive technique and gives the spatial details of the flow field over a plane of interest. There is some flexibility in the choice of the measuring plane. The measurement process can be repeated in time to yield temporal evolution of the flow field. The ability to make global velocity measurements makes PIV a special tool in experimental fluid mechanics. With PIV, it is possible to acquire practically instantaneous velocity fields with high spatial resolution. The spatial resolution is limited by the thickness of the laser sheet and the choice of the interrogation spot during analysis. The latter is about 8 or 16 pixels. The smallest length scale that can be detected depends on the size of the pixel, and hence the spatial resolution of the camera. Depending on the camera speed, a time series of images can be recorded during experiments. The ensemble average of the instantaneous velocity vectors yields the time-averaged velocity field. This includes zones of reversed flow that cannot be dealt with by hotwire and pitot probes. Once the velocity field is obtained, other quantities such as vorticity, strain rates and momentum fluxes can be estimated. With developments in lasers, camera and high speed/low cost computers it is now possible to use PIV regularly for research and industrial applications.
The picture of the PIV setup is shown in Figure 3.9 and the photograph of important hardware of PIV is shown in Figure 3.10. In the present experiments, PIV measurements were carried out at selected planes perpendicular and parallel to the cylinder axis. A double pulsed Nd:YAG laser of wavelength $\lambda = 532\text{nm}$ and $15 \text{mJ/pulse}$ with a maximum repetition rate of $15\text{Hz}$ per laser head was used. The light sheet had a maximum scan area of $10 \times 10 \text{cm}^2$. The sheet thickness was about 1 mm to minimize the effect of the out-of-plane velocity component. The assembly of Peltier-cooled 12 bit CCD camera and frame grabber with a frame speed of $8\text{Hz}$ was used for acquisition of PIV images. Figure 3.11 shows geometric diagram of PIV measurements. A cross section of the flow is illuminated with a thin light sheet, and the tracer particles in the light sheet are projected onto a recording medium (CCD) in the image plane of a lens as shown in Figure 3.11. The intensity of the light sheet thickness $\Delta Z_0$ is assumed to changes only in the $Z$ direction. The magnification of particle image depends upon the position of the imaging lens. The CCD consisted of an array of $1280 \times 1024$ pixels. A Nikon 50 mm manual lens with $f^* = 1.4$ was attached to the CCD camera for covering the field of interest. Both the camera and laser were synchronized with a synchronizer controlled by a dual processor PC. The field of view employed in the present set of PIV measurements was 40 mm by 35 mm. Velocity vectors were calculated from particle traces by the adaptive cross-correlation method. The final interrogation size was $16 \times 16$ pixels starting from an initial size of $64 \times 64$. Thus, 5561 velocity vectors were obtained in the imaging area with a spatial resolution of 0:5 mm. Inconsistent velocity vectors were eliminated by local median filtering and subsequently replaced by interpolated data from adjacent vectors. The laser pulse width was $20 \mu\text{s}$ and the time delay between two successive pulses was varied from 40 to 200 $\mu\text{s}$ depending on the fluid velocity (Keane and Adrian, 1990). The time-averaged velocity field was obtained by averaging a sequence of 200 velocity vector images, corresponding to a total time duration of 50 seconds. Laskin nozzles were used to produce seeding particles from corn oil. The mean diameter of oil particles was estimated to be $2 \mu\text{s}$. Data generated from PIV carries superimposed noise. Noise is introduced during recording of PIV images (optical distortion, light sheet non-homogeneity, transfer function of the CCD, non-spherical particles, and speckle) and during data processing (peak fitting algorithm, image interpolation and peak deformation). The validation of the PIV technique was carried out by comparing velocities with pitot static tube and hotwire anemometry, as discussed in later sections.
Seedling arrangement for PIV

One of the most important steps in PIV measurements is seeding of the flow. In order to consider PIV as a non-intrusive technique, it is necessary that the addition of tracer particle does not alter the flow properties. Proper seeding is essential to capture complicated flow details, for example, the recirculation zone. Seeding should be homogeneous (spatially uniform) and sufficient (of high enough density). The injection of tracer particle has to be done without significantly disturbing the flow, but in a way and at a location that ensures homogeneous distribution of the tracers. Particles should be of small diameter so that they follow the original local air velocity without causing any disturbance. The particle density should ideally match that of the fluid to eliminate velocity lag. This issue is adequately taken care of by micron-sized particles for which surface forces are in excess of body forces.

For the present investigation, tracer particles (namely, droplets of corn oil) were added to the main air flow by a number of copper tubes upstream of the honeycomb section. A large number of tiny holes, 0.1 mm diameter were drilled along the length of the copper tubes to make the seeding uniform over the entire test section. The seeding density was adjusted through an air pressure control valve. Laskin nozzles were used to produce oil droplets as tracers. For the range of frequencies in the wake, an expected slip velocity error of 0.3% to 0.5% relative to the instantaneous local velocity is expected in the present study (Adrian, 1991).

Laskin nozzles are widely used as atomizers of non-volatile liquids due to simplicity of design and the resulting uniform particle size distribution. The picture of the Laskin nozzle seed generator has been shown in Figure 3.12. A detailed schematic drawing of the Laskin nozzle seed generator is shown in Figure 3.13. The particles should be small in size, spherical in shape, of appropriate density and refractive index, and non-volatile. Above all, the liquid should be non-toxic and of low cost. The particles should be efficient scatterer of the illuminating laser light. This largely decides the illuminating laser type and the recording hardware i.e. camera. For example, if a given particle scatters weakly, then one would have to employ more powerful lasers or a more sensitive camera, both of which can drive up costs, as well as the associated safety issues. Corn oil was used for the present work, in view of its high surface tension required for producing small particles along with favorable light scattering properties.
Fig 3.12: Picture of the Laskin nozzle seed generator
Fig 3.13: Schematic of the Laskin nozzle used for the seeding generation

An important source of error in velocity measurement is the particle weight. The following analysis ascertains that particle weight is not a major consideration in the present experiments in the sense that particles would follow the main flow without excessive slip. The approach is to find the settling velocity of the particles under a gravity field. Assuming that Stokes law of drag is applicable, the settling velocity \( u_\infty \) is given by

\[
   u_\infty = \frac{g d_p^2 (\rho_p - \rho_f)}{18 \mu}
\]

Here \( d_p \) and \( \rho_p \) are the particle diameter and density respectively, and \( \mu \) and \( \rho_f \) are the fluid viscosity and density respectively. Particles are suitable as long as \( u_\infty \) is negligible compared to actual fluid velocity. For the present set experiments, \( u_\infty \) was estimated to be 0.014 m/s.


Module 3: Velocity Measurement
Lecture 12: Introduction to PIV

**Particle Dynamics**

The particle dynamics as outlined by Adrian (1991) for successful PIV measurements is discussed in this section. The PIV technique measures in principle the Lagrangian velocities of the particle, \( \nu \). If the particle velocity is being used to infer Eulerian fluid velocity \( u(x,t) \), one must consider the accuracy with which the particle follows the fluid motion. With subscript \( p \) denoting particle-level properties, the equation of motion of a single particle in a dilute suspension is a balance between inertia and drag force is written as:

\[
\rho_p \frac{\pi d_p^3}{6} \frac{d\nu}{dt} = C_D \frac{\rho\pi d_p^2}{4} |\nu - u| (\nu - u)
\]  

(1)

The above equation requires a correction for the added mass of the fluid, unsteady drag forces, pressure gradients in the fluid, and nonuniform fluid motion. In gaseous flows with small liquid particles, we may ignore all these terms except the static drag law with drag coefficient \( C_D \). This term incorporates finite Reynolds number effects.

Particle response is often described in terms of the flow velocity and a characteristic frequency of oscillation. The first question is, how fast can the flow be, before the particle lag \( |\nu - u| \) creates an unacceptably large error. An appropriate approach is to evaluate the particle slip velocity as a function of the applied acceleration. For the simplified drag law of the above equation, one has

\[
|\nu - u| = \left[ \frac{2 \rho_p \frac{d_p}{3}}{\rho \ C_D} |\dot{\nu}| \right]^{\frac{1}{2}}
\]  

(2)

This shows that the slip velocity for finite particle Reynolds number, where \( C_D \sim \) constant, is only proportional to the square root of the acceleration. In the limit of small particle Reynolds number \( |\nu - u| d_p / \nu \leq 1 \), Stokes' law may be used to evaluate \( C_D \) resulting in

\[
|\nu - u| = \frac{\rho_p d_p^2 |\dot{\nu}|}{36 \rho u}
\]  

(3)
The time separation $\Delta t$ is the single most important adjustable variable in a PIV system, as it determines the maximum and minimum velocities that can be measured. The duration of the light pulses $\delta t$, determines the degree to which an image is frozen during the pulse exposure. The accuracy of velocity measurements depends upon one's ability to determine the displacement of the particle, $\Delta x$ over a certain time interval from measurements of the displacement of the image $\Delta X$.

**Fig 3.14:** Distributed hole arrangement for uniform seeding distribution.
Generating a light sheet

For PIV measurement a high intensity light source is required for efficient scattering of light from tracer particles. Light sheet is generated from a collimating laser beam using cylindrical lens and spherical lens. The effective intensity of a light sheet can be increased by sweeping a light beam to form sheet thereby concentrating the energy by a factor equal to the height of the light sheet divided by the height of the beam. Figure 3.15 shows the schematic of a light sheet formation. A combination of cylindrical and spherical lens is used. A negative focal length lens is first used to avoid focal line. The cylindrical lens causes the laser beam to expand in one direction only, i.e. it "fans" the beam out. The position of the minimum thickness is determined by the focal length of the cylindrical lens. The spherical lens causes the expanding beam to focus along the perpendicular direction, at a distance of one focal length downstream to the beam waist.

Synchronizer

In order to make PIV measurements, different components of the PIV system need to be time coordinated, for example, the camera, the laser flash lamps and its Q-switches. The synchronizer controls the time sequence. A part of the functions is executed automatically, while others have to be defined by the user. The synchronizer thus manages all the timing events needed for doing PIV measurements.

![Fig 3.15: Light sheet formation using spherical and cylindrical lens for PIV.](image-url)
The frame grabber needs 40 ns to lock onto the trigger signal. Afterwards, the control data can be transferred to the camera. The exposure time is controlled by the external trigger from the synchronizer in a user-defined range between 100 ns and 1 ms. Before the second exposure, the camera has a frame straddling time of 200 ns or 1 μs which depends on the parameter settings of the cross correlation function. Before the next double exposure can be started, data of the first image pair is transferred to the frame grabber.

The laser must be synchronized to the double exposure mode of the camera. For emitting a laser pulse, a high energy must be generated in the laser cavity. The laser cavity has a Nd:YAG rod that is pumped with energy from a flash lamp. There is a nonlinear relation between the time the cavity is pumped and laser power emitted. During the pumping procedure, the mirror at the far end of the cavity is closed by a Q-switch. The success of PIV measurements depends crucially on the time correlation between laser pulse generation and camera recording achieved by the synchronizer unit. Figure 3.16 shows the timing diagram for the pulsed laser with double shutter CCD camera.

**Figure 3.16:** Timing diagram for CCD camera and double pulsed laser (PIV Manual, Oxford Lasers).