Gravitational Waves

Physics, Technology, Astronomy and Opportunities

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The Structure of Physics: **Matter, Fields, Space and Time**

$$\vec{a} = \vec{F} / m$$ and Principle of relativity

Laws of Motion

Physical Understanding

Laws of Fundamental Forces

$$F = -GMm / r^2$$

Conservation Laws (Constraints)

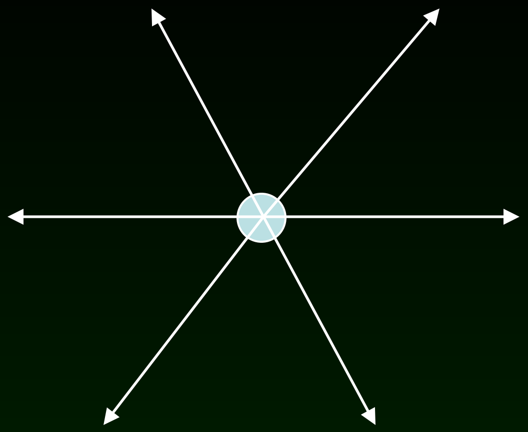
Conservation of momentum. Energy etc…

Four Fundamental Interactions and several fundamental particles

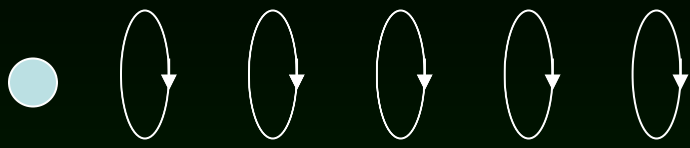
Gravity, Electromagnetism, Weak interactions and sub-nuclear strong interactions.

Electromagnetism
Weak Interaction
Strong Interaction

Gravity



Electric Field



Magnetic Field

Charges and Currents are fundamental, fields are ‘theoretical’.

Current → Magnetic field

Electromagnetic waves need electric AND magnetic field for generation and propagation.

$$-\frac{\partial B}{\partial t} = \nabla \times E, \quad \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = \nabla \times B \quad \ldots Maxwell$$

Unobservables in Physics

Fields, Wavefunction, Space and Time…

GRAVITY, ITS FIELDS and THEIR WAVES

$$F = -GMm / r^2$$

$$\vec{a} = \vec{F} / m$$

$$a_g = -GMm / r^2 m = -GM / r^2$$

Independent of the mass of the falling body

$$F = kQq / r^2$$

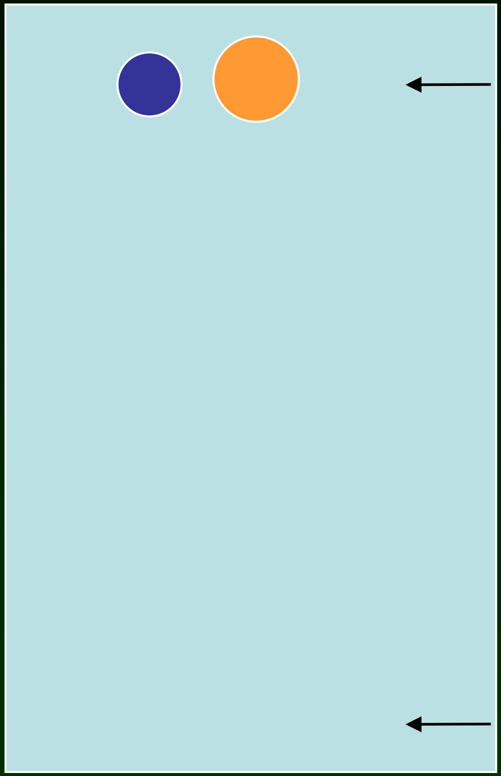
$$\vec{a} = \vec{F} / m$$

$$a_{em} = kQq / r^2 m = \frac{kQ}{r^2}\left(\frac{q}{m}\right)$$

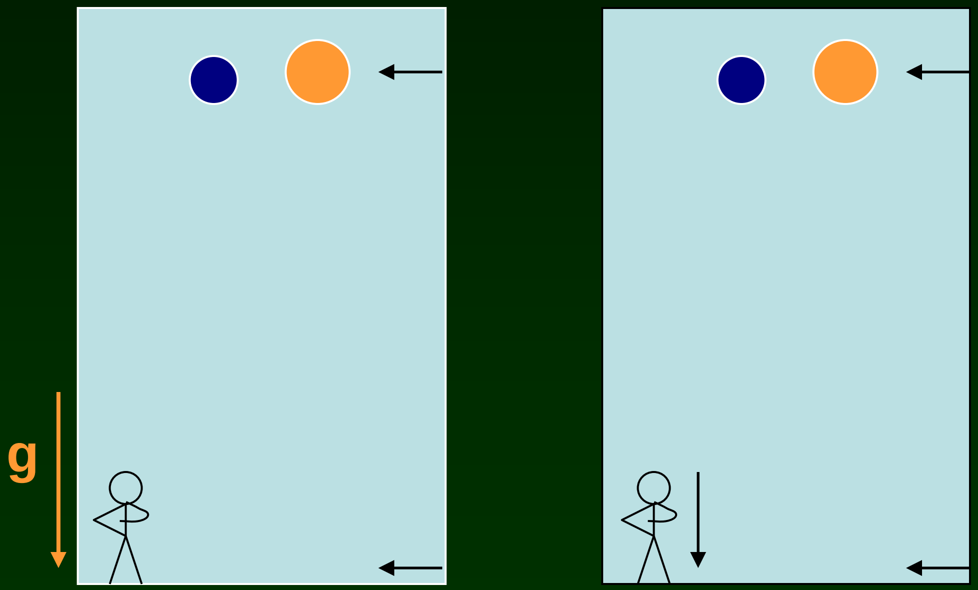
Gravity seems to be a special interaction

Inertia turns out to be identical to the gravitational charge – Equivalence Principle (physics of gravity identical to physics in accelerated frames)

Universality of Free-Fall



$$\delta a < 10^{-12} m/s^2 \rightarrow \frac{\delta a}{a} < 10^{-13}$$

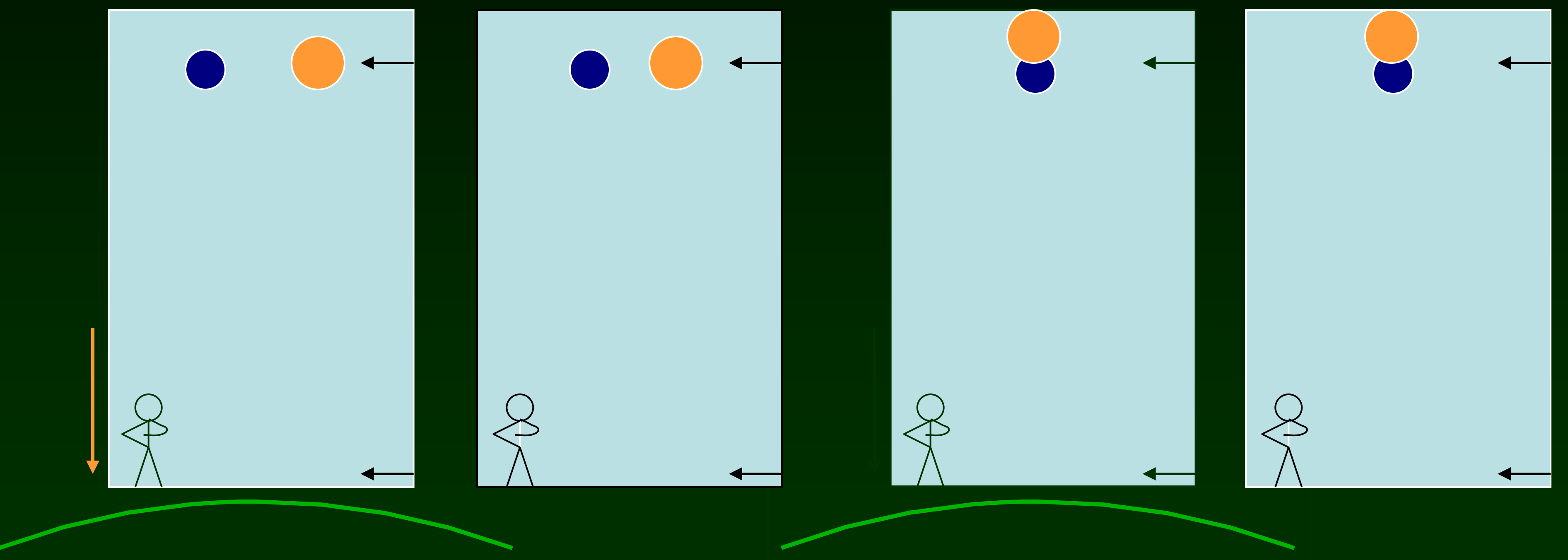


So, gravitational field ‘g’ and acceleration ’-a=g’ seem equivalent

This is called the Equivalence Principle

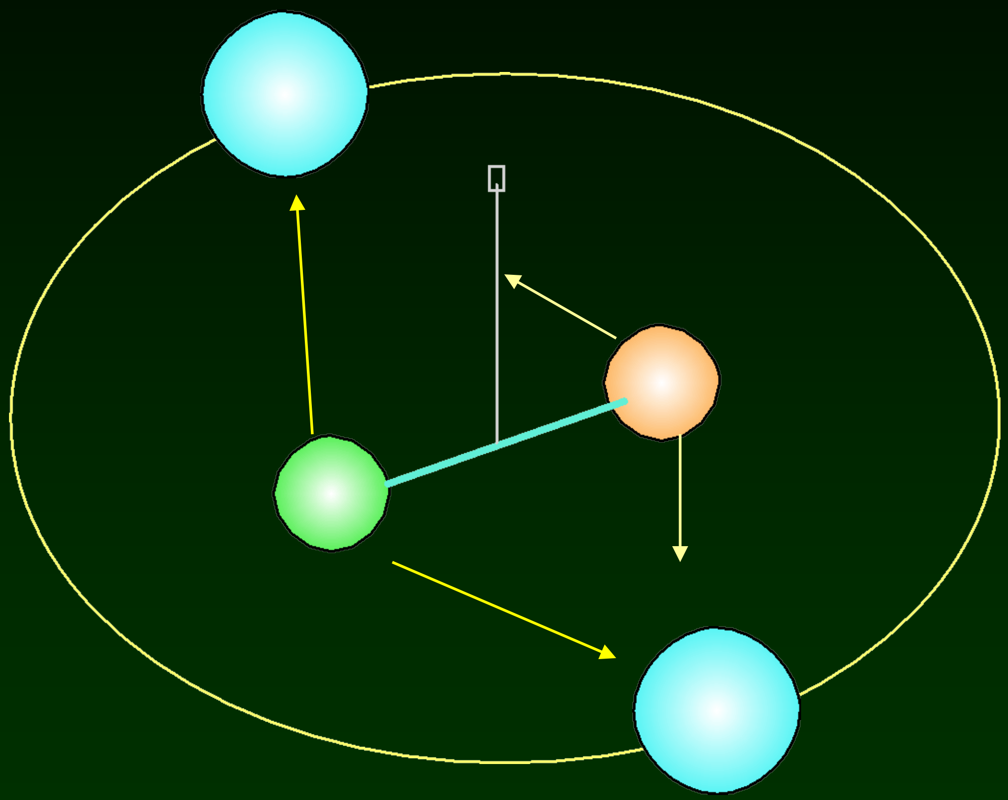
This is the same as saying that in free-fall, there is no gravitational field

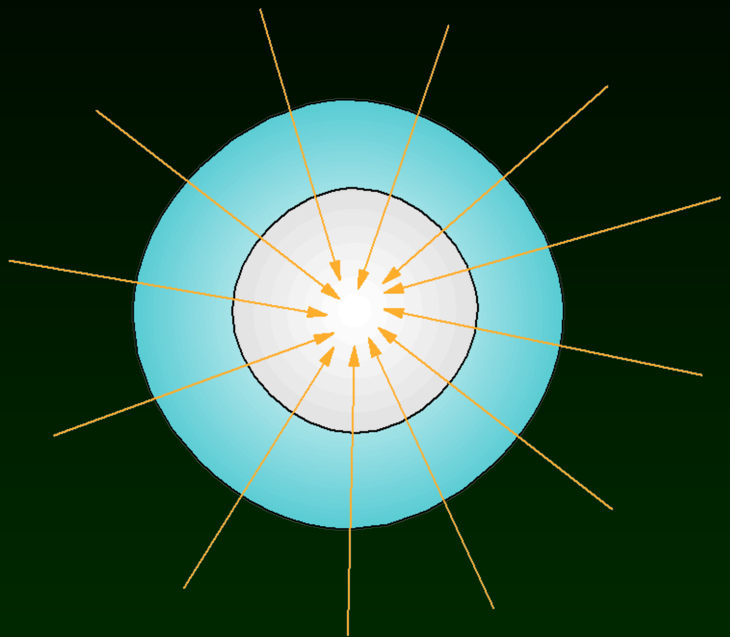
But it does not mean that in free-fall there is no gravity!



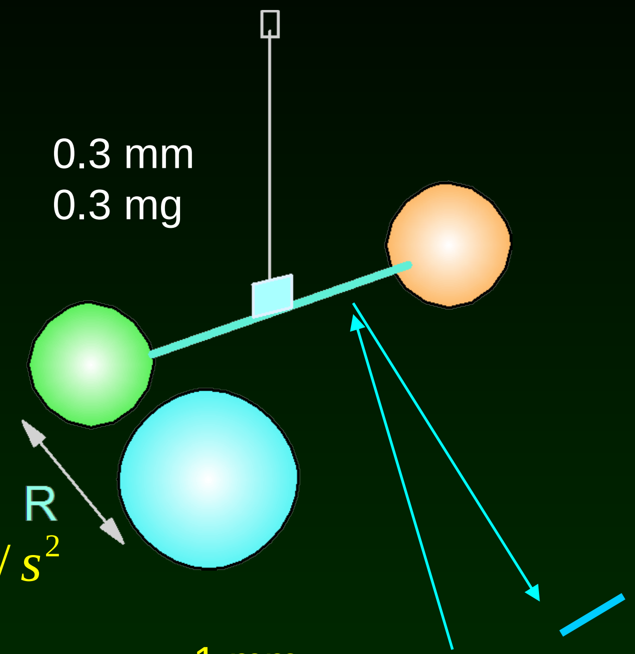
So, tidal deviations cannot be eliminated by free-fall
(description in the General Theory of Relativity)

Torsion balance: Harmonic potential
Ultra-sensitive





$$g \approx 5\times10^{-8}\ cm/s^2$$

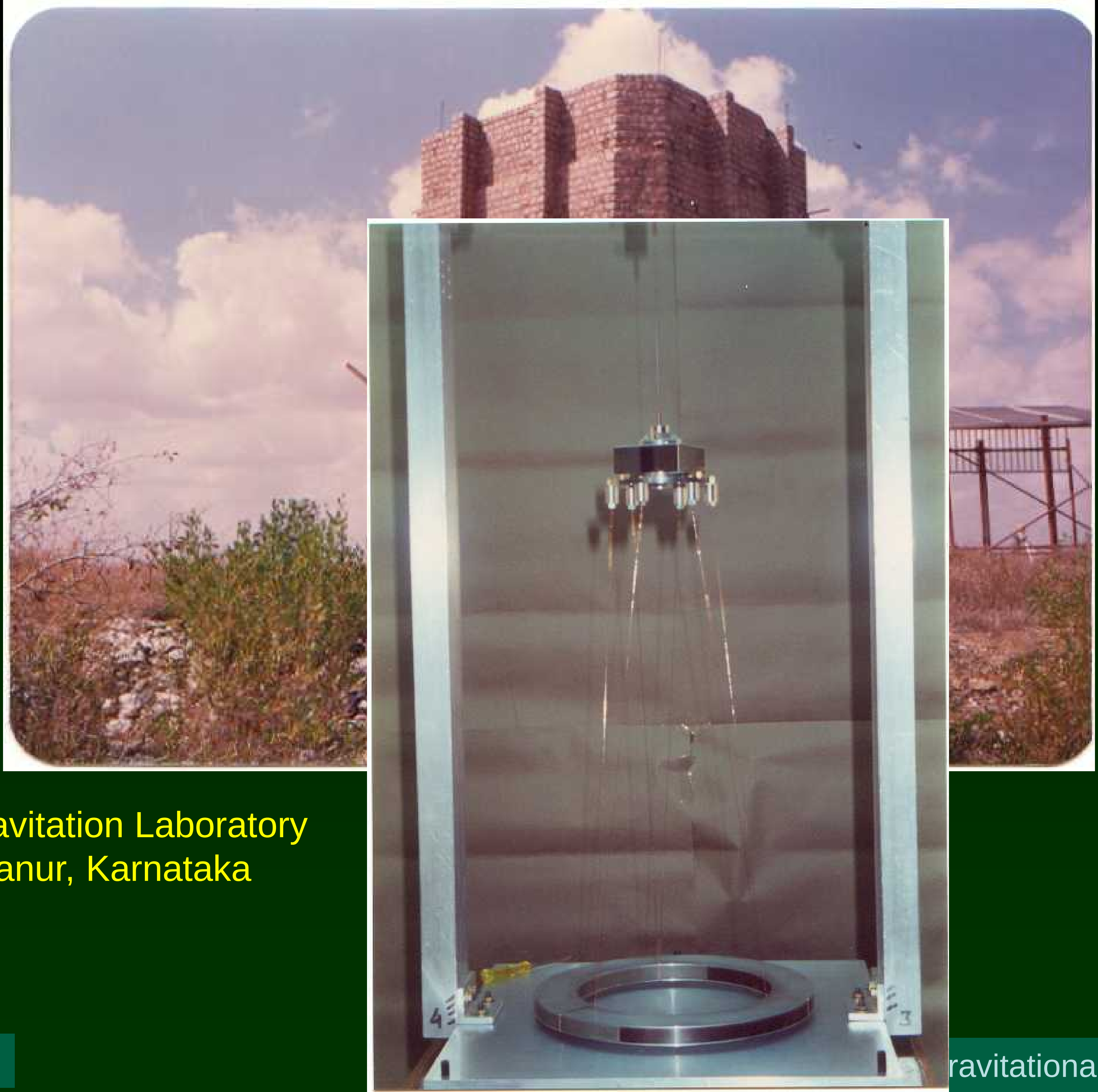


$$Flux-density \propto 1/R^2$$

(For 3D space)

What if space is higher dimensional at some tiny scales (micrometers or less) ?

Inverse-square law for 'g' will change!



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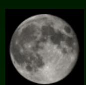
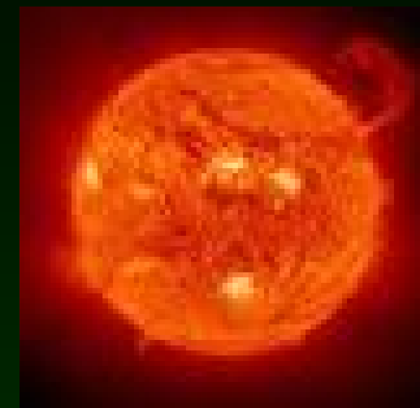
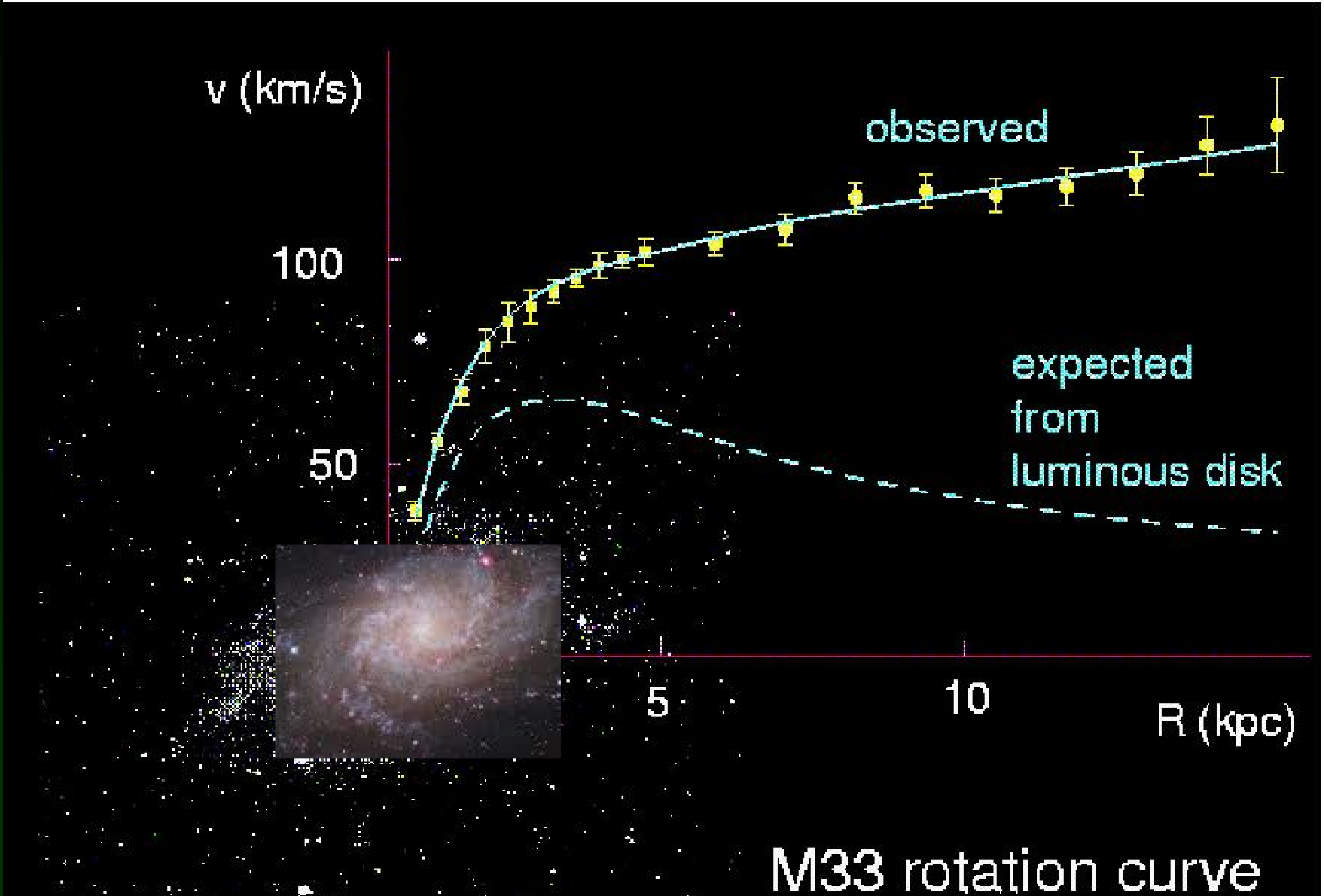


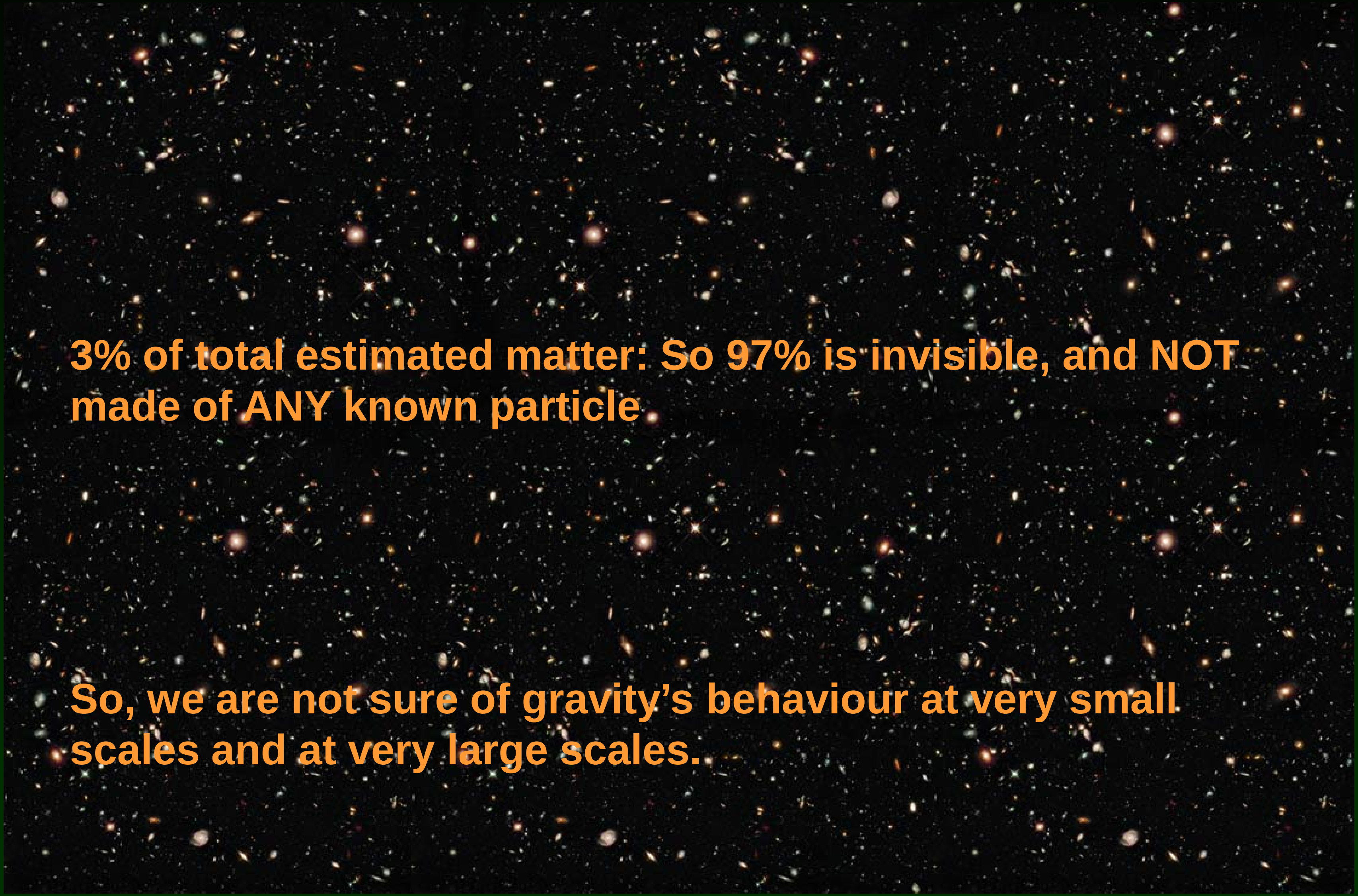
‘g’ of Sun ~0.6 cm/s^2



$$\delta a(earth - moon) < 10^{-14} cm/s^2$$

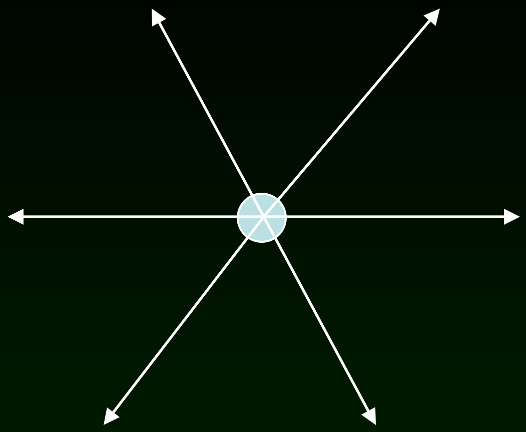

v (km/s)
100
50
observed
expected
from
luminous disk
5
10
R (kpc)
M33 rotation curve

Dark matter

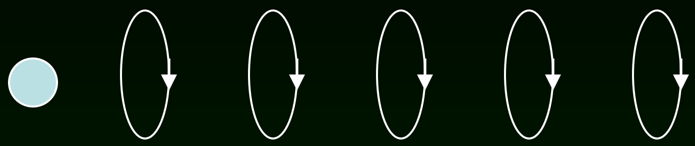


3% of total estimated matter: So 97% is invisible, and NOT made of ANY known particle

So, we are not sure of gravity's behaviour at very small scales and at very large scales.



Electric Field



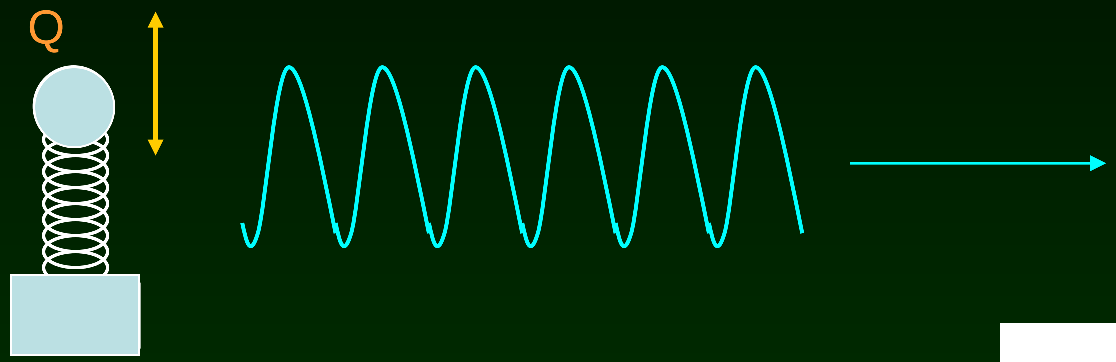
Magnetic Field

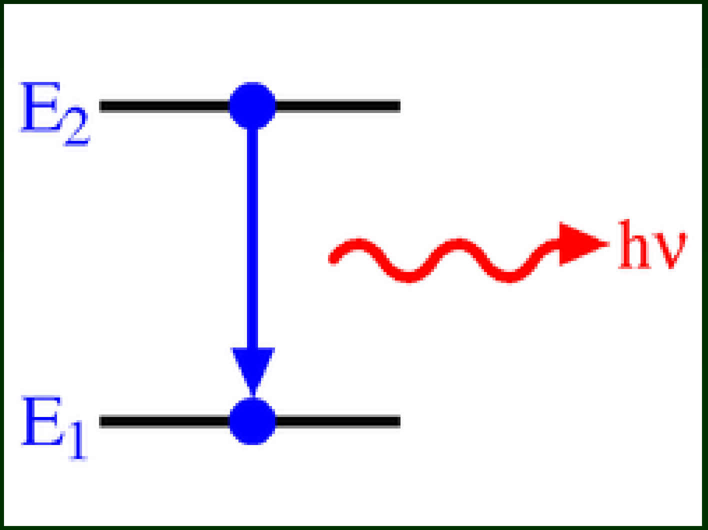
Current → Magnetic field

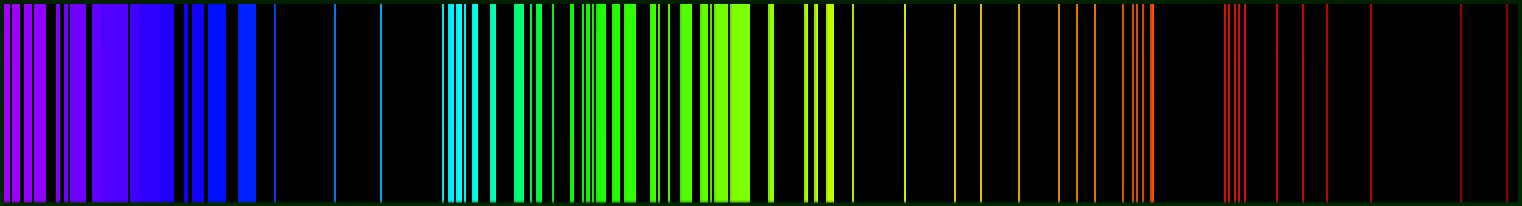
Electromagnetic waves need electric AND magnetic field for generation and propagation.

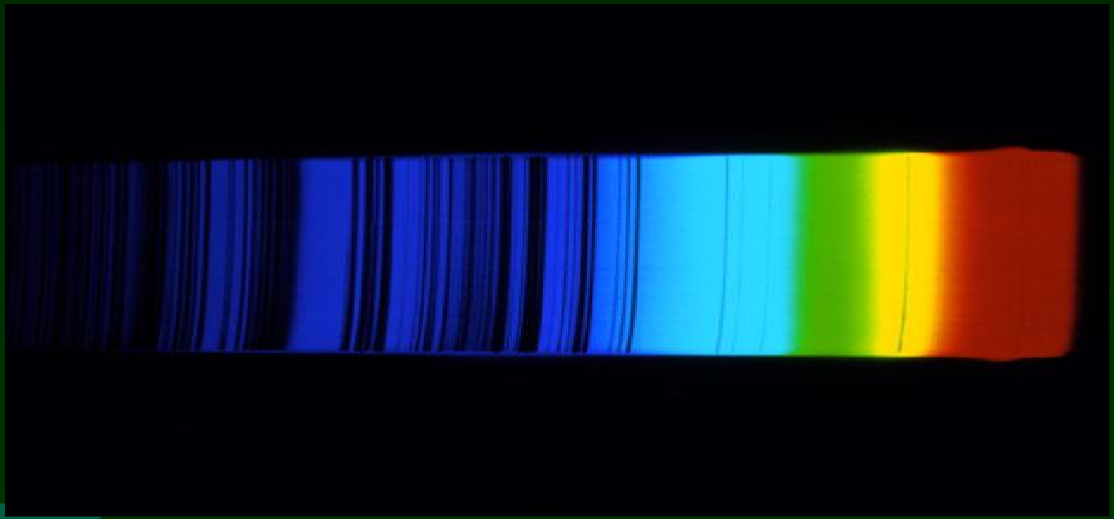
$$-\frac{\partial B}{\partial t} = \nabla \times E, \quad \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = \nabla \times B \quad \ldots Maxwell$$

What is an electromagnetic wave?

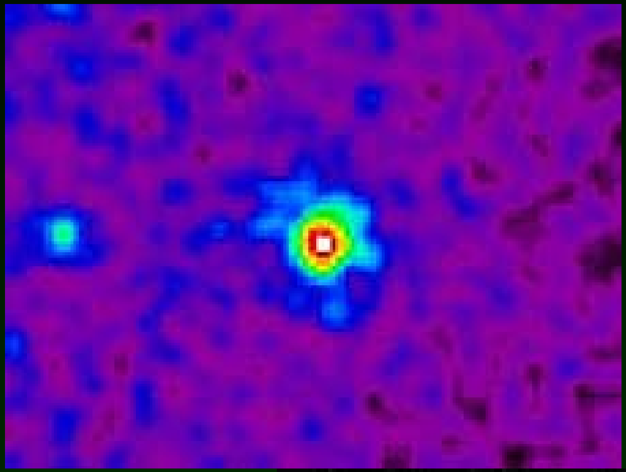


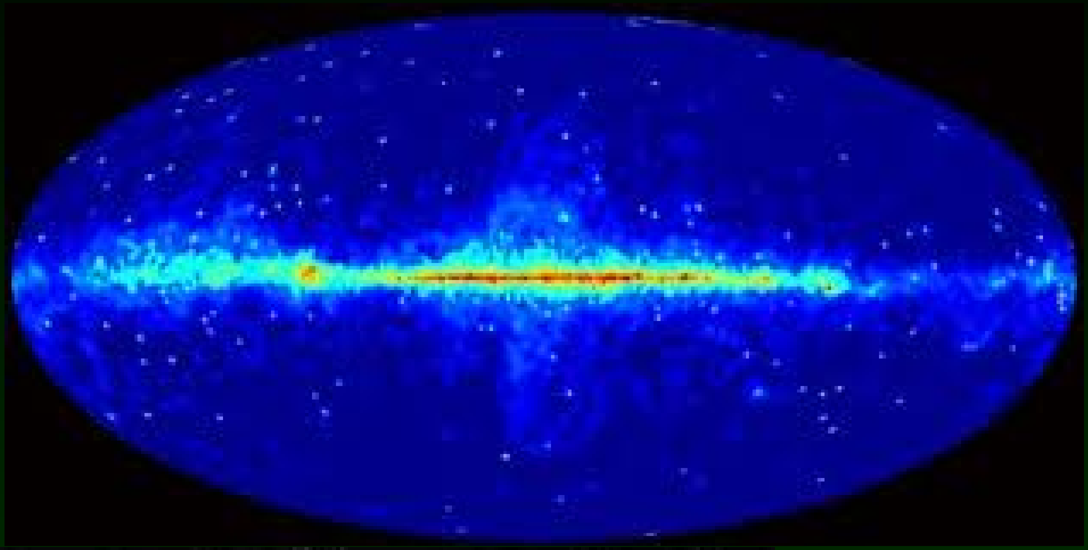


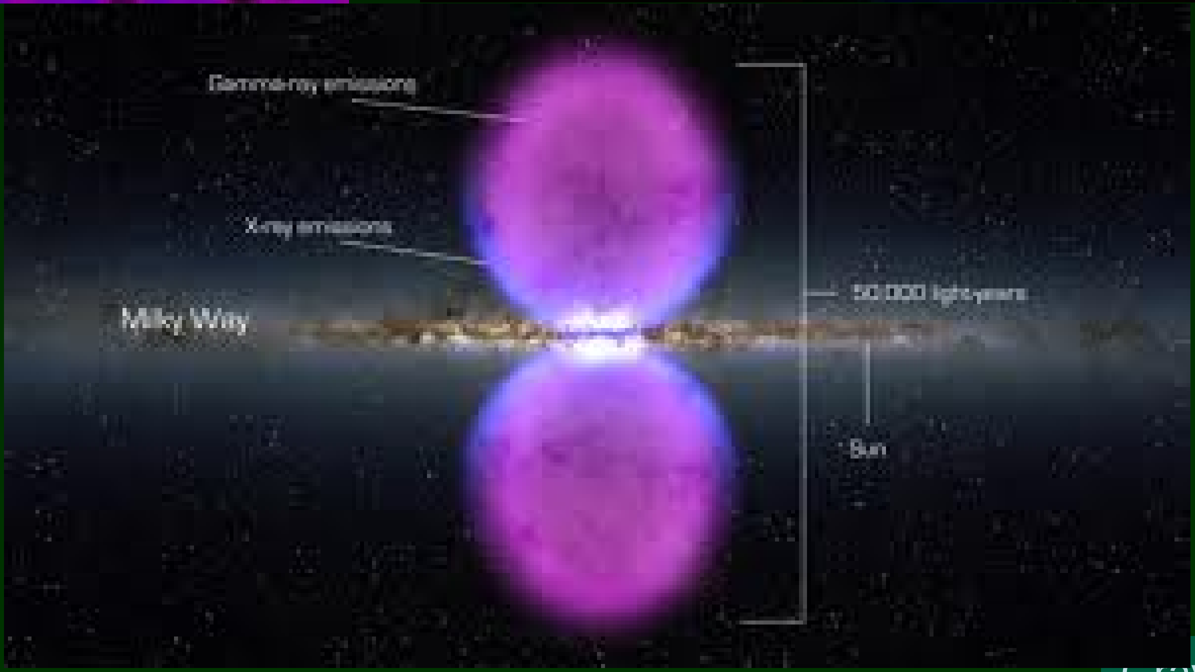




Gamma rays and galaxies



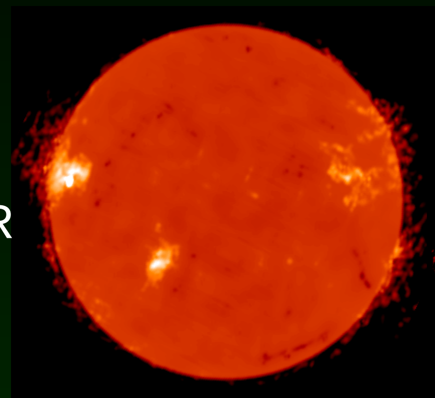


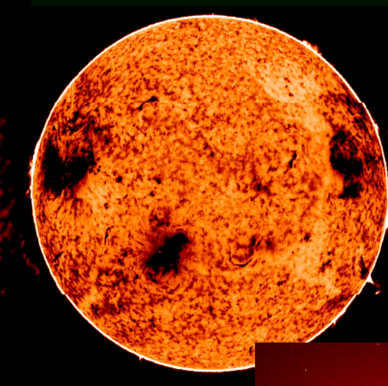


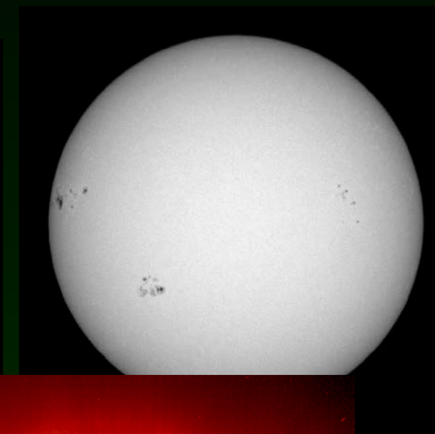
Spectral view – multi-wavelength

Radio

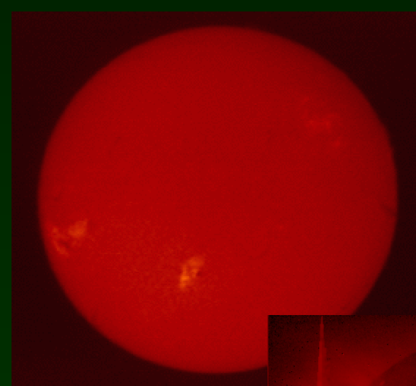
IR

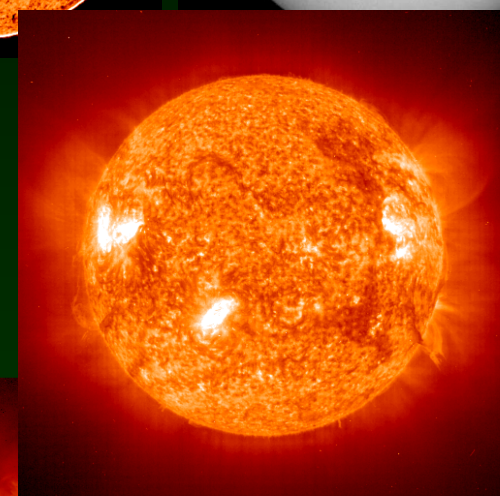






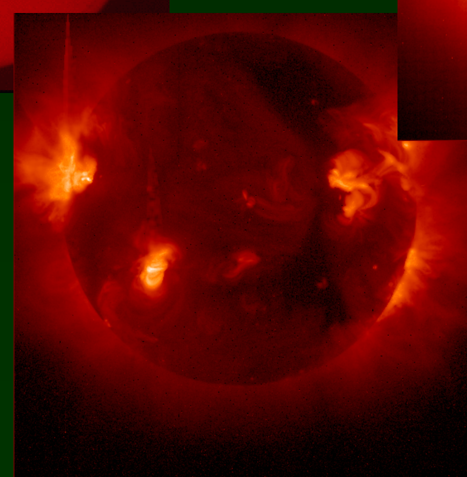
H-α





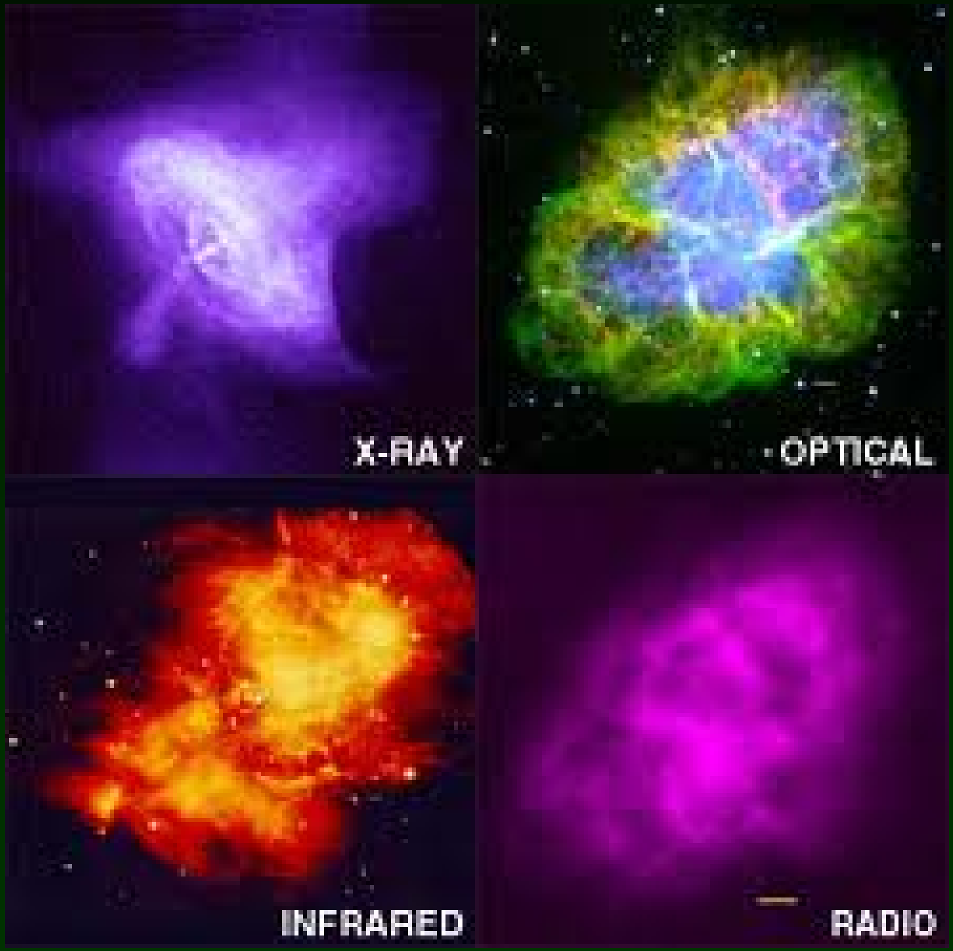
UV

X-ray

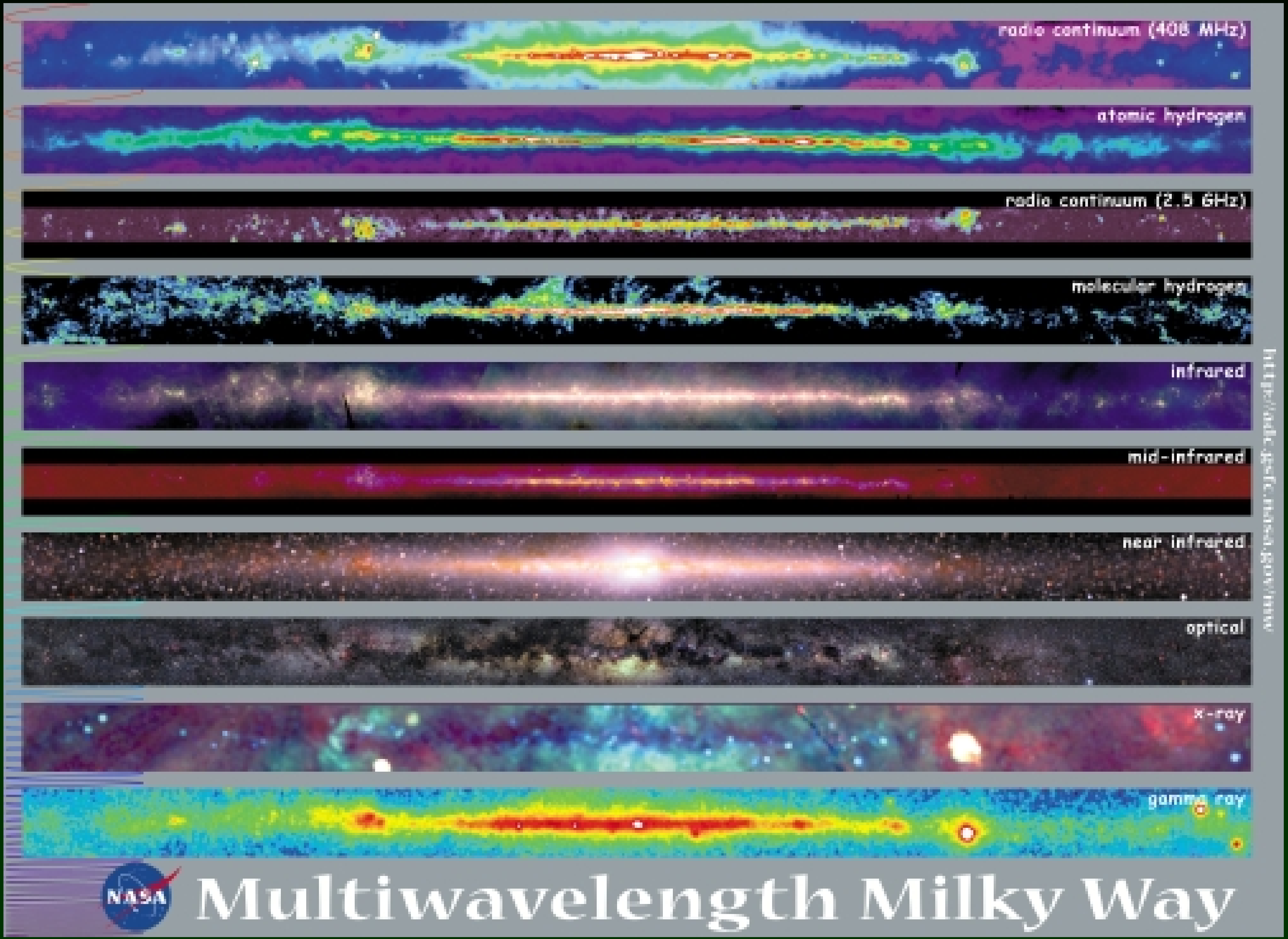


Spectral view – multi-wavelength

Crab Nebula



Multi-wavelength galaxy



Gravity and Electromagnetism

Both have ‘electric’ and ‘magnetic’ parts → Charges and currents.

Mass is the ‘charge’ of gravity and Spin is its ‘gravito-magnetic moment’

One important different between the two is that while electric and magnetic fields have no electric charge, gravitational field has gravitational charge!

With $m=E/c^2$, all forms of energy is equivalent to mass, and hence generate gravity. Therefore, all fields including the gravitational field, which carry energy, also generate gravitational fields. This is one reason why the theory of gravity (The General Theory of Relativity) is complicated to work with.

Gravity and electromagnetism

Charges (static): Coulomb force – electric fields
Currents (motion): Ampere's force – magnetic fields

Electromagnetic Waves

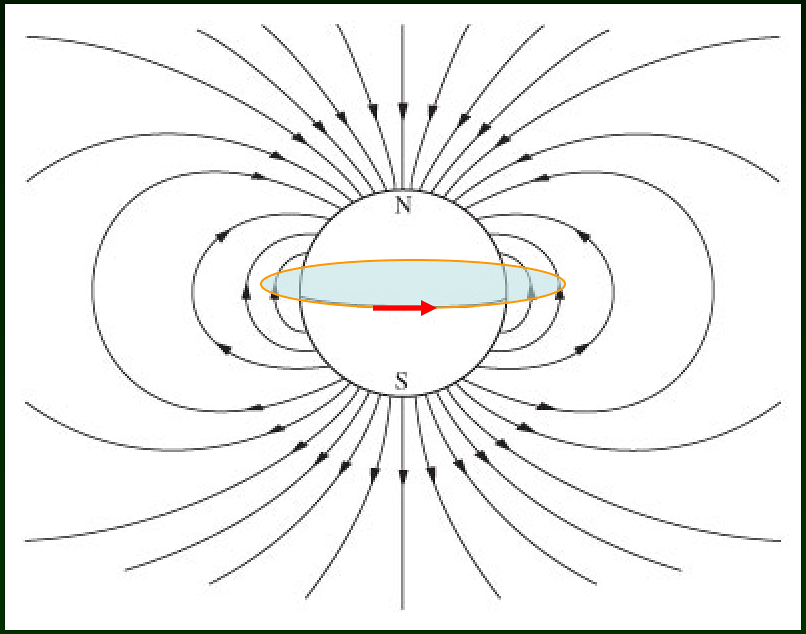
What about relativistic gravity?

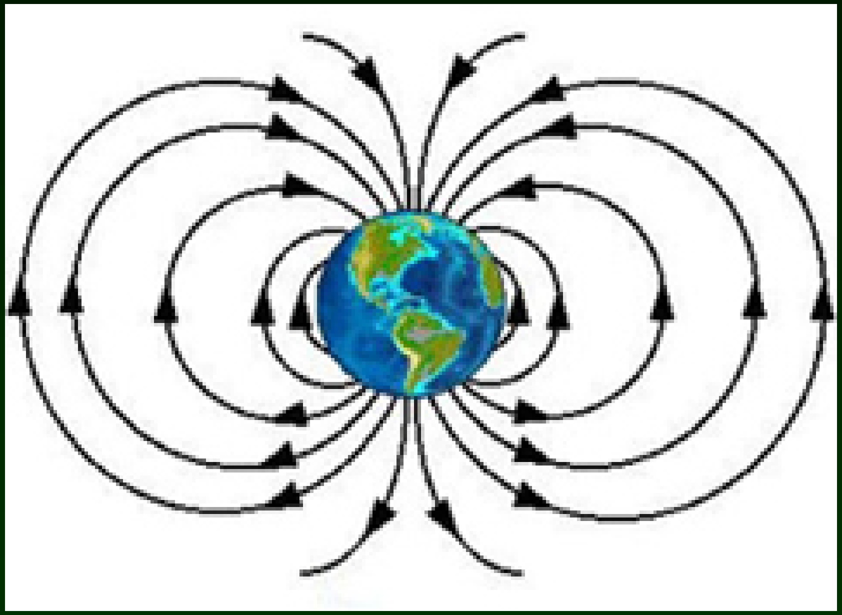
We know static gravitational charge (mass/energy) generates g-field.
Does moving and rotating masses generate a gravito-magnetic field?!

If so, then there is a possibility of gravitational waves…

Gravito-magnetism

A natural consequence of relativistic gravity, and yet, was not detected experimentally till recently.

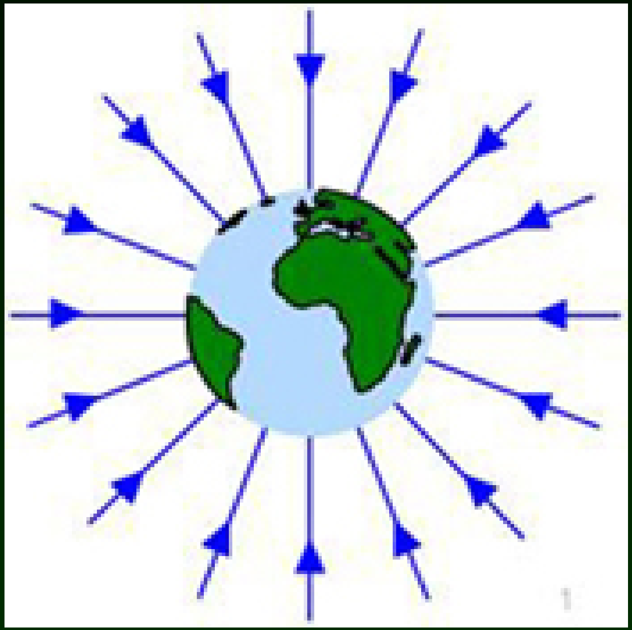


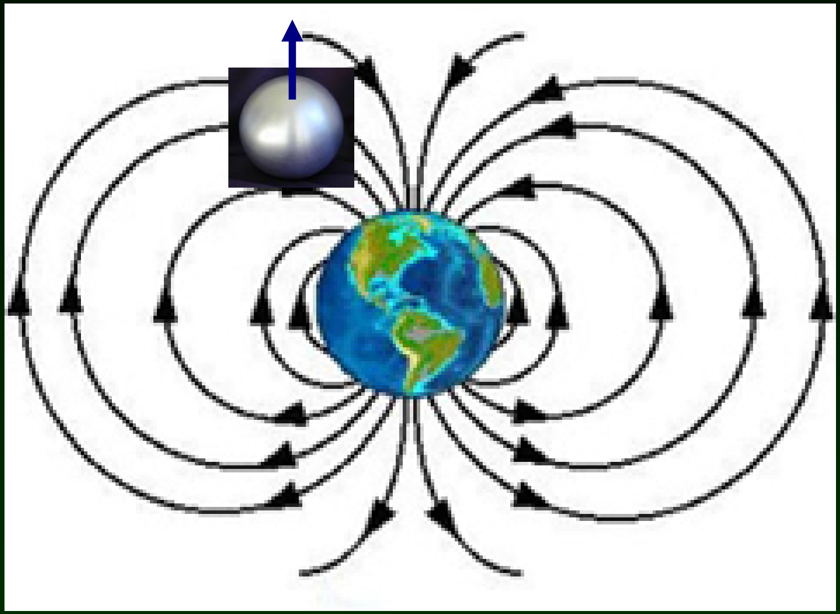


$$B = \frac{\mu_0 M}{r^3}$$

$$B_g = \frac{GJ}{c^2 r^3} = \frac{GI\Omega}{c^2 r^3} \rightarrow \frac{GM}{c^2 r}\Omega$$

The real gravitational field near the earth



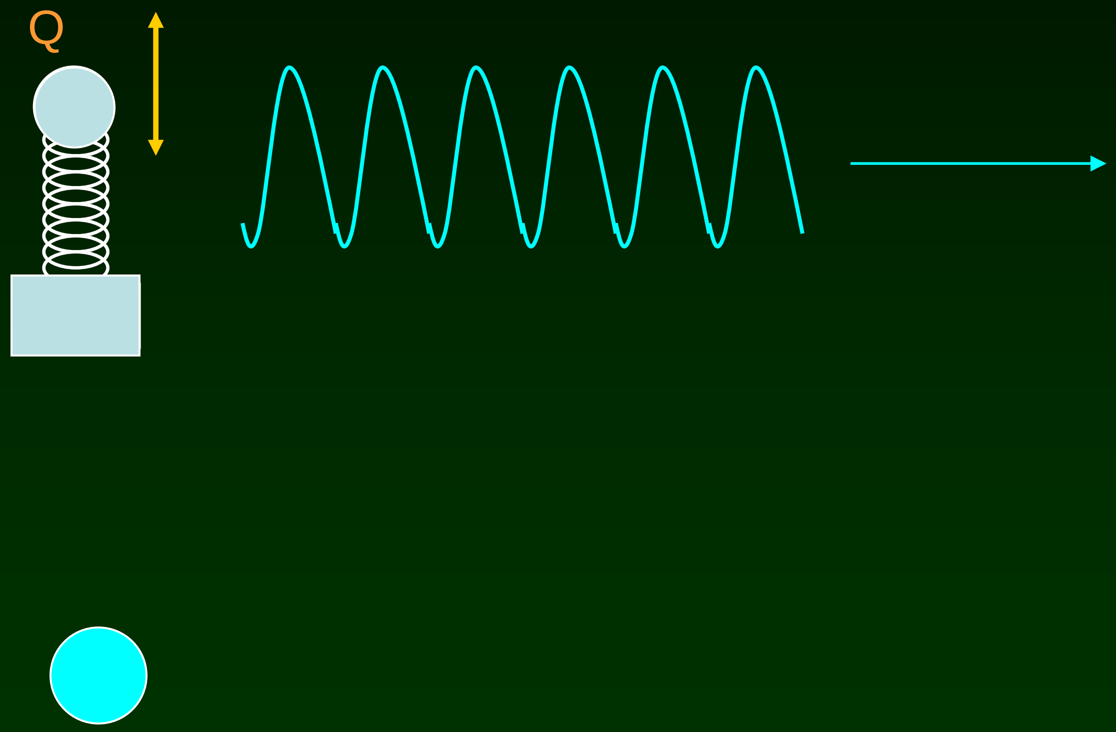


$$\omega \approx \boxed{\frac{GM}{c^2 r}}\Omega \approx 5\times10^{-14}\, rad/s$$

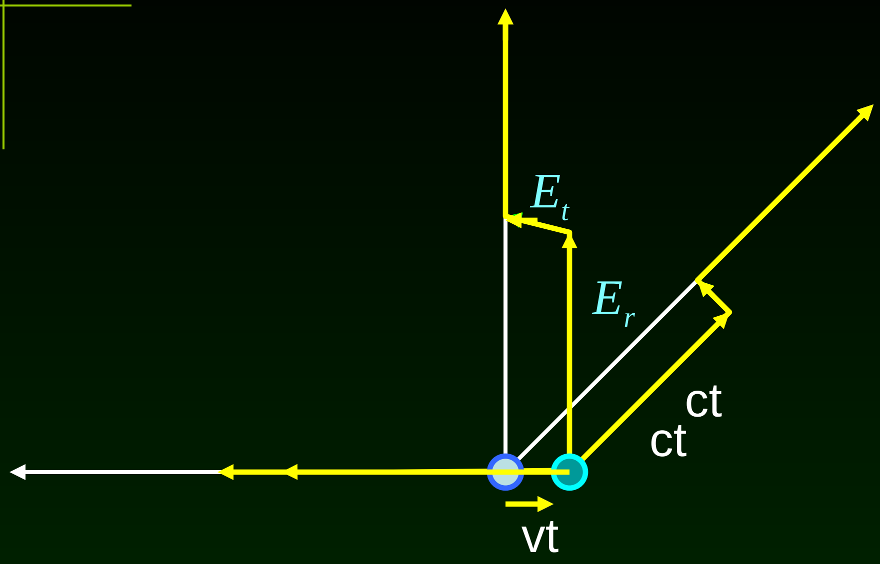
Gravity Probe – B (Stanford U.)

Physics of Gravitational Waves

What is an electromagnetic wave?



So, can we expect gravitational waves from oscillating (accelerated) masses?

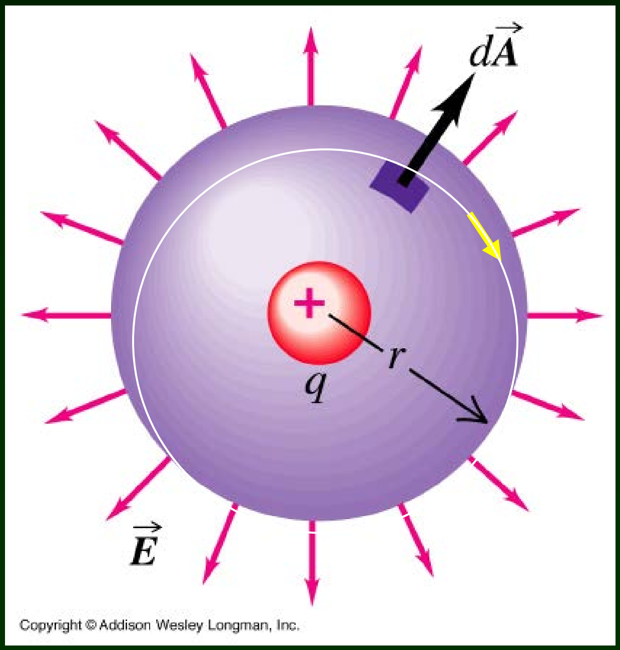


Radiation

$$\frac{E_t}{E_r} = \frac{vt}{ct} \approx \frac{at}{c} = \frac{aR}{c^2} \text{ and } E_r = q/R^2$$

$$E_t = \frac{q}{R^2}\frac{aR\sin\theta}{c^2} = \frac{q\ddot{r}\sin\theta}{Rc^2}$$

$$qr = d \rightarrow E_t = \frac{\ddot{d}\sin\theta}{Rc^2}$$

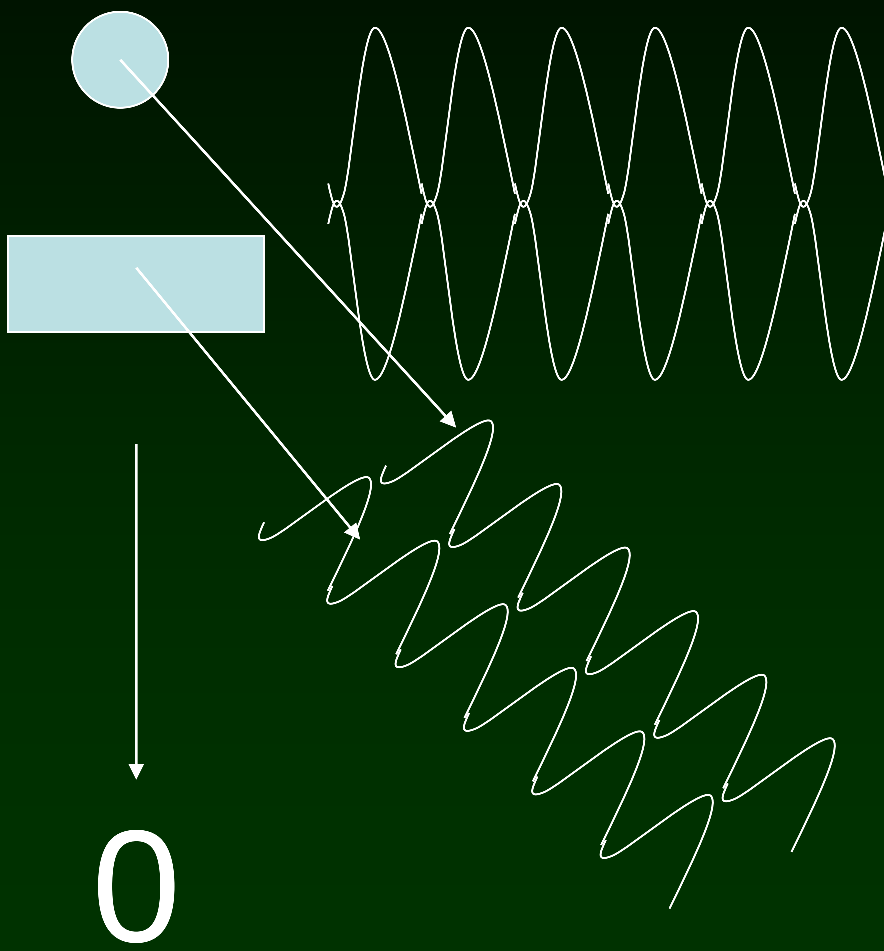


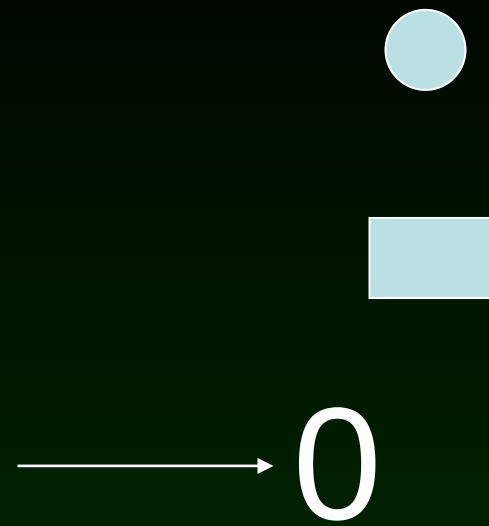
Since area=R^2, number of flux lines/area → $1/R^2$

So, radial Electric field (flux/area) ~ $1/R^2$

What about the Transverse Electric field (radiation)?

Since, circumference of a great circle on the sphere increases only as R, transverse radiation field decrease as 1/R.





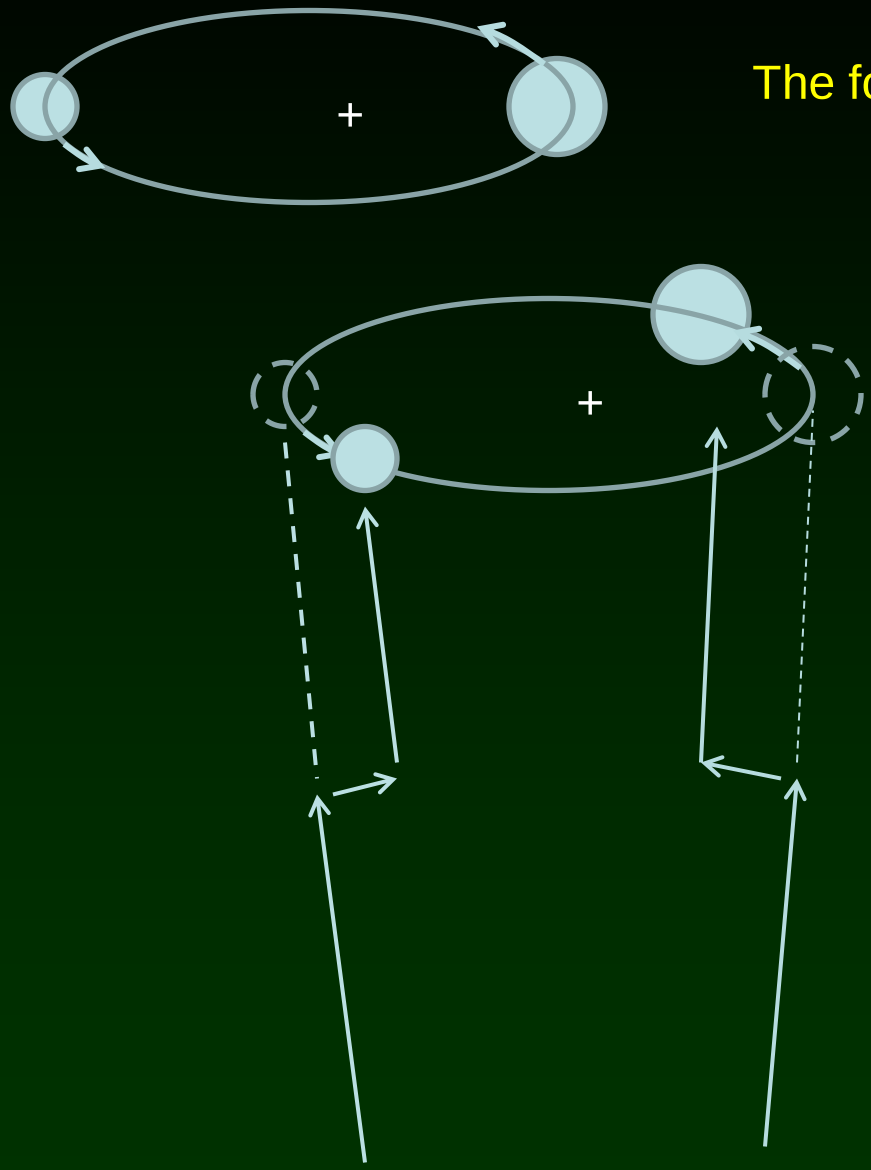
Phase diff.

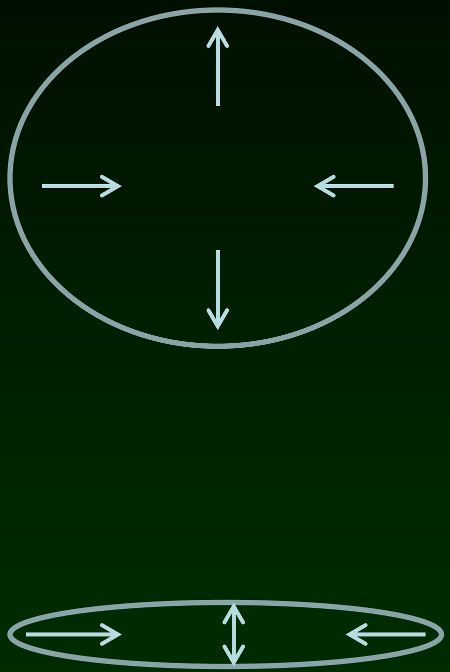
$$E_{gt} = \frac{GM\,\ddot{r}\sin\theta}{Rc^2} \times \vec{k}\cdot\vec{r}$$

$$= \frac{GM\,\ddot{r}\sin\theta}{Rc^2} \times \frac{\omega r\cos\theta}{c}$$

$$= \frac{GMr^2\omega^3}{c^3 R}\sin\theta\cos\theta$$

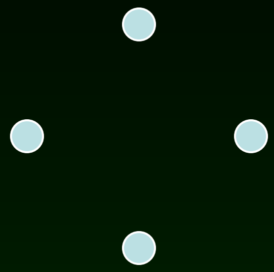
The force-field of gravitational waves

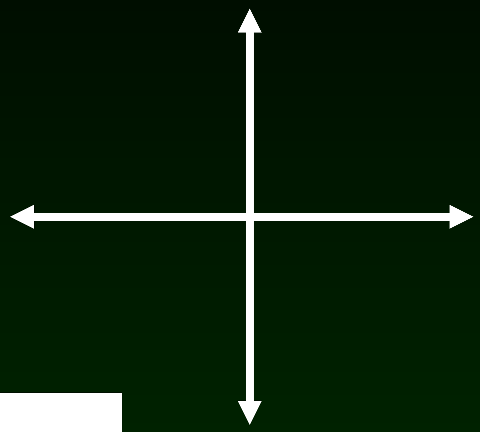


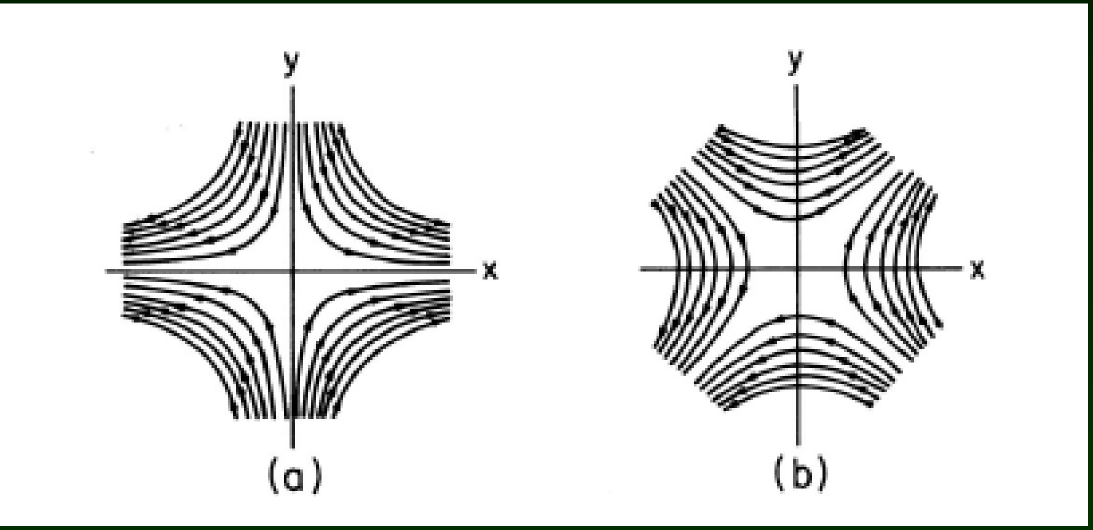


The relation between spin of the field and polarization of the force field

What is the physical effect of a passing gravitational wave?







Quadrupole Radiation formula

$$\text{Strain} \;\; h = \frac{\Delta L}{L} \approx \frac{G}{c^4}\frac{Mr^2\omega^2}{R} \approx \frac{G}{c^2 R}\frac{Mv^2}{c^2}$$

Are we confident that Gravitational Waves exist, apart from the belief in the correctness of the theory?

ON GRAVITATIONAL WAVES.

Jl. Franklin Inst. 1937

BY

A. EINSTEIN and N. ROSEN.

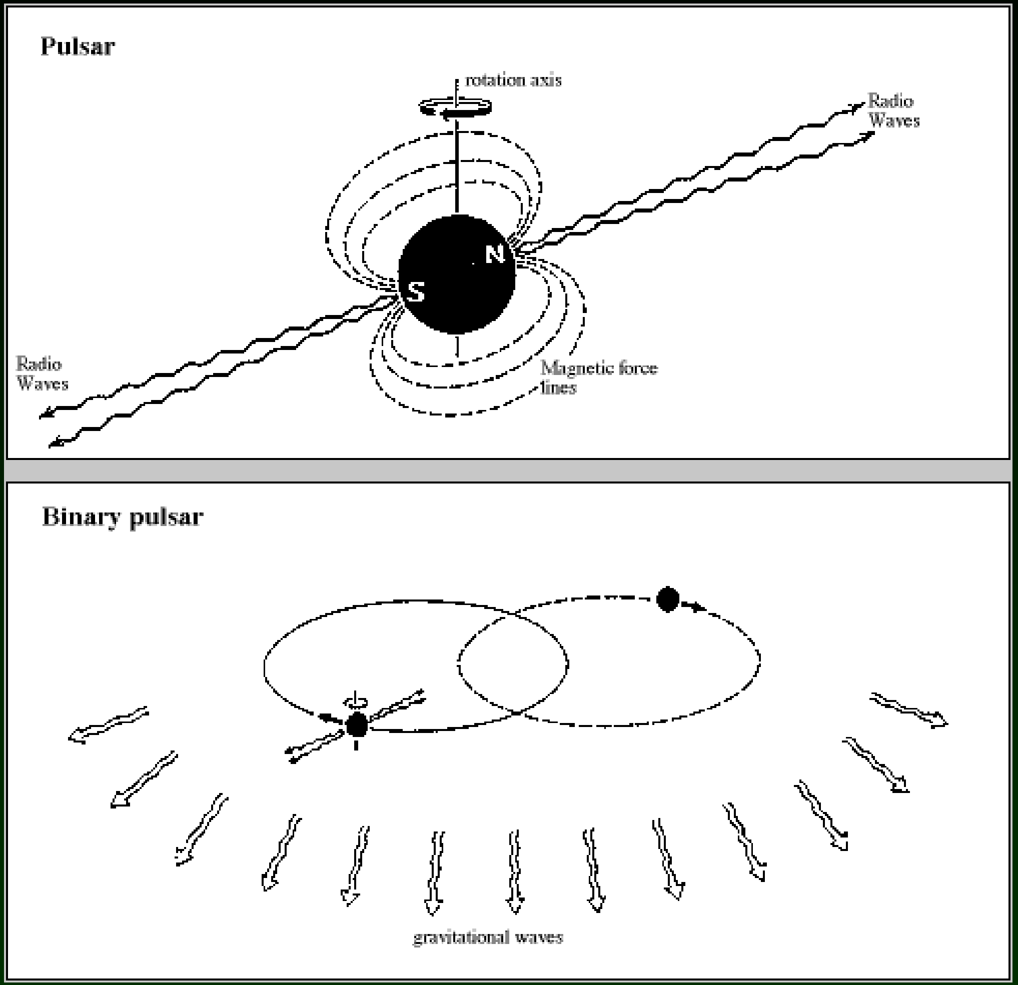
ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of *rigorous* solutions for undulatory gravitational fields, we investigate rigorously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

Note.—The second part of this paper was considerably altered by me after the departure of Mr. Rosen for Russia since we had originally interpreted our formula results erroneously. I wish to thank my colleague Professor Robertson for his friendly assistance in the clarification of the original error. I thank also Mr. Hoffmann for kind assistance in translation.

A. EINSTEIN.

Binary Pulsar 1913+16 (Hulse-Taylor)

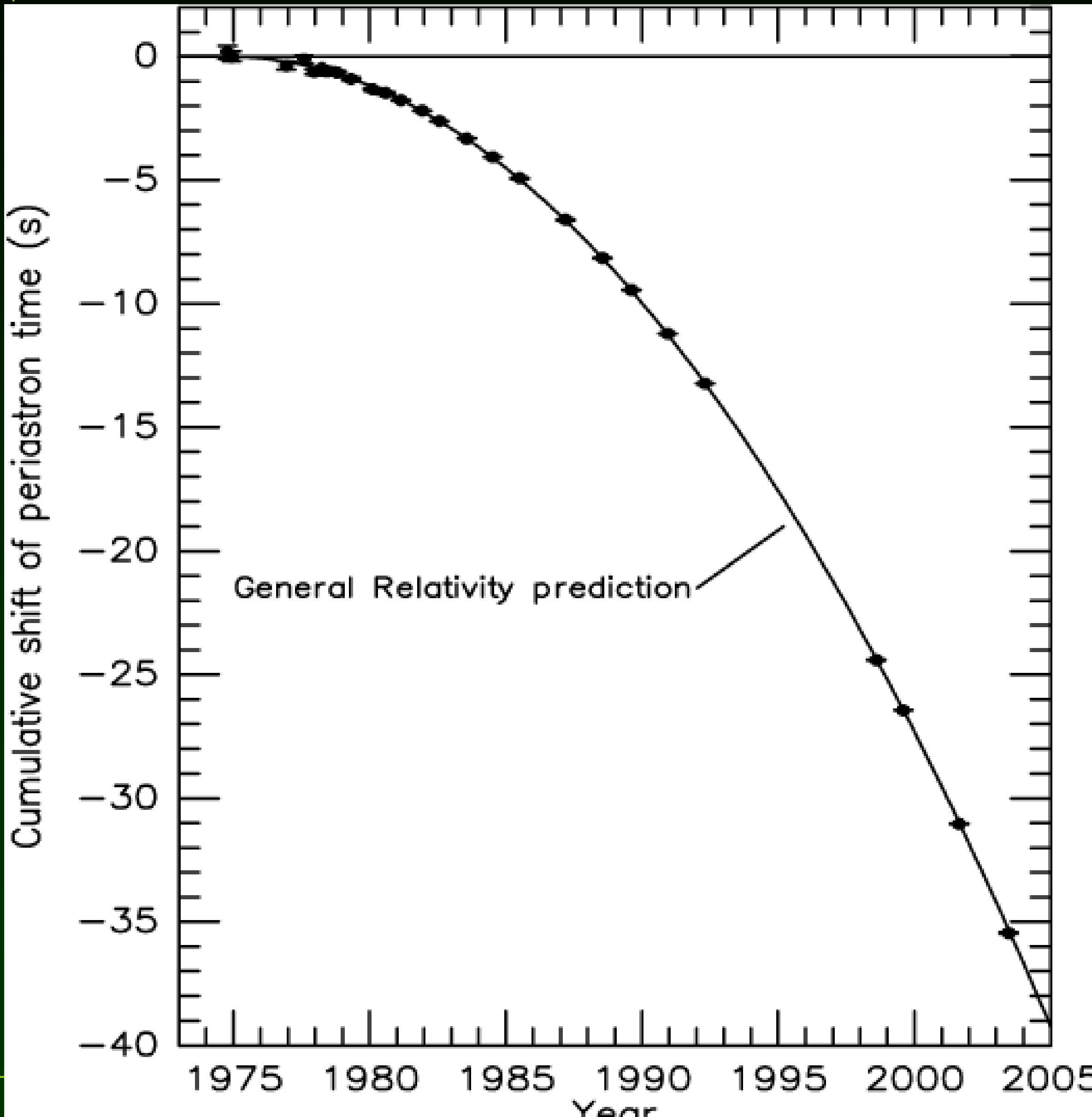


$$\dot{E}_G = \frac{dE_G}{dt} \approx \frac{32G}{c^5} M^2 \omega^6 r^4$$

$$\omega^2 \approx GM / a^3 \rightarrow \nu \approx \tfrac{1}{2\pi}\sqrt{G\rho}$$

$$t_c \approx 3 \times 10^8 \; yrs$$

Orbital decay and speeding up of the binary pulsar:



$$\dot{E}_G = \frac{dE_G}{dt} \approx \frac{32G}{c^5} M^2 \omega^6 r^4$$

Signal Strength at Earth for neutron star spiral in milky way:

Distance: 10 kpc ~ 10^{20} meters

$$\text{Strain } \ h \approx \frac{G}{c^4}\frac{Mv^2}{R} \approx 10^{-64} Mv^2$$

$$v \approx \sqrt{\frac{Gm}{2r}} = 0.1c \ (3\times 10^7 \ \text{m/s}) \text{ for neutron stars at } r \approx 100 \text{ km}$$

With M~10^{30} kg, v~3x10^7 m/s, $\text{Strain } \ h = \frac{\Delta l}{l} \approx 10^{-19}$

If the event happens in another galaxy, 100 Mpc (10^{24} m away),

$$\text{Strain } \ h \approx 10^{-23}$$

This small strain requires the measurement of <10^{-20} meters in a detector of size 1 km! (almost million times smaller than the atomic nucleus).

Is it a mad venture trying to make a ‘detector’?

EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

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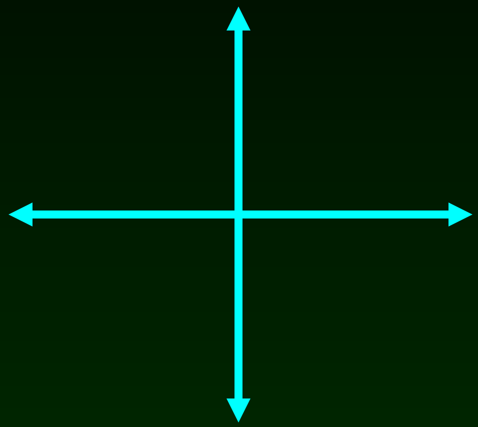
(Received 29 April 1969)

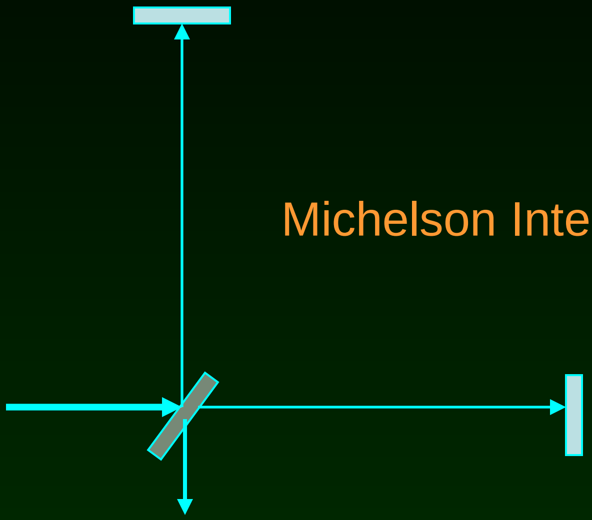
Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.

$$h \approx 10^{-17}$$

A modern cryogenic resonant mechanical detector

When these waves reach earth, what can they do to free masses?





$$\text{Strain } h \approx \frac{G}{c^2 R}\frac{Mv^2}{c^2} \rightarrow \frac{6.7\cdot 10^{-11}\times 3\cdot 10^{30}}{\left(3\times 10^8\right)^2\times 10^{20}} 0.05^2 < 10^{-19}\, m/m!$$

Much less than the size of the nucleus.
This is the primary device for gravitational wave detection

Signal
1/R, G/c^2,
Random source…

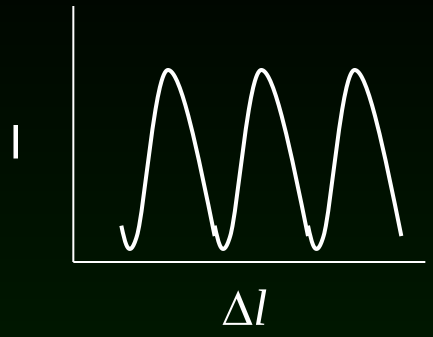
<<

Thermal noise,
Seismic Noise,
Quantum Noise
Tidal Noise,
All instrument Noise,
Any Noise one can think of…

Signal
1/R, G/c^2,
Random source…

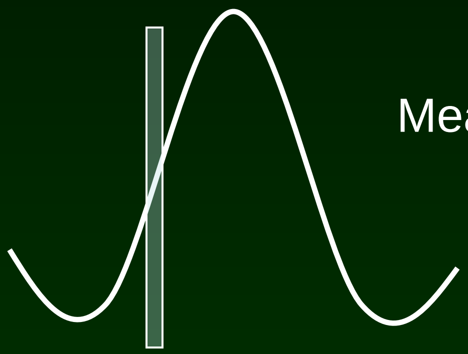
> or ~

Lower Limits of all these noises allowed by Physics and today's technology…



Signal in the interferometer

The general problem of ‘fringe splitting’
(centroid, locking…)



Mean number of photons @ intensity I= N, $N = P/h\nu$

Change in the number of photons, $\Delta N \approx \dfrac{N\Delta\phi}{\pi}$

Noise $= \sqrt{N} \approx \Delta N_{\min}$

With 1 W of optical power, $\mathrm{N}=10^{19}\ / s$,

$$\Delta\phi_{\min} \approx \frac{\pi\Delta N_{\min}}{N} = \frac{\pi}{\sqrt{N}} \approx 10^{-9}$$

There is another equivalent way to talk about photon shot noise that explicitly brings out the basic feature of quantum mechanics involved.

The energy-time uncertainty relation $\Delta E \Delta t \geq \hbar$

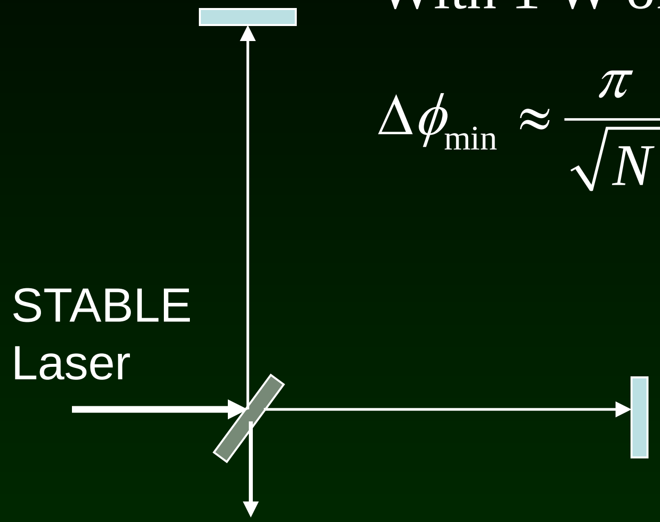
$$\Delta E \Delta t = \hbar \omega (\Delta N)\, \Delta \phi / \omega \approx \hbar$$

$$\rightarrow \Delta N\, \Delta \phi \geq 1 \rightarrow \Delta \phi \approx \frac{1}{\Delta N} = \frac{1}{\sqrt{N}}$$

Detection of gravitational waves requires the measurement of movements 10^{-17} to 10^{-20} meters in a detector of size 1 km.

With 1 W of optical power, $N = 10^{19}/s$, and 10 ms (100 Hz),

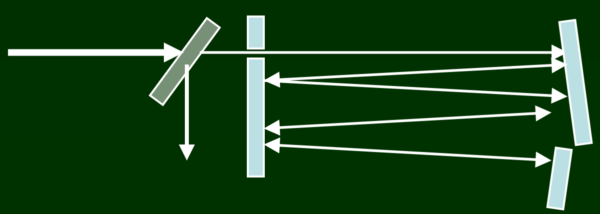
$$\Delta\phi_{\min} \approx \frac{\pi}{\sqrt{N}} \approx 10^{-8} \rightarrow \Delta L = \Delta\phi_{\min} \times \lambda \approx 10^{-14}\ m$$



1) Increase Laser Power $10\ kW \rightarrow \Delta L \approx 10^{-16}\ m$

2) Increase Length up to 4 km: Not much gain, though very important (1/R).

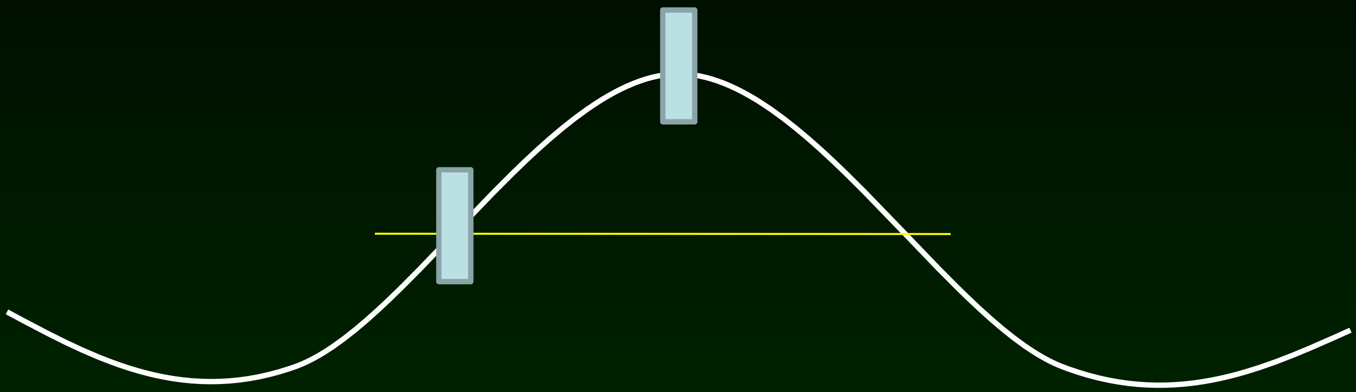
3) Fold optical path



$$n = 10 \rightarrow 10^{-17}\ m$$

Reaching there, but not comfortable yet!

Optimal length of the Interferometer arm:



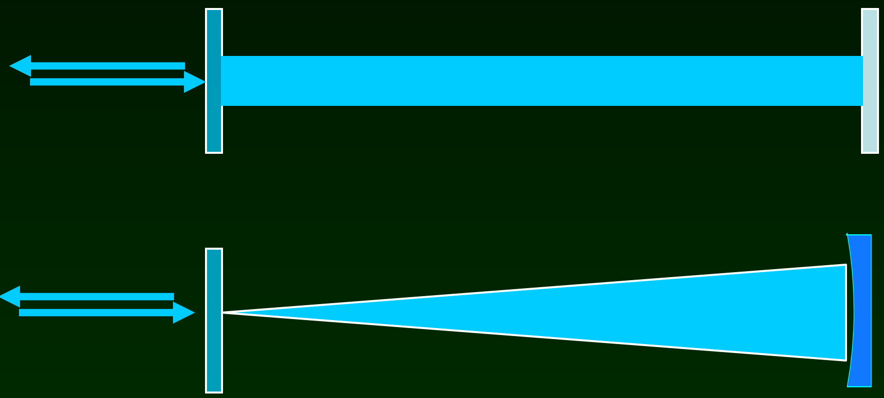
$$L_{opt} \approx \frac{\lambda_g}{4} \approx \frac{c/\nu_g}{4} \approx \frac{3\times10^8 m/100Hz}{4} = 750\,km!$$

This is the optimal distance the light should travel for maximal signal. In other words light should be in the interferometer for an optimal duration of about 750 km/c seconds or a quarter of the GW period of 10 ms or so. This is achieved by multiple bounces with average time equal to about quarter of the GW period such that 4 km x n_B = 750 km. So, the number of bounces is about 200.

Detection of gravitational waves requires the measurement of movements 10^{-17} to 10^{-19} meters in a detector of size several km.

Improvements:

1) Folding → Fabry-Perot Cavity



Finesse ~ n : 300 +

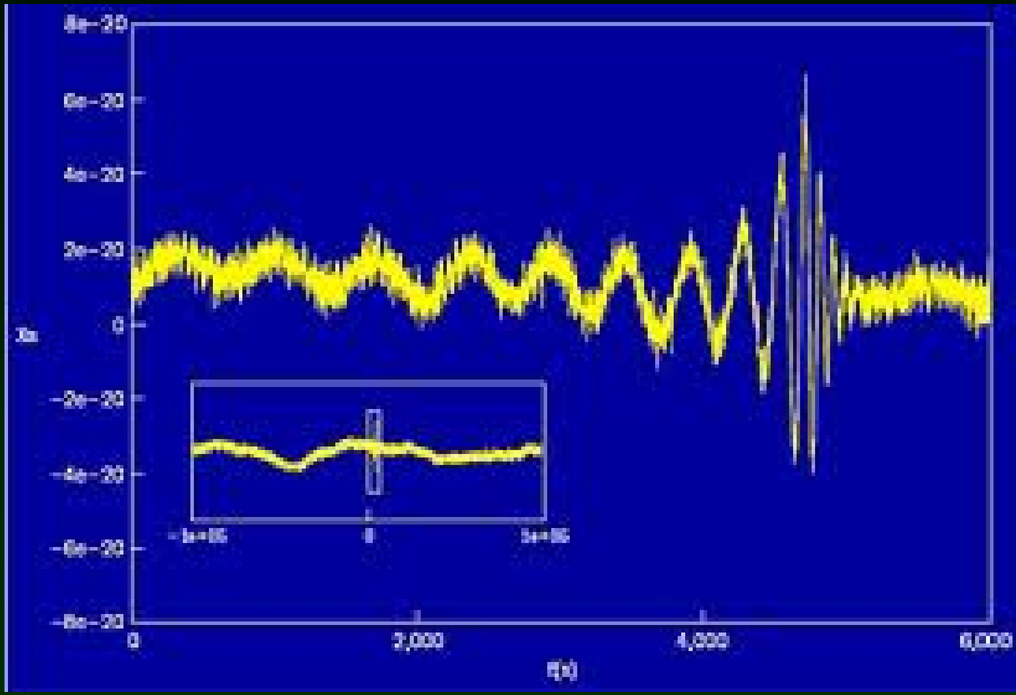
$$10\ kW,\ 4\ km \rightarrow 10^{-16}\ m \rightarrow 3\times10^{-19}\ m \text{ with } F \approx 300$$

Intra-cavity power > 1 MW !

Radiation Pressure Noise and Thermal Lensing are problems


Large Interferometer VIRGO at Pisa, Italy (3 km)

LIGO-HO



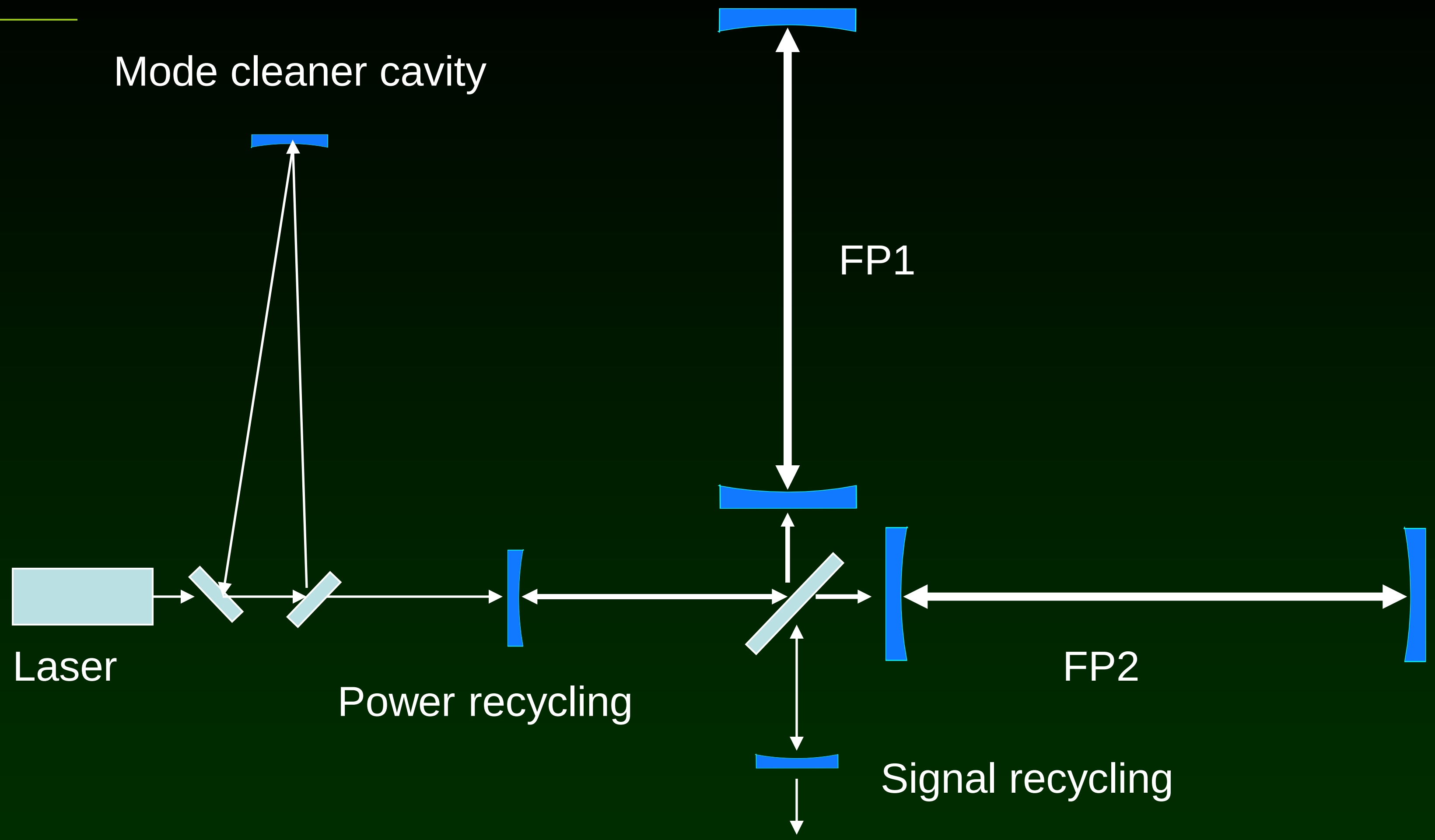
Strain $h = \dfrac{\Delta L}{L} \approx \dfrac{G}{c^4} \dfrac{Mr^2\omega^2}{R}$

$\omega^2 \approx GM / a^3 \rightarrow \nu \approx \frac{1}{2\pi}\sqrt{G\rho}$

But, every bit counts… because waves strength is 1/R

If sensitivity is increased by factor X, then the distance reach increases by X, and the number of astrophysical sources increases as X^3!

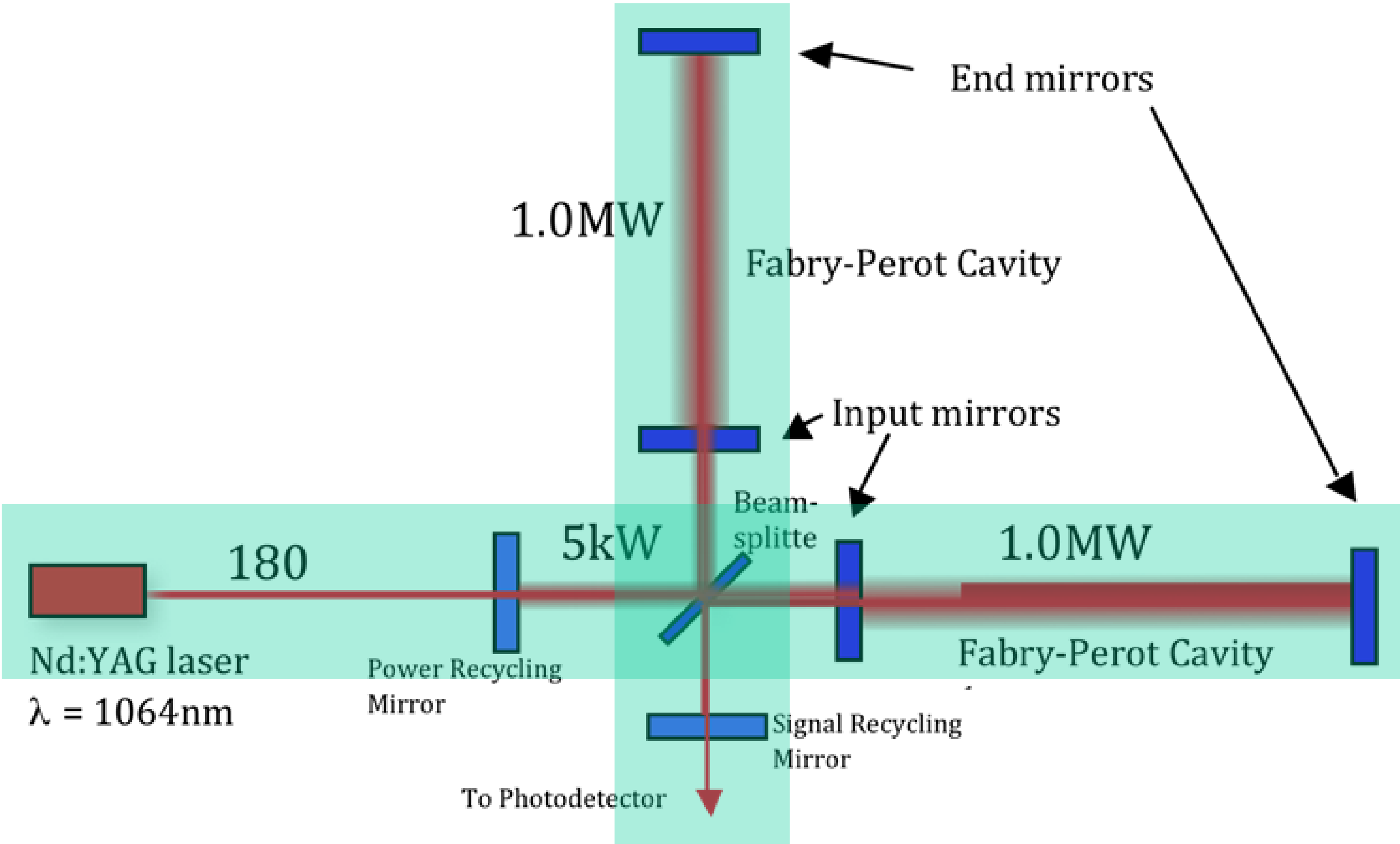
So, a factor of 10 in sensitivity means a factor of 1000 in number of possible detections.



Scheme of the Interferometric detector

3m scale prototype being built in TIFR – can measure $<10^{-17}$ m

Schematic of Advanced LIGO detectors



Large scale Ultra high Vacuum to be fabricated in India
10 mega -litres at nano-torr

Noise from Light:

$$\Delta l = \lambda \Delta\phi_{\min} = \frac{\lambda\pi}{\sqrt{N}}$$

$$\sqrt{N} = \sqrt{P_i / \left(hc/\lambda\right)}$$

Photon Shot Noise: $h_{sn} = \alpha\sqrt{\frac{1}{P_i}}$

Radiation Pressure Noise $F_{rad} = \sqrt{N}\ \left(h/\lambda\right) = \sqrt{\frac{hP_i}{c\lambda}}$

Movement noise due to this force: $h_{rp} \propto F_{rad} / m\omega^2 = \beta\sqrt{P_i}$

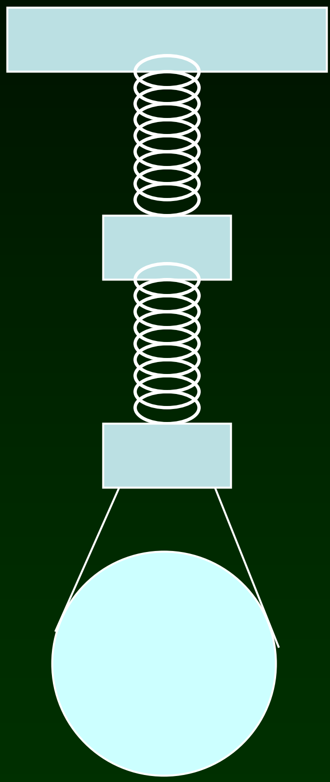
Standard Quantum Limit

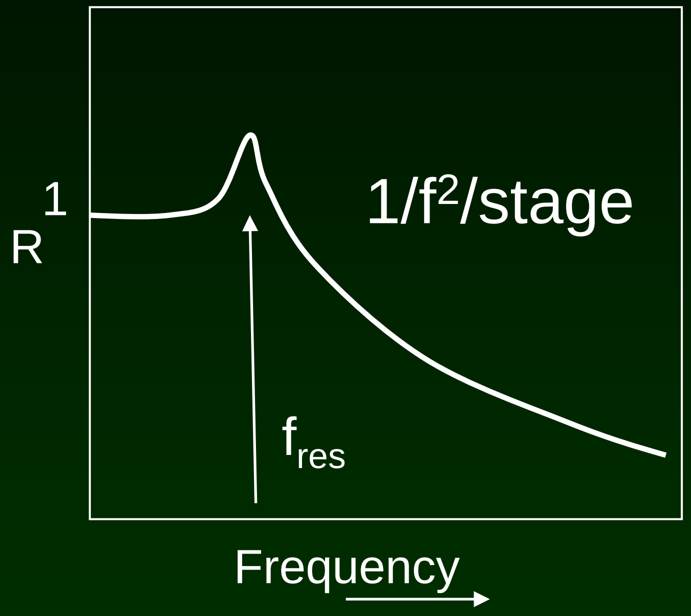
$$h_{total} = \sqrt{h_{sn}^2 + h_{rp}^2} \rightarrow h_{\min} = h_{SQL} = \frac{1}{\pi L f}\sqrt{\frac{\hbar}{m}}$$

How can we hope to measure 10^{-19} m when the ground vibrations are like 1 micrometer?!

Immunity to vibrations needed by a factor of 10^{14}!

Solution:

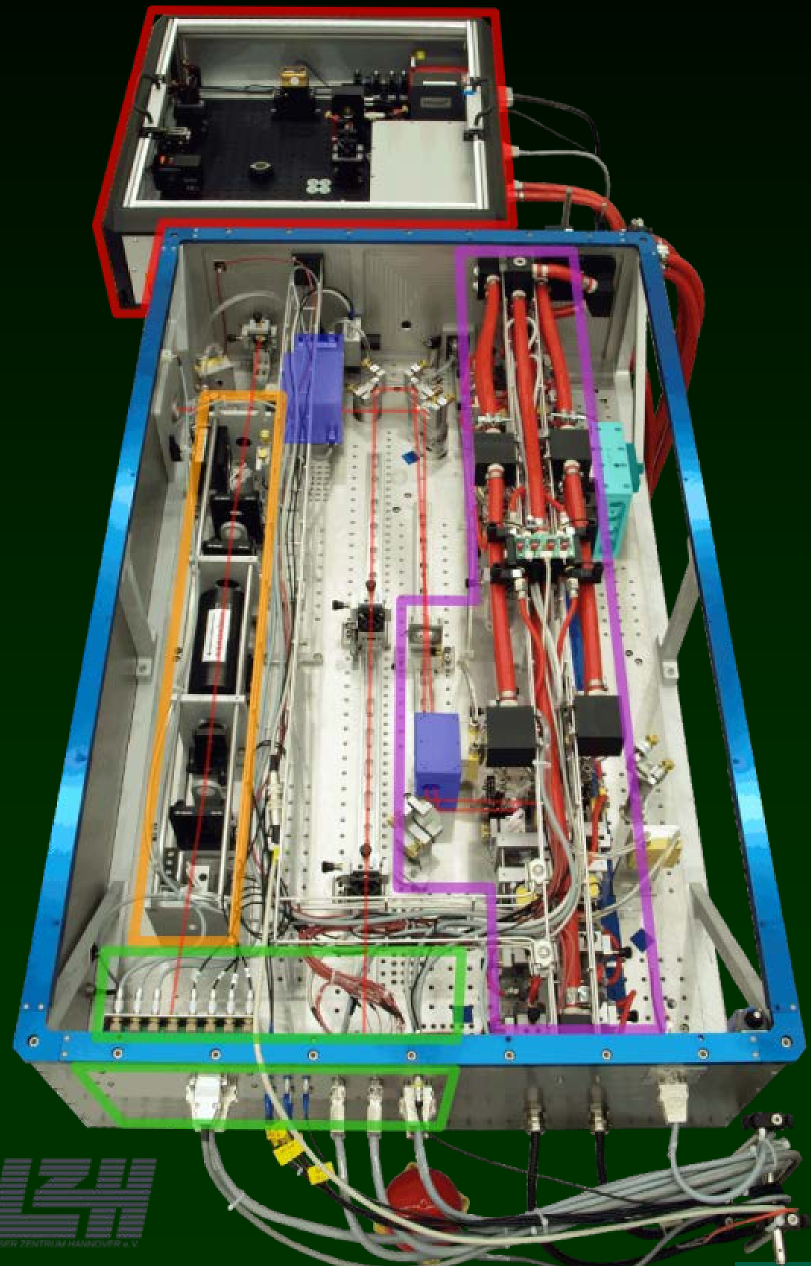


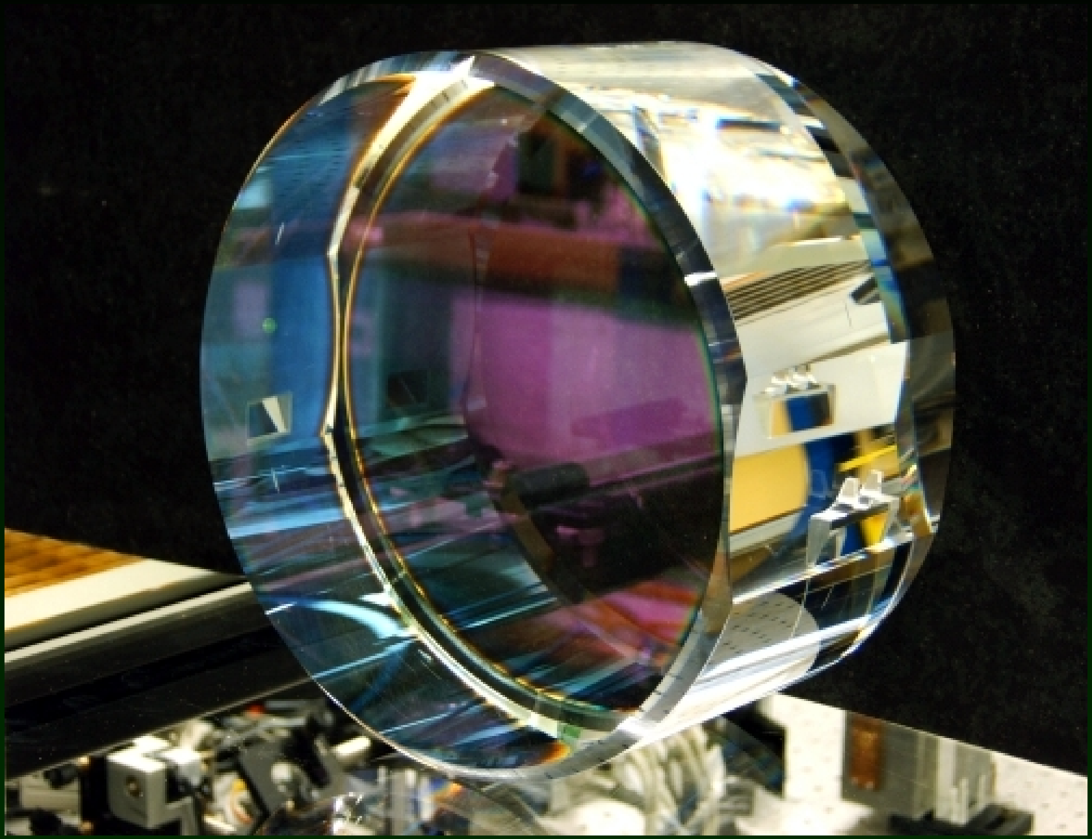


Ground vibrations: 10^{-6} m at 1 Hz, 10^{-9} m at 30 Hz

3 stages of springs and pendulum with each resonance at around 1 Hz
→ Response down by a factor $(10^3)^3 = 10^9$ at 30 Hz!

Possible to isolate from vibrations at the 10^{-20} m level at 100 Hz with 3-4 stages.

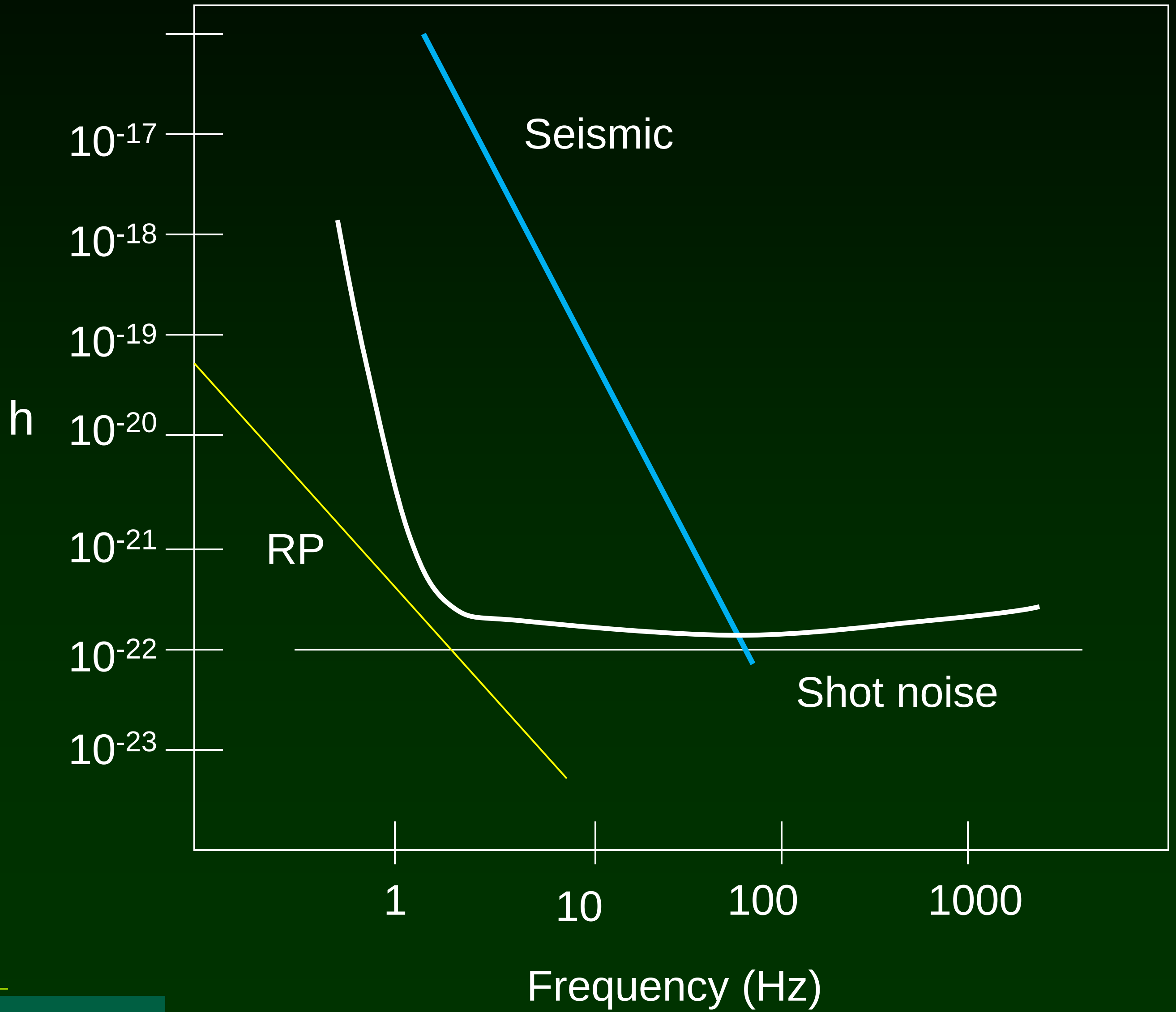




Absorption/Scattering → ppm level

GEO600

$$h_{total} = \sqrt{h_{sn}^2 + h_{rp}^2} \rightarrow h_{\min} = h_{SQL} = \frac{1}{\pi L f}\sqrt{\frac{\hbar}{m}}$$



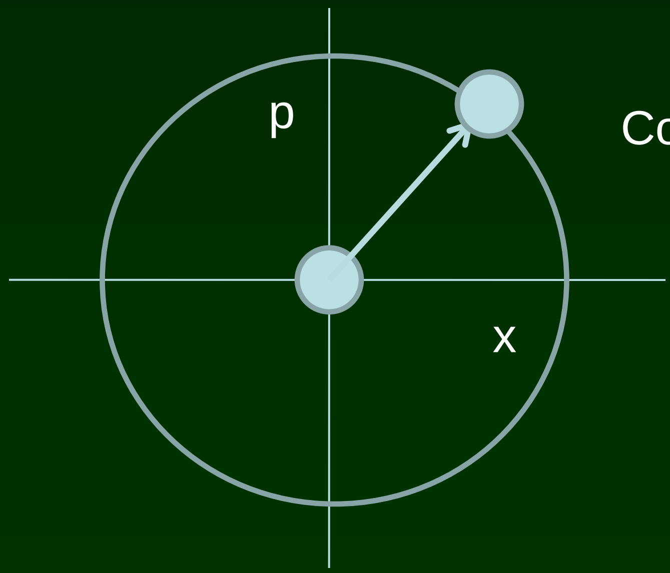
A sample of Vacuum chambers (LIGO): < 10^{-8} mbar, all 4x2 km

Quantum Noise

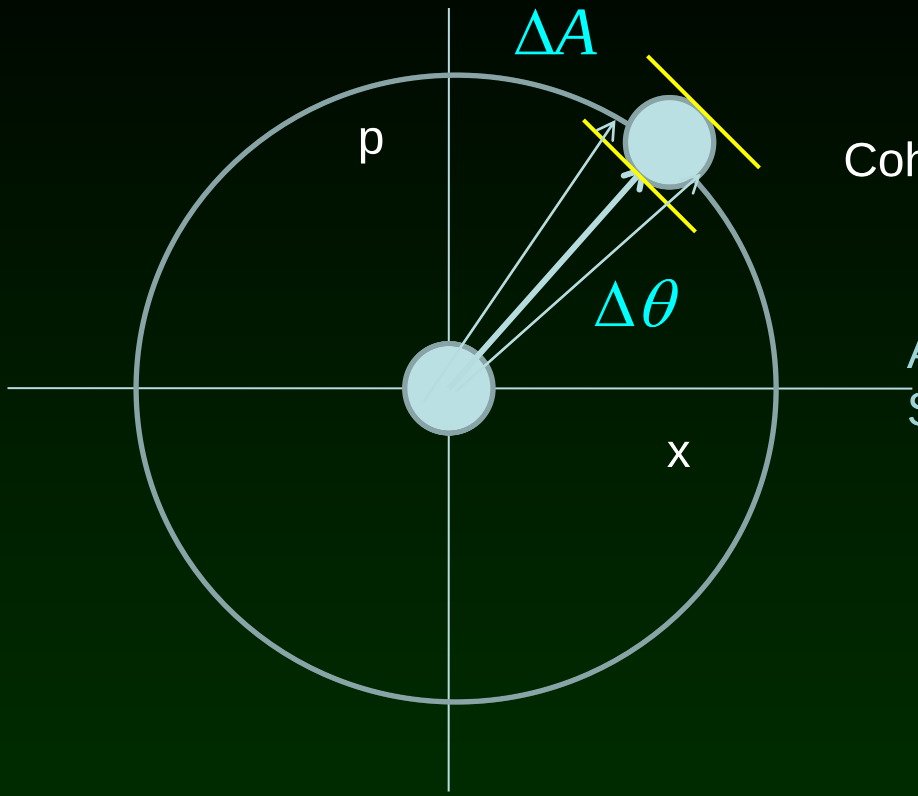
$$\Delta x \Delta p \geq \hbar$$

Light: Energy $\mathrm{E} \approx E^2 + B^2 \rightarrow p^2 + q^2$ Harmonic Oscillator

Main features: Zero point energy, equally quantized energy levels interpreted as number states of photons



Coherent state of light - laser



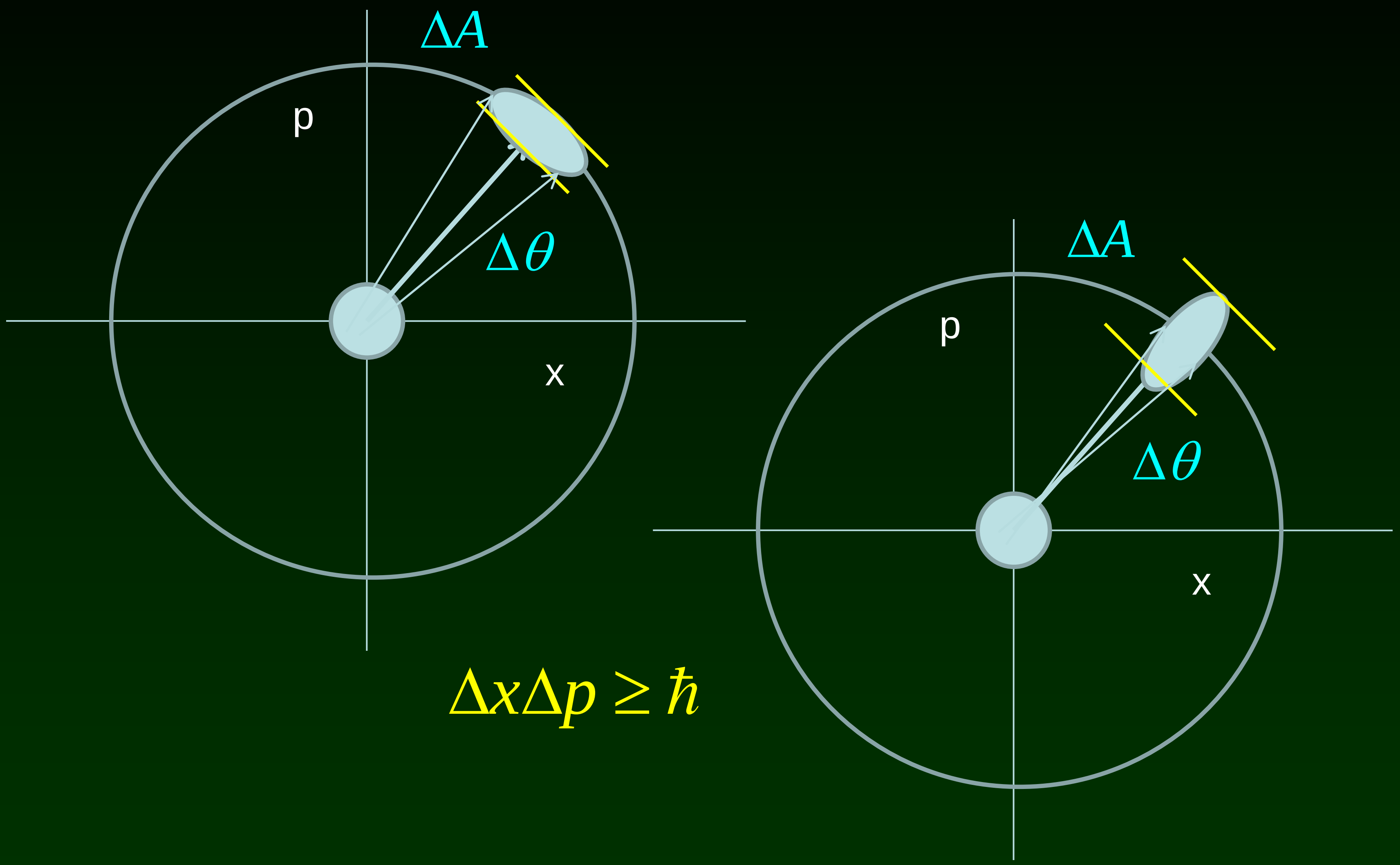
Coherent state of light - laser

Amplitude and Phase quadratures
Same Heisnberg uncertainty

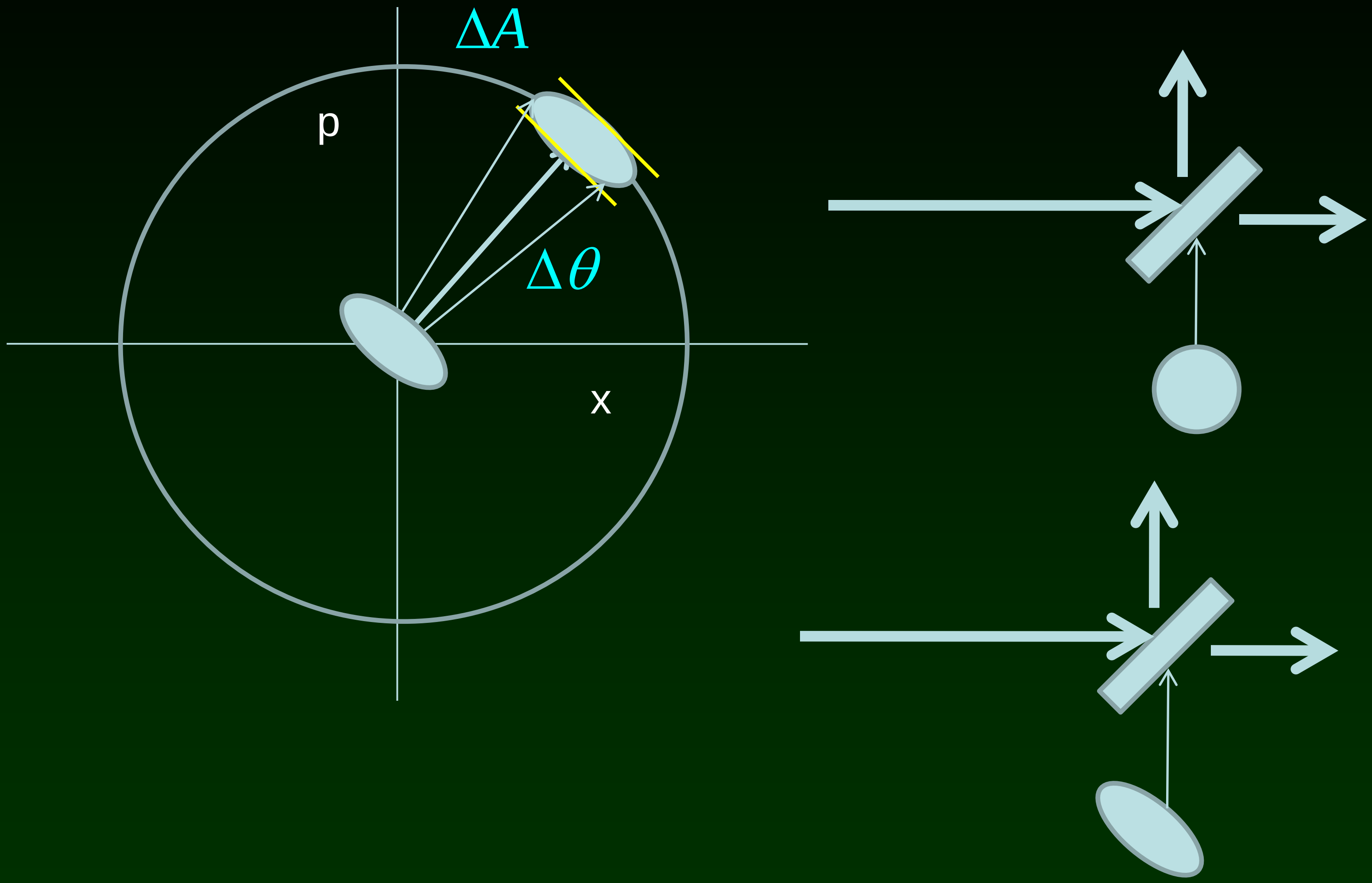
Amplitude Noise: $\dfrac{\Delta A}{\sqrt{N}}$

Phase Noise: $\dfrac{\Delta\theta}{\sqrt{N}}$

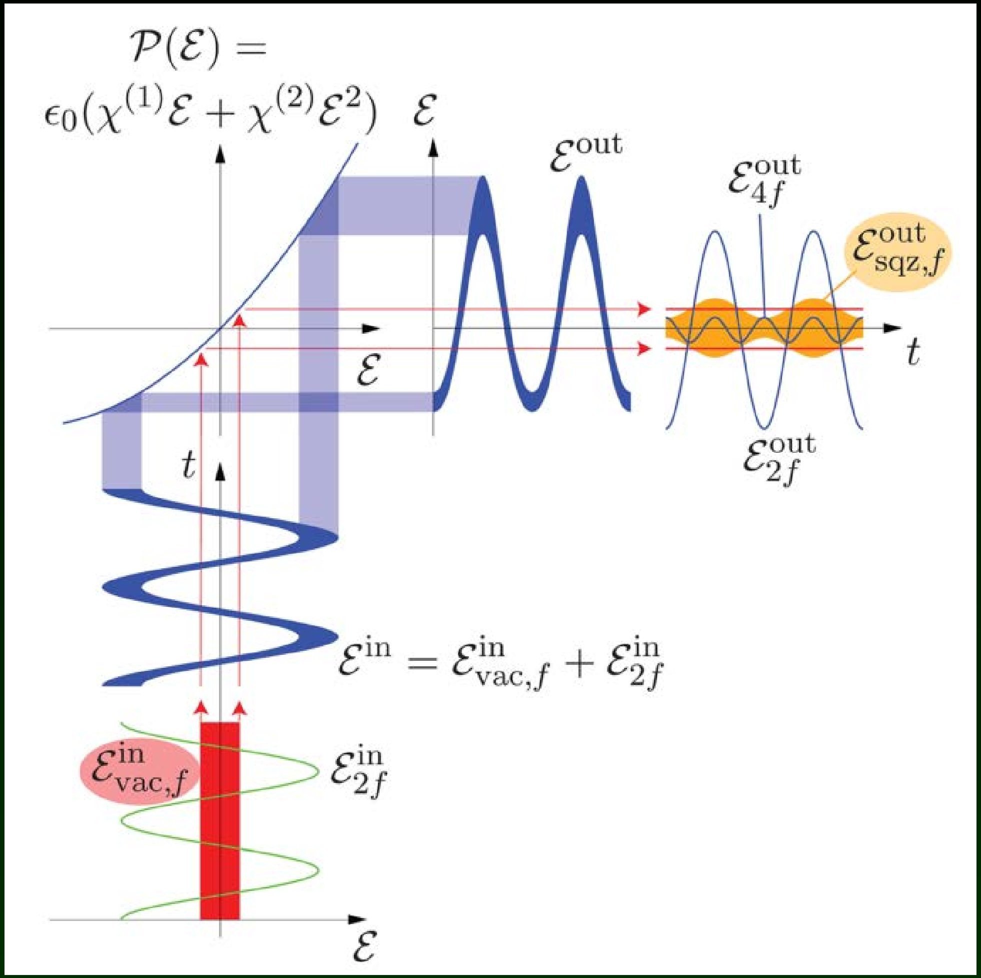
Therefore, if we can somehow SQUEEZE the uncertainty is amplitude or phase, we can improve measurements by the squeezing factor.



$$\Delta x \Delta p \geq \hbar$$



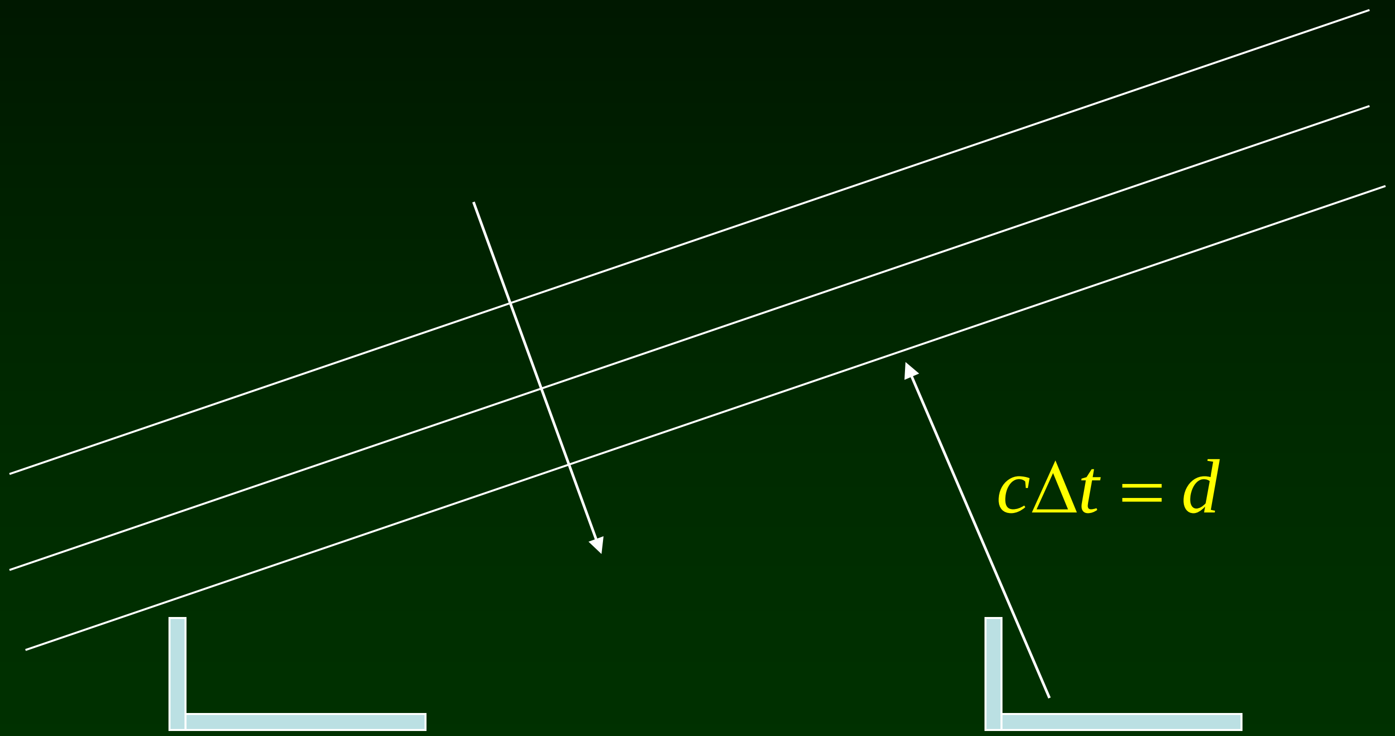
If the injected ‘vacuum’ is squeezed by a factor 10, then the quantum noise is reduced by the same factor, and this is equivalent to increasing the laser power by a factor 100, because noise reduces only as square root of power!



Roman Schnabel
American Jl. Physics 2013

Why do we need more detectors?

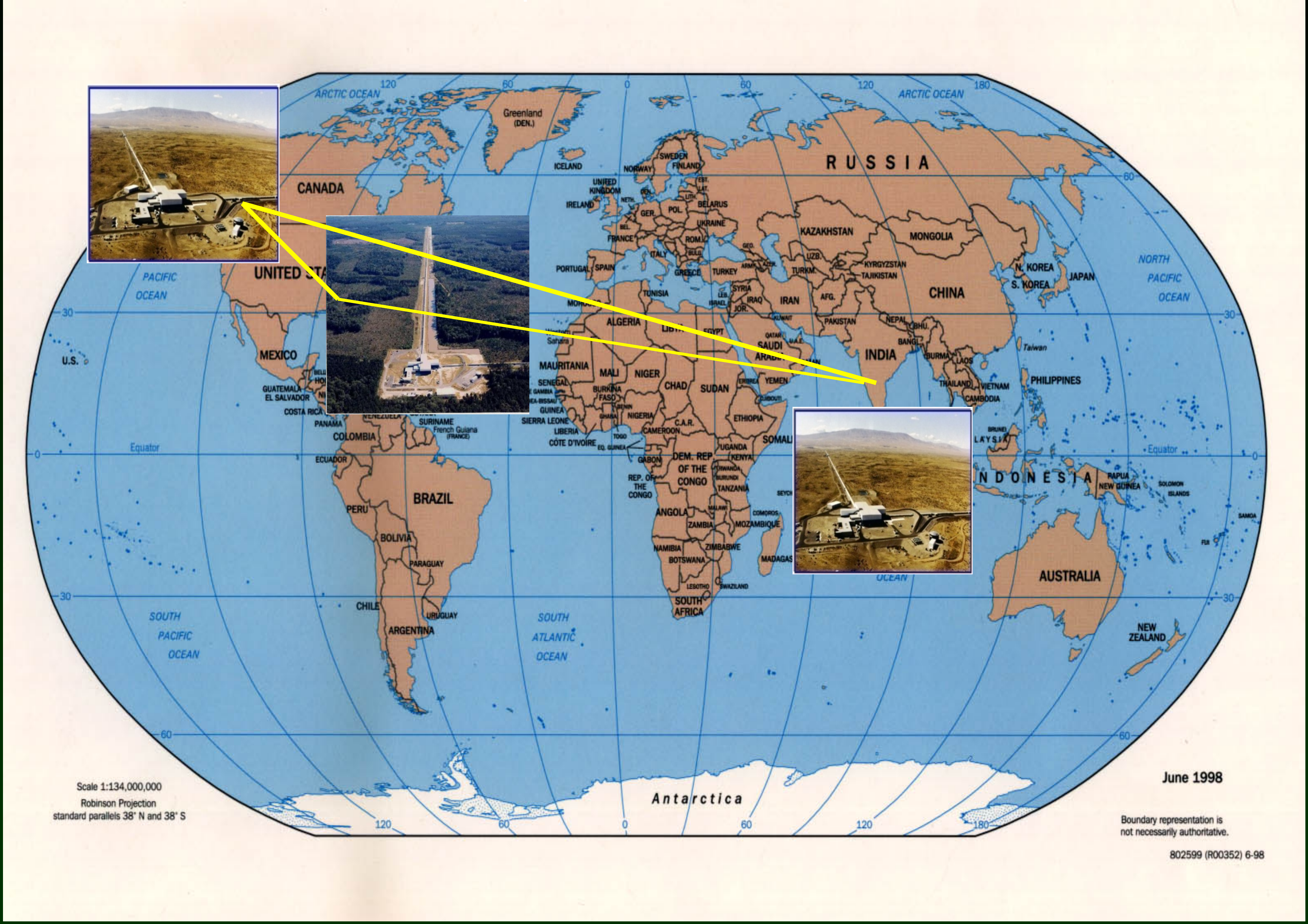
GW detectors are not telescopes. They cannot pin-point a source.

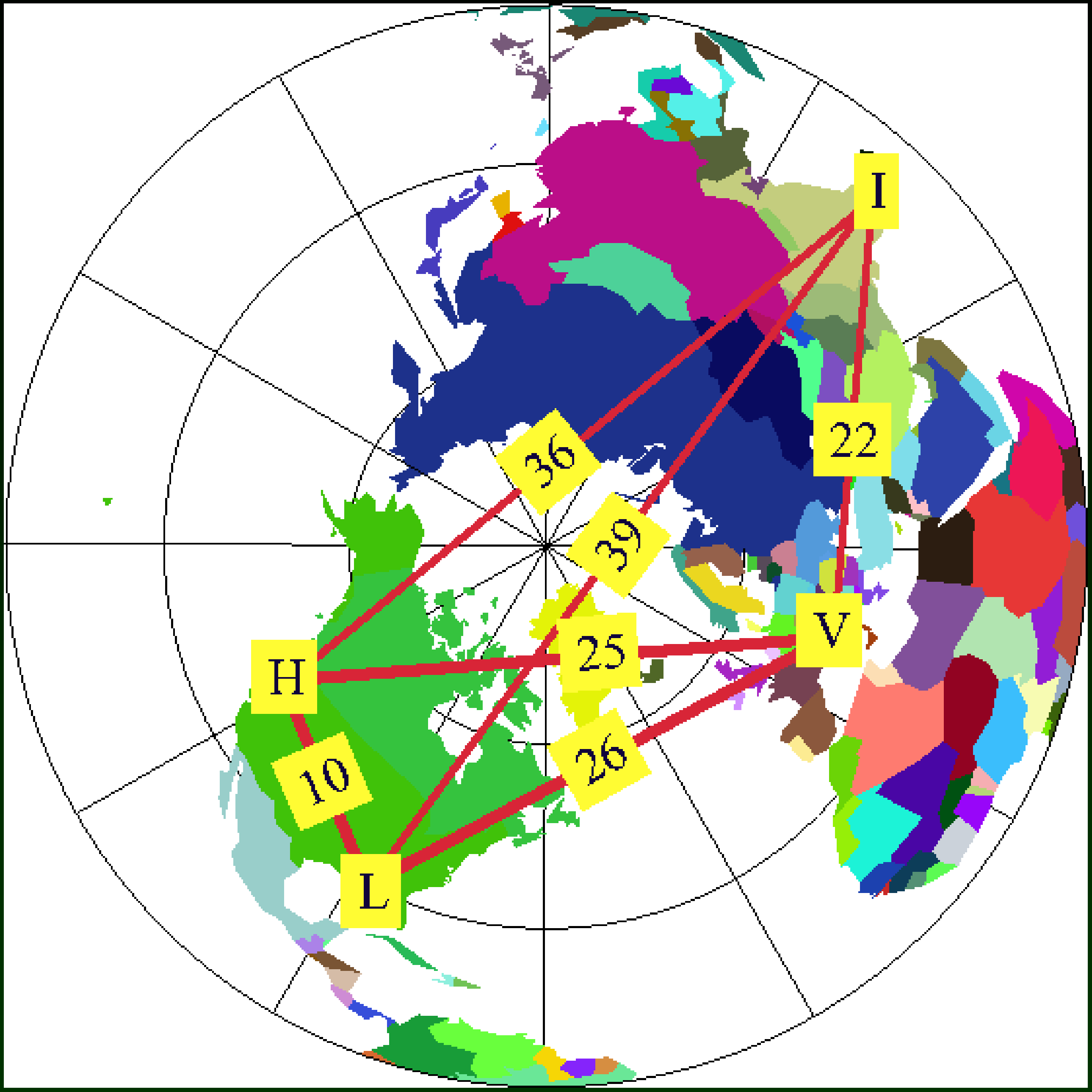


Timing (and only timing) can fix a direction

Need at least 3

The LIGO-India Concept





Good angular resolution

Base-lines to Japan comparable

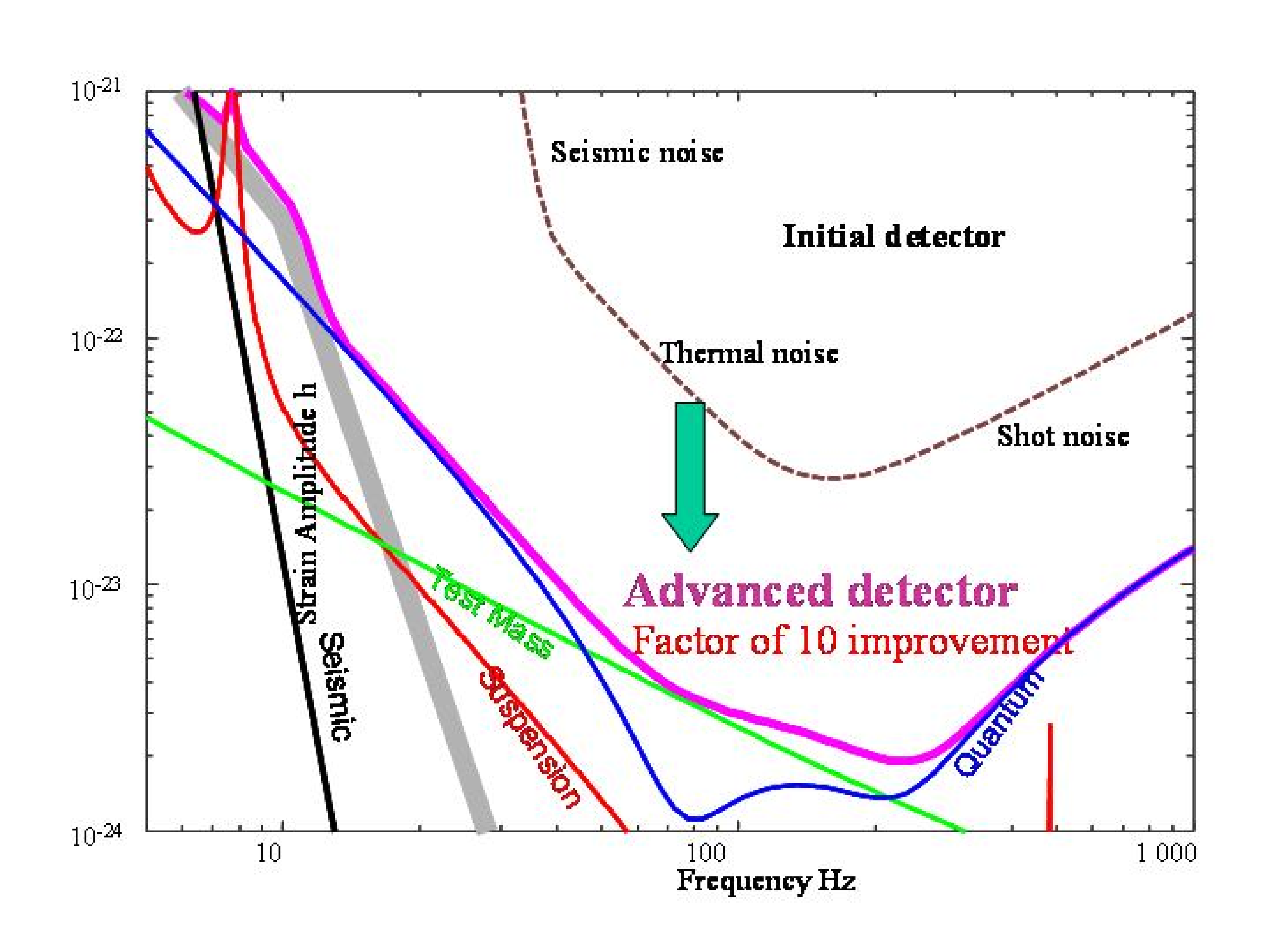
The LIGO-India Idea and Opportunity

The LIGO-India proposal envisages the hardware meant for one of the LIGO detectors (out of 3) to be given to India and Indian scientists and engineers will build and operate the detector at a suitable site in India. Thus the LIGO-India detector will be the third vertex of the LIGO network, working like a large gravitational wave telescope.

The Dept. of Atomic Energy, Dept. of Science and Technology and the Planning Commission have approved the Rs.1300 Crore (15 years) proposal. A cabinet approval is required and awaited. Site selection is in progress.

indigo

www.gw-indigo.org



10^{-21}
10^{-22}
10^{-23}
10^{-24}
Seismic noise
Initial detector
Thermal noise
Shot noise
Strain Amplitude h
Seismic
Test Mass
Suspension
Quantum
Advanced detector
Factor of 10 improvement
10
100
1 000
Frequency Hz

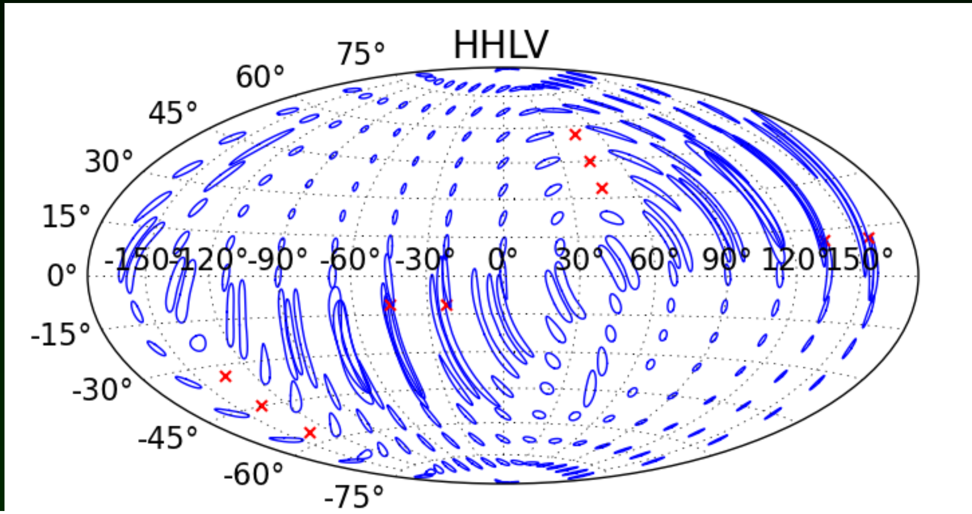
Science Gain from Strategic Geographical Relocation

Source localization error

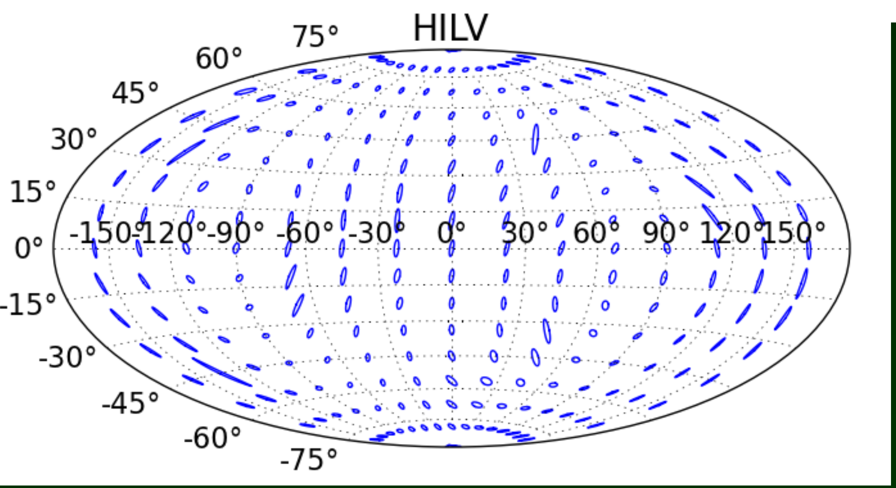
Also Polarization

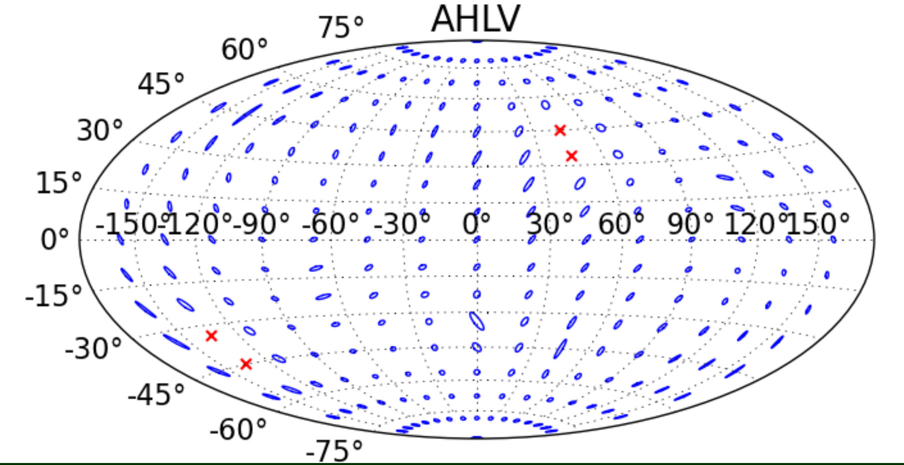
S. Fairhurst
arXiv:1205.6611v2

B. S. Sathyaprakash et al., LIGO document T1200219-v1



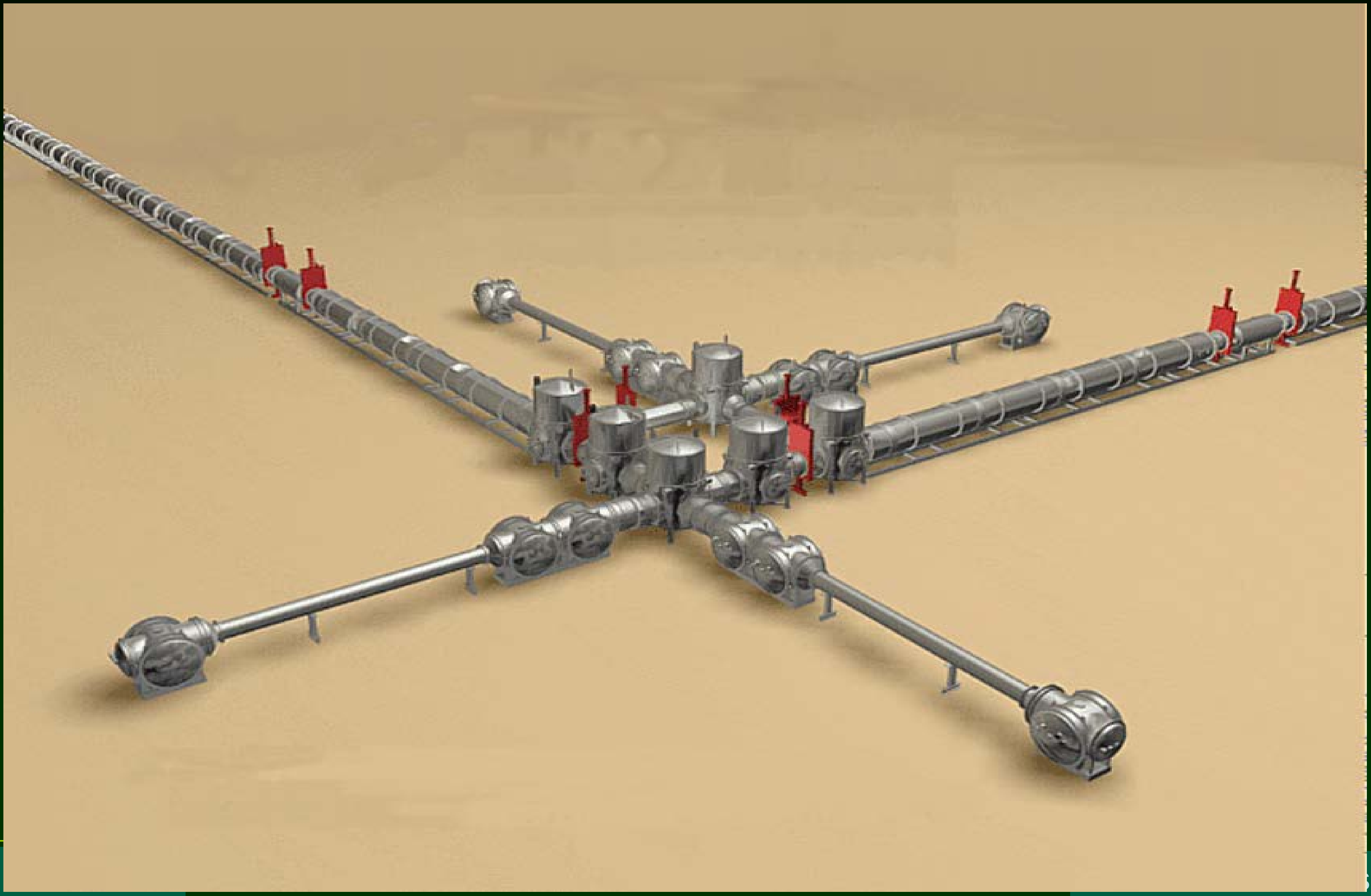
Original Plan
2 +1 LIGO USA+ Virgo

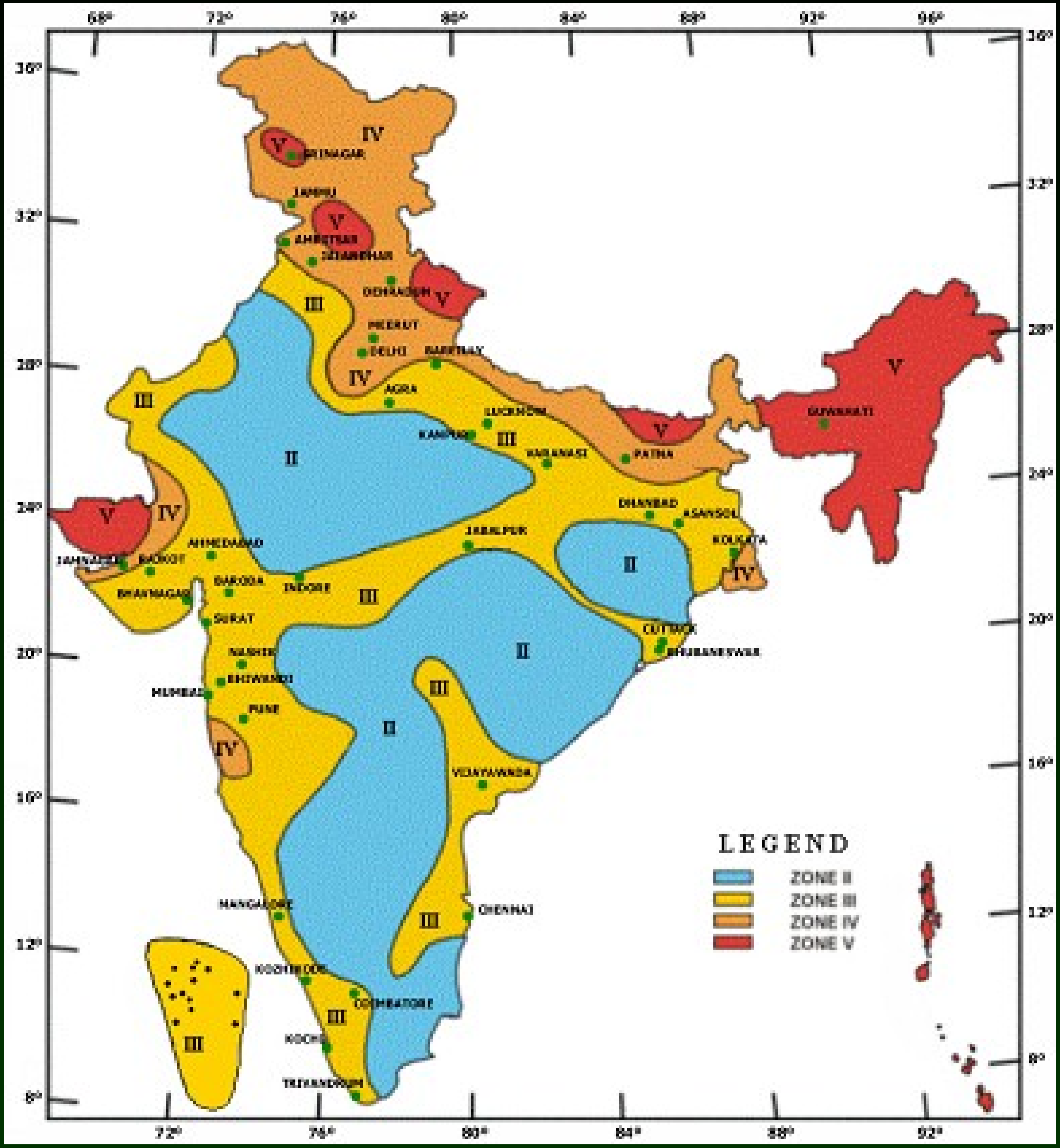




LIGO-India plan
1+1 LIGO USA+ Virgo+ LIGO-India

LIGO-Aus plan
1+1 LIGO USA+ Virgo+ LIGO-Aus





SRINAGAR
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DEHRADUN
MEERUT
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LUCKNOW
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DHANBAD
ASANSOL
KOLKATA
JABALPUR
AHMEDABAD
RAJKOT
BHAVNAGAR
BARODA
INDORE
SURAT
NASHIK
BHIWANDI
MUMBAI
PUNE
VIJAYAWADA
CUTTACK
BHUBANESWAR
CHENNAI
MANGALORE
COIMBATORE
TRIVANDRUM
LEGEND
ZONE II
ZONE III
ZONE IV
ZONE V


4 km

Gravitational Waves: The Indian Initiative
LIGO-India Project

There are great possibilities and bright future for gravitational wave-based astronomy… if we manage to detect gravitational waves with these detectors

The IndiGO Consortium

www.gw-indigo.org