Review of some points

Yesterday we compared electromagnetism and gravity and derived expression for the propagation of the transverse ~~elec of~~ gravitational field resulting from the motion of masses.

1) No dipole radiation $\ddot{d} = 0 \rightarrow \dot{p} = 0$

(2) (No monopole radiation either!)

3) The next order is quadrupole. $\ddot{I}_{/R} \sim \frac{1}{R}\frac{d^2}{dt^2}(Mr^2) \sim \frac{Mr^2\omega^2}{R} \cdot \left(\frac{G}{c^4}\right)$

The transverse field was.

$$E_{gt} = \frac{GM}{c^2 R} r\omega^2 \sin\theta \cdot \frac{2\pi}{\lambda_g} \cdot r\cos\theta$$

$$= \frac{GMr^2\omega^2}{c^3 R} \omega \sin\theta \cos\theta \quad \Big\} \; \lambda \sim c \cdot \frac{\pi r}{v} \sim 3\times 10^6 m \gg r$$

Strain: $m \cdot \frac{E_{gt}t^2}{m} \sim at^2$ But universal!

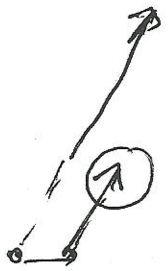
So only tidal effects matter

$$\Delta L = \Delta a t^2 = \left(\frac{\partial E}{\partial x} \cdot \cancel{\delta x} L\right) t^2 \rightarrow \left(\frac{E}{\lambda_g} \cdot L\right) \frac{1}{\omega^2}$$

$$\Delta L \approx \frac{GMr^2\omega^2}{c^4 R} \cdot L$$

$$h = \frac{\Delta L}{L} = \frac{GMv^2}{c^4 R}$$ Quadrupole formula.

Yesterday there was a question about Continuity of the field lines and necessity of connecting ~~past~~ old and ~~present~~ new with a transverse field. Let us address that better.



$\nabla\cdot E \neq 0$!

$$\nabla\cdot E = \rho$$

$\hookrightarrow 0 \rightarrow \Delta\cdot E = 0$

Now we turn to the description of the gravitational waves in terms of the metric for spatial ~~&~~ and time intervals (I ~~do~~ deliberately avoid saying space-time)

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

$$= \eta_{ij}\, dx^i dx^j \qquad \eta_{ij} = \begin{bmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}$$

~~In~~ When a gravitational field is present (or when this is a coordinate transformation)

$$\eta_{ij} \longrightarrow g_{ij}\,(x, t)$$

$$g_{ij} = \eta_{ij} + h_{ij} \hookrightarrow \text{very small.}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_{ij} = 0\,, \qquad h\,(\omega t - k\cdot x)$$

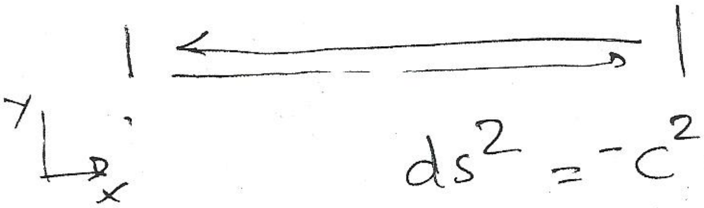
Since the waves are transverse ~~they~~ planes will be in the x-y direction, 0 in t and z direction

$$h_{ij} \rightarrow \begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & 0 & 0 & 0 \\ x & 0 & a & b & 0 \\ y & 0 & b & -a & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$$

Traneverse, Traceless TT

Quadrupole — Spin-2 etc.

Anticipating that we are going to use light to mark out distance, as in the interferometer, let us see how phase shift is related to change in the metric h_{ij}.



$$ds^2 = -c^2 dt^2 + dx^2 = 0$$

$$(dx = c\,dt)$$

$$g_{ij}\, dx^i dx^j = 0$$

$$(\eta_{ij} + h_{ij})\, dx^i dx^j = 0$$

$$-c^2 dt^2 + \left(1 + h_{11}(\omega_g t - k_g \cdot x)\right) dx^2 = 0$$

$$dt = \frac{1}{c}\sqrt{1+h_{11}}\, dx \approx \frac{1}{c}\left(1 + \frac{h_{11}}{2}\right) dx.$$

$$\int dt = \frac{2L}{c}\left(\frac{h}{2}\right)$$

$$\Delta T = \frac{2L}{c} h, \qquad \Delta\phi = \omega_\ell \Delta T = h(t)\left(\frac{2L}{c}\right) \cdot \frac{2\pi c}{\lambda_g}$$