LIGO-G1401146-v1

Control system in Gravitational Wave Detectors

Koji Arai / Stan Whitcomb – LIGO Laboratory / Caltech

Introduction ~ Control?

- Gravitational wave detection
  Laser displacement sensor
  Requires linear displacement detection

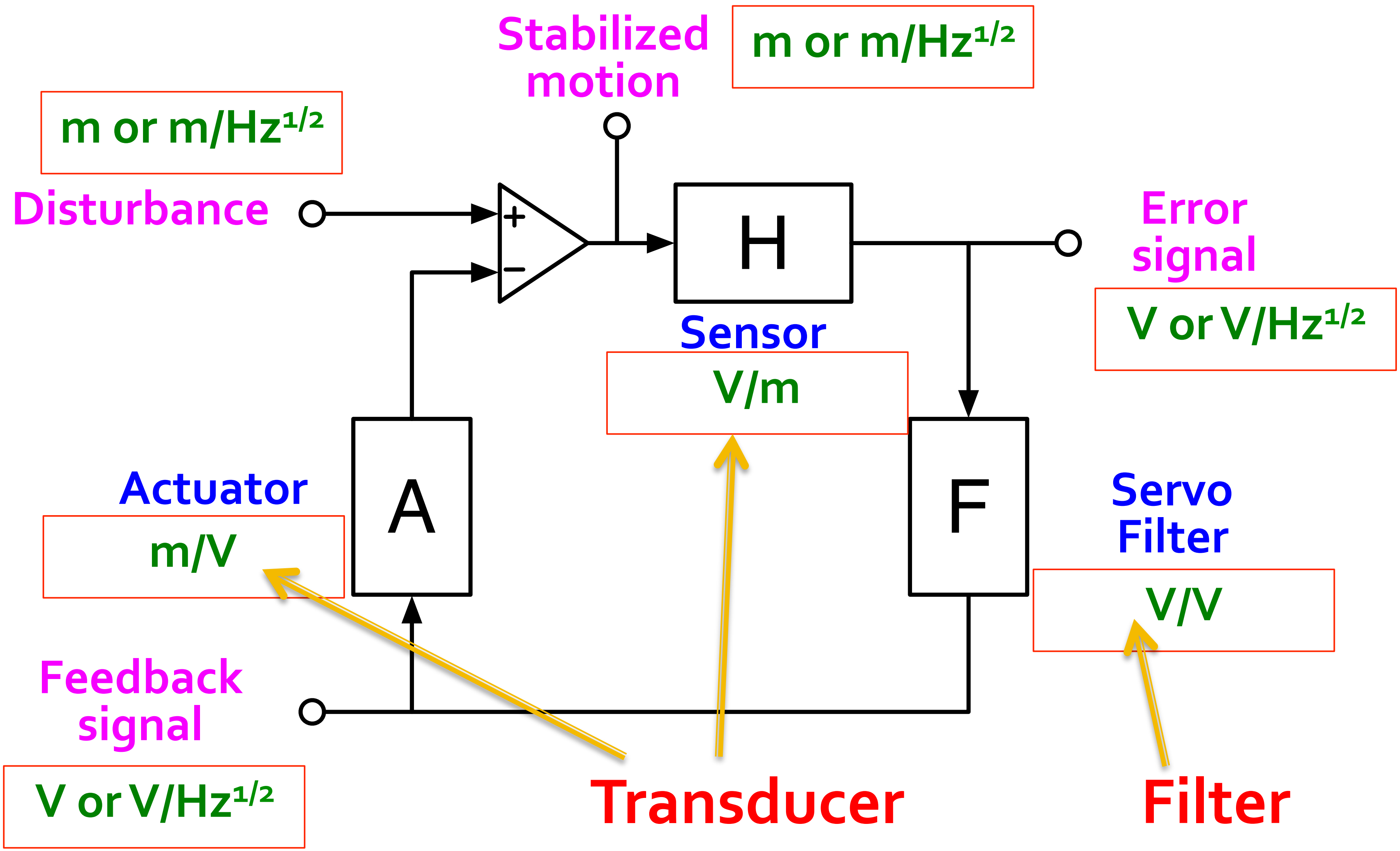
- Control for measurement
  Laser interferometer = nonlinear device
  Feedback control => linearization

Introduction ~ Control?

- What is the feedback control?
  - A scheme to monitor and modify output(s) of a system by changing the input(s) depending on the output(s)
- Examples
  - Shower temperature
  - Car driving
  - Tight rope walking
  - Air conditioning
  - Bike riding
  - Inverted bar on a hand
- Imagine what happens
  - If the response is too slow?
  - If the response is too fast?

Introduction ~ Control?

- Elements of a feedback loop



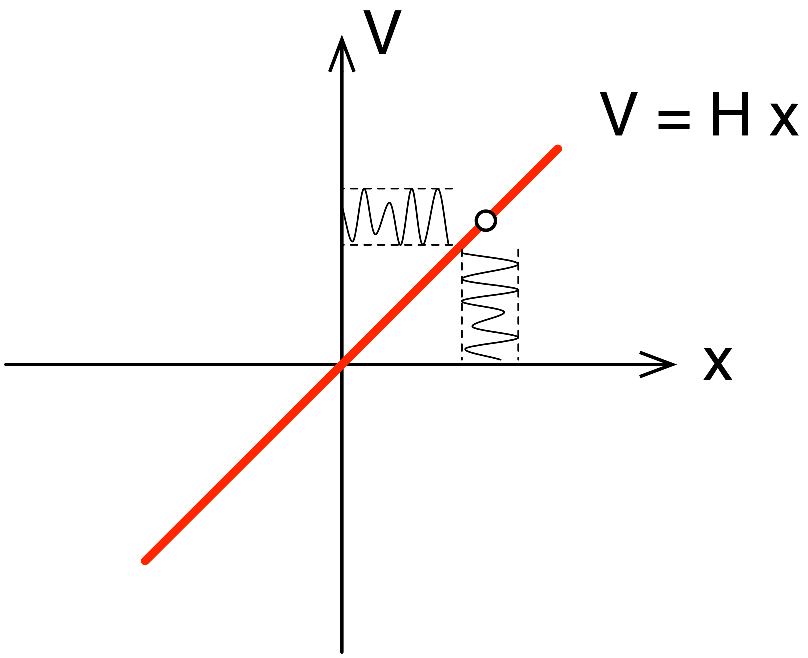
Introduction ~ Control?

- **Sensor:**

  Transducer for displacement-to-voltage conversion

- **<u>If the sensor is completely linear</u>**

  (and has or no frequency dependence)

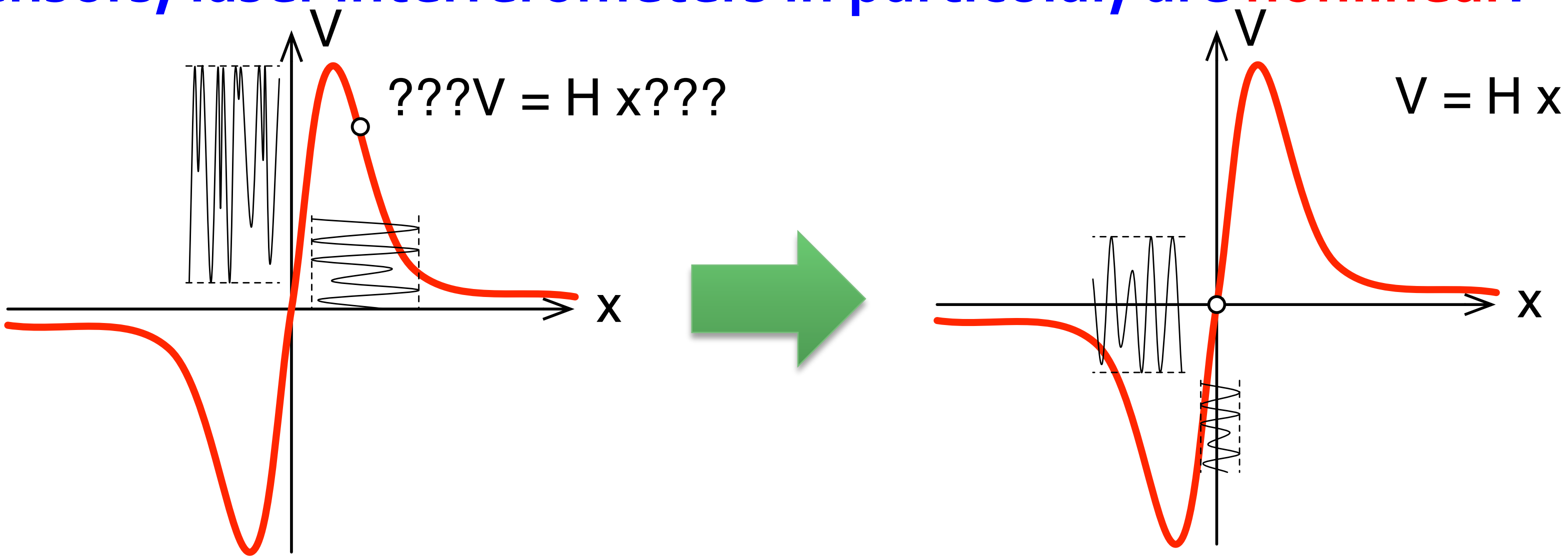


<u>We don’t need feedback control!</u>

Introduction ~ Control?

- **In reality:**

  Sensors, laser interferometers in particular, are nonlinear!



- **<u>Enclose the operating point in the linear region</u>**

  => The system recovers linearity

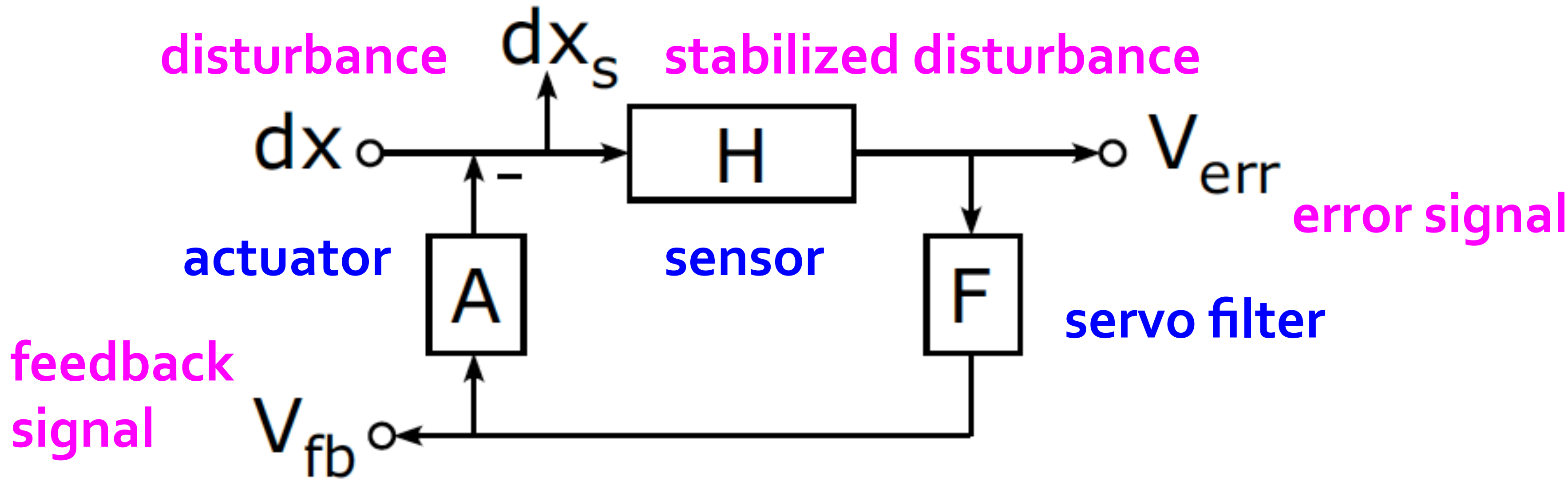
- **Was the displacement modified by the feedback?**

  => Precise knowledge of the control system

  for signal reconstruction

Introduction ~ Control?

- **Elements of a feedback loop**



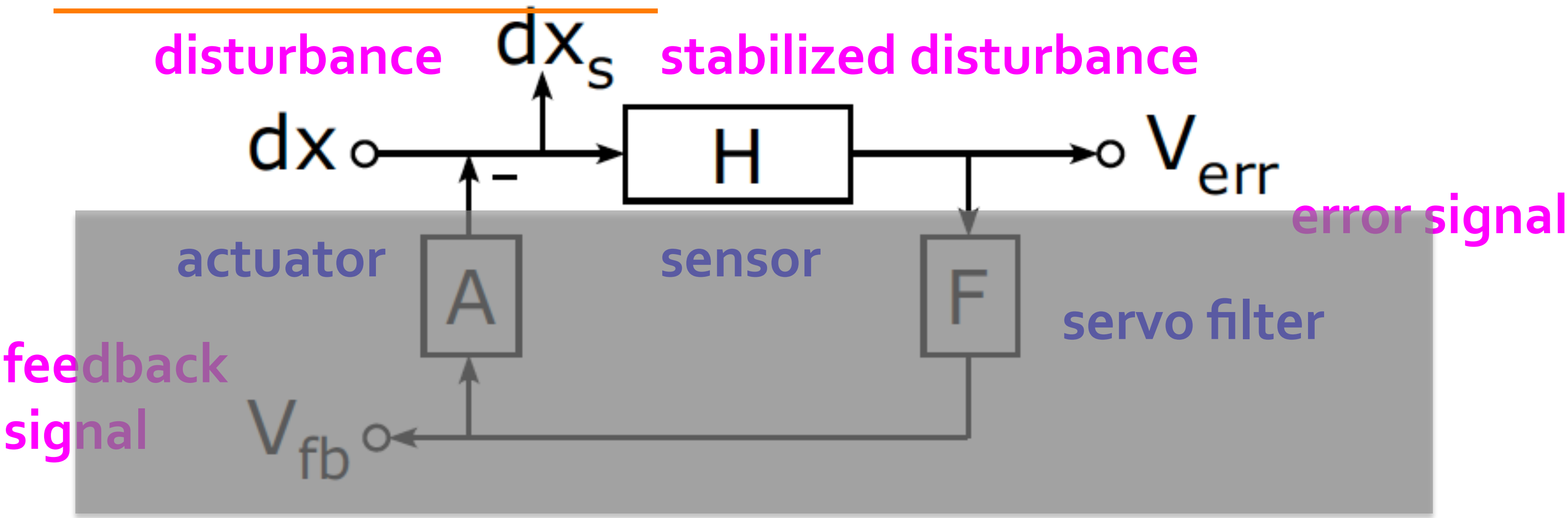
$$dx_s = dx - G\, dx_s$$
$$\Rightarrow dx_s = dx \,/\, (1+G)$$
$$\Rightarrow dx = V_{err}\, (1+G) \,/\, H$$
$$dx = V_{fb}\, A\, (1+G) \,/\, G$$

Open loop transfer function

$$\boxed{G \overset{def}{=} H\, F\, A}$$

Introduction ~ Control?

- **When G is small:**



$$dx_s = dx - G\, dx_s$$
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$$\Rightarrow dx = V_{err} \; (1+G) \,/\, H$$
$$dx = V_{fb}\, A\, (1+G) \,/\, G$$

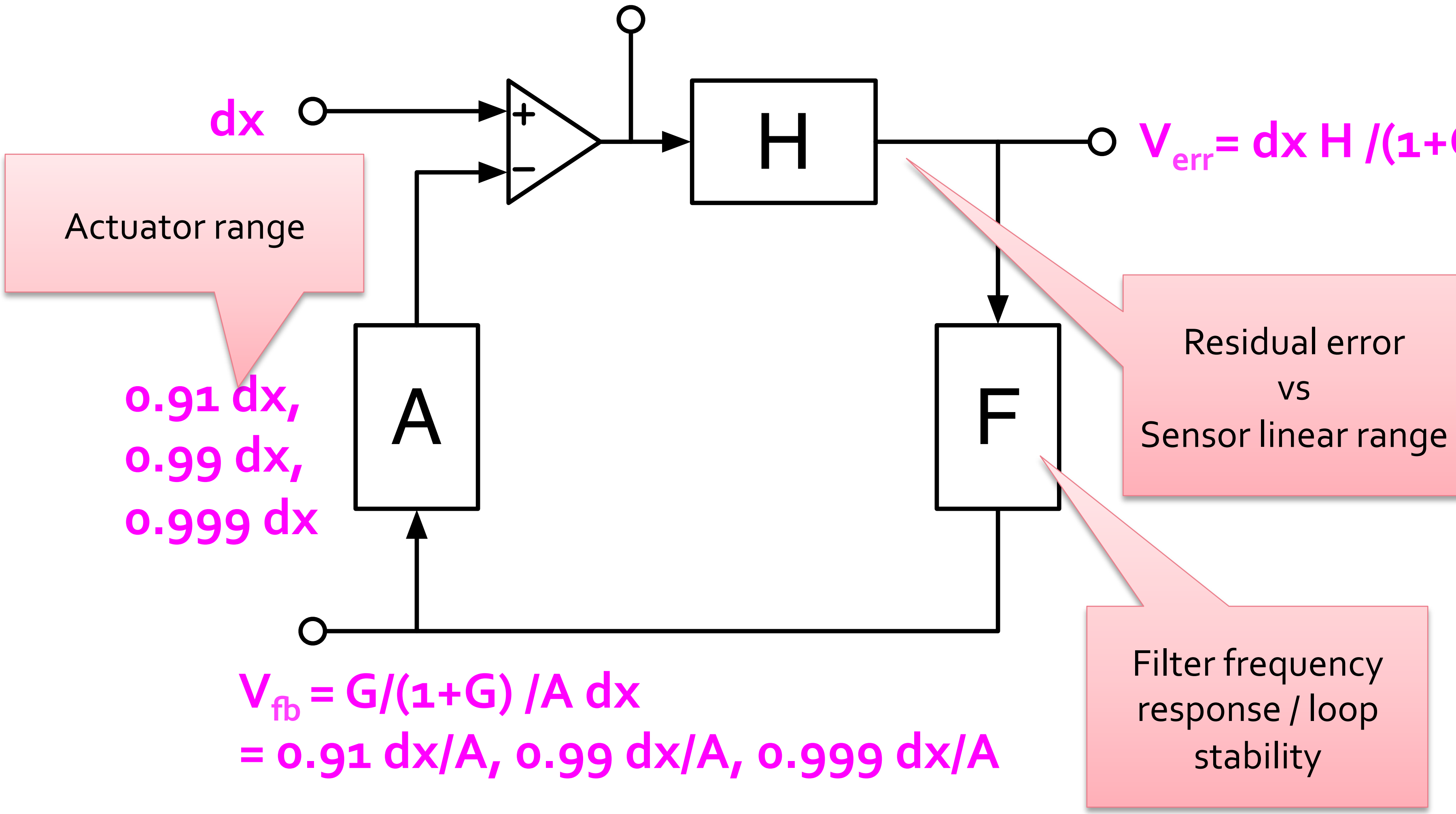
Open loop transfer function

$$\boxed{G \overset{\text{def}}{=} H\, F\, A}$$

Introduction ~ Control?

- When G is big: e.g. G = 10, 100, or 1000

$dx_s = dx/(1+G) = 0.09\,dx,\ 0.01\,dx,\ 0.001\,dx$



$V_{err} = dx\,H/(1+G)$

0.91 dx,
0.99 dx,
0.999 dx

$V_{fb} = G/(1+G)\,/A\,dx$
$= 0.91\,dx/A,\ 0.99\,dx/A,\ 0.999\,dx/A$

Introduction ~ Control?

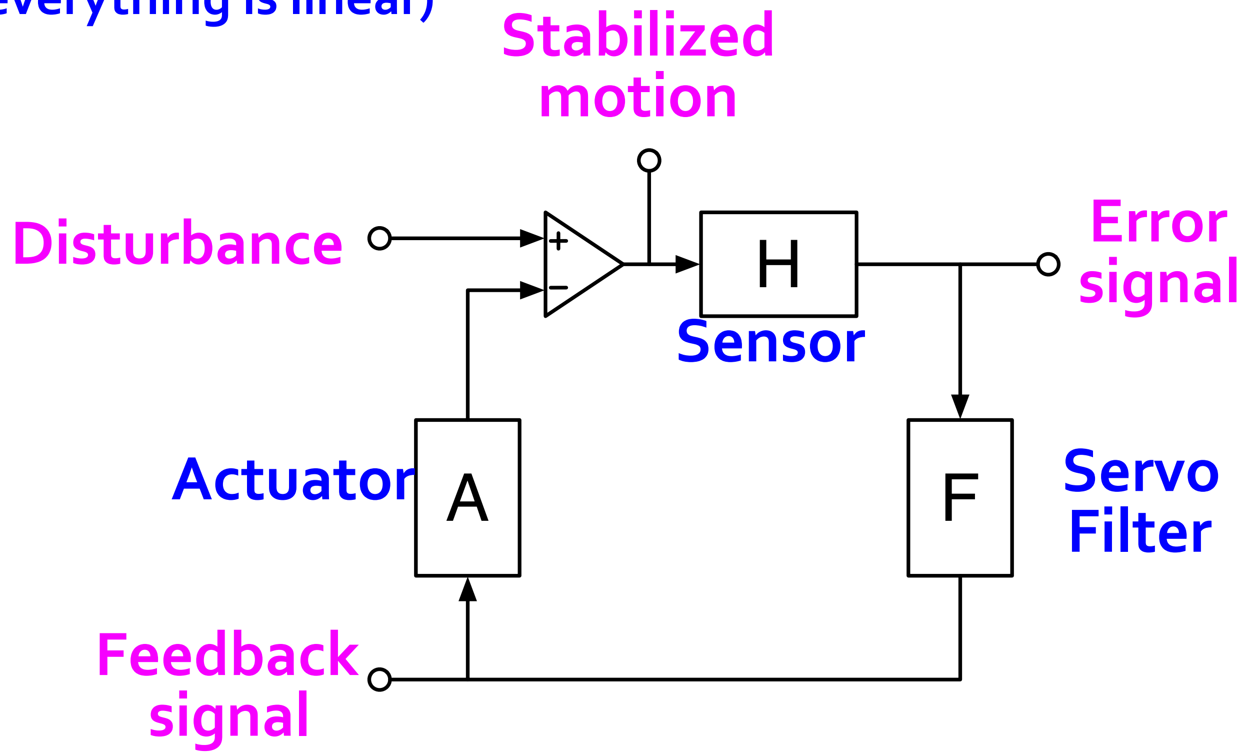
- When the openloop gain G is >>1, the error signal gets suppressed

- "Wow! our sensor signal became smaller!"
  Is our system more sensitive now?
  - No. We are just moving the actuator so that the error signal looks smaller.
    The signal and noise are equally suppressed in the error signal.
    Thus the SNR does not change.

- OK... So can we still measure gravitational waves
  even if the error signal is almost zero?
  - Yes. We should be able to recover the original signal by compensating the effect of the control i.e. (1+G)
  - And we can also use the feedback signal in order to reconstruct the original signal with appropriate compensation i.e. (1+G)/G

Introduction ~ Control?

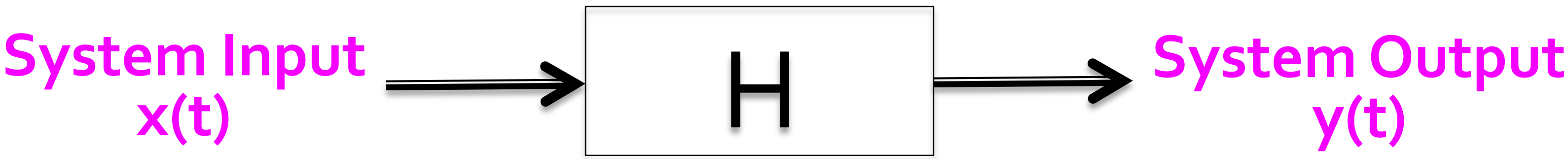
- **Important difference between**
  - **"Feedback control for stabilization" and "Feedback control for measurement"**
  - **Feedback control changes the stabilized motion but reconstructed Disturbance is not modified by the loop* (*if everything is linear)**



Linear systems and their stability

Linear systems and their stability

- **A deterministic and time-invariant system: H**



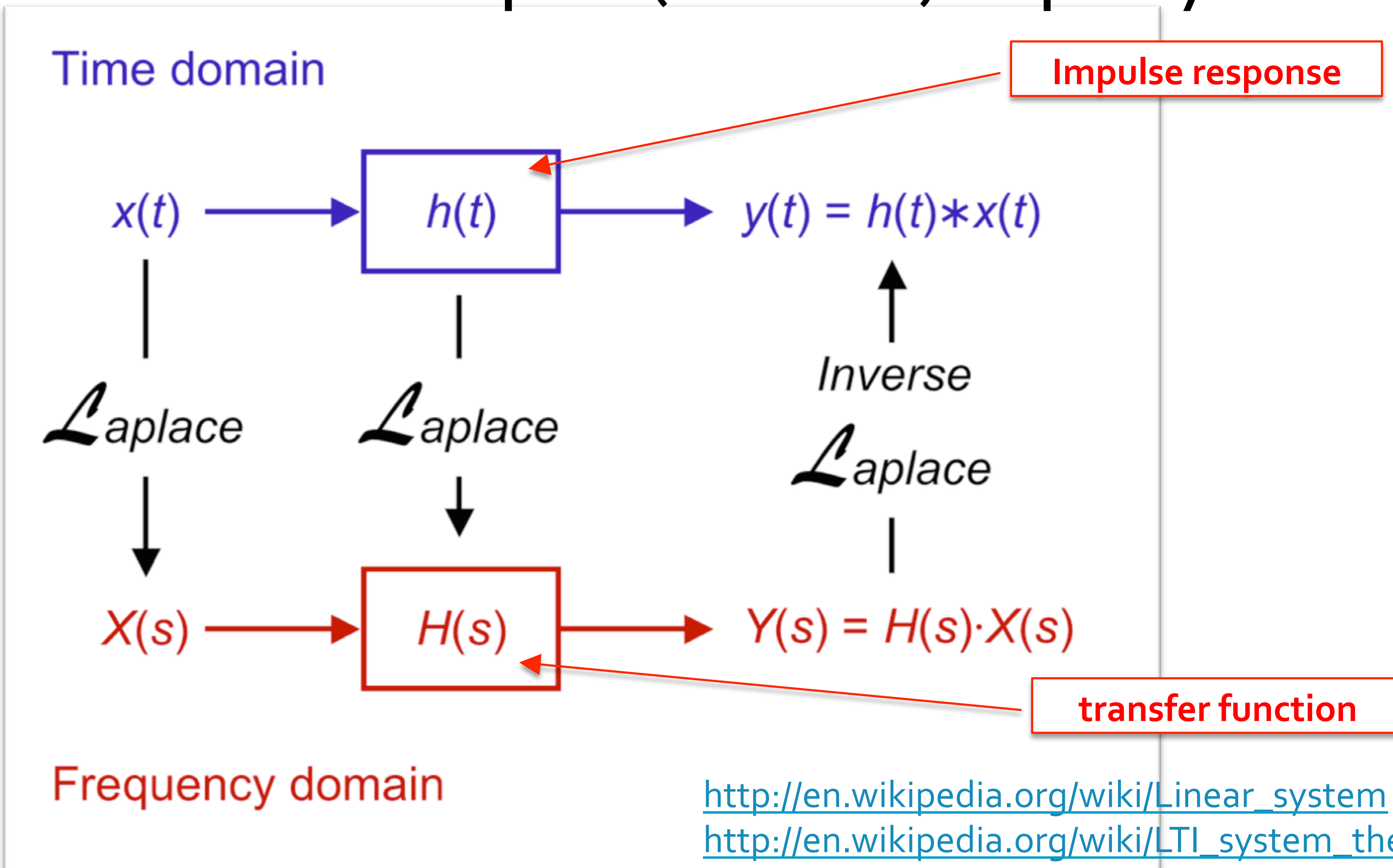
- **The system H is LTI (linear & time-invariant) when**

$$y_1(t) = H\left\{x_1(t)\right\}$$
$$y_2(t) = H\left\{x_2(t)\right\}$$
$$\Longrightarrow \alpha y_1(t) + \beta y_2(t) = H\left\{\alpha x_1(t) + \beta x_2(t)\right\}$$

- **We can deal with such a system using Laplace transform (or almost equivalently Fourier Transform)**

Linear systems and their stability

- **Time domain vs Laplace (or Fourier) frequency domain**



http://en.wikipedia.org/wiki/Linear_system
http://en.wikipedia.org/wiki/LTI_system_theory

Linear systems and their stability

- It is easy to convert from an ordinary differential equation to a transfer function

$$\frac{d}{dt} \Longrightarrow s$$

Laplace Transform

$$\Longrightarrow i\omega = i2\pi f$$

Fourier Transform

- e.g. Damped oscillator

$$m\ddot{x}(t) = -kx(t) - \gamma\dot{x}(t) + F(t)$$

$$ms^2X(s) = -kX(s) - \gamma sX(s) + F(s)$$

$$\boxed{H(s) \equiv \frac{X(s)}{F(s)} = \frac{1}{ms^2 + \gamma s + k}}$$

Linear systems and their stability

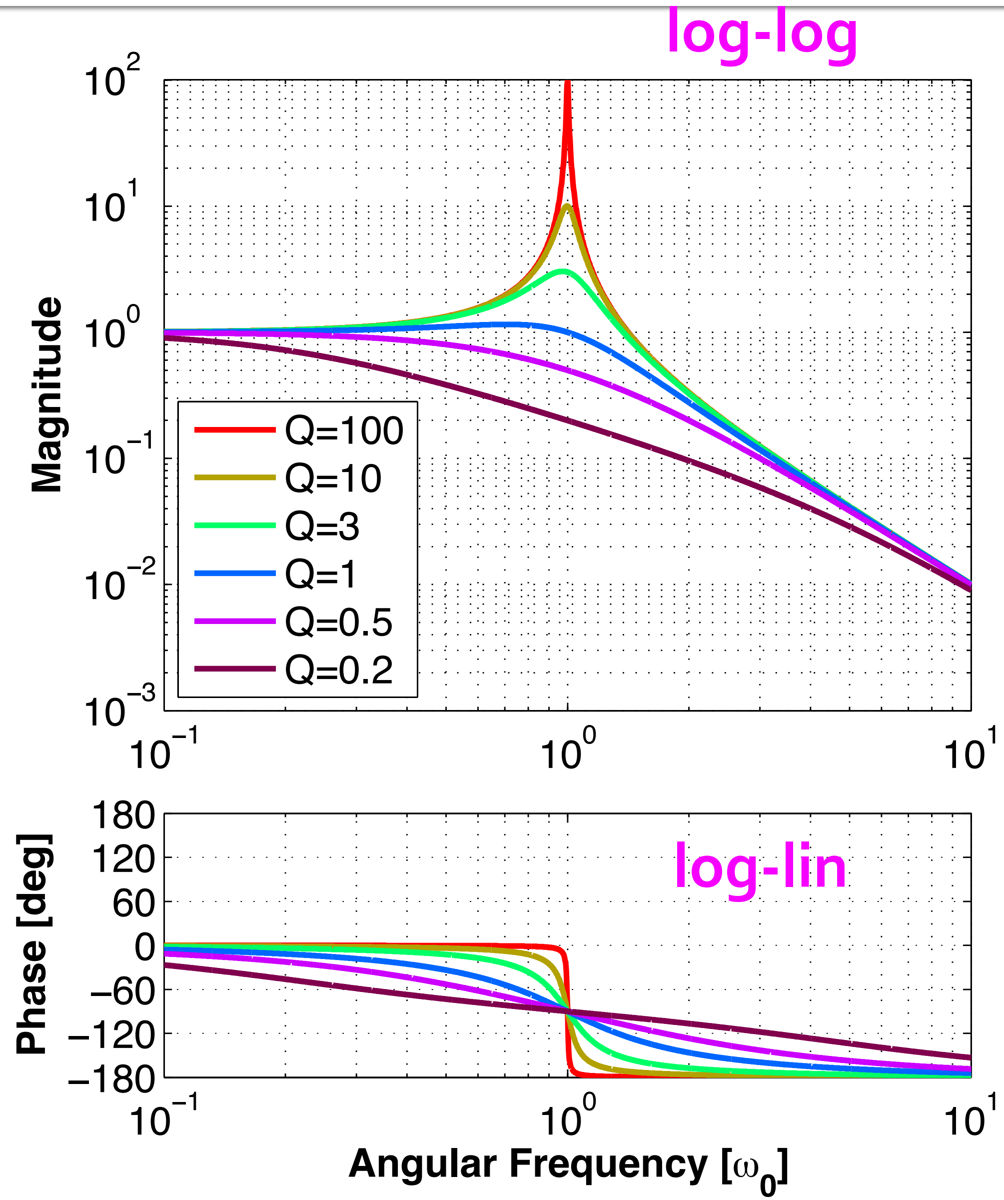
- **e.g. Damped oscillator**

$$H(s) = \frac{1}{ms^2 + \gamma s + k}$$

$$H(s) = \frac{1}{m}\frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(\omega) = \frac{1}{m}\frac{1}{-\omega^2 + i\frac{\omega_0}{Q}\omega + \omega_0^2}$$

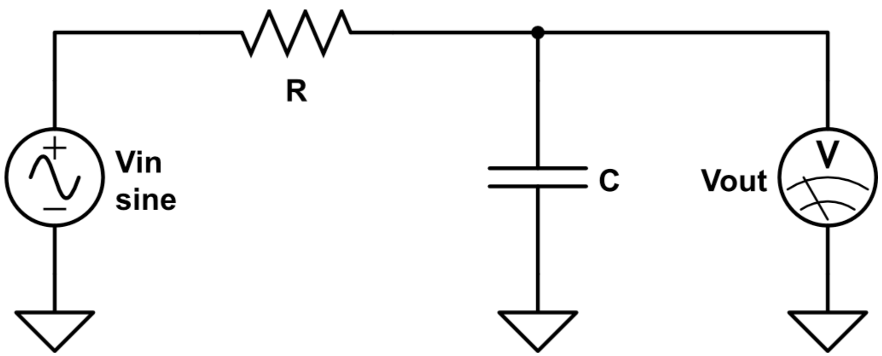
$$\omega_0 = \sqrt{k/m}, \ \ \gamma = m\omega_0/Q$$



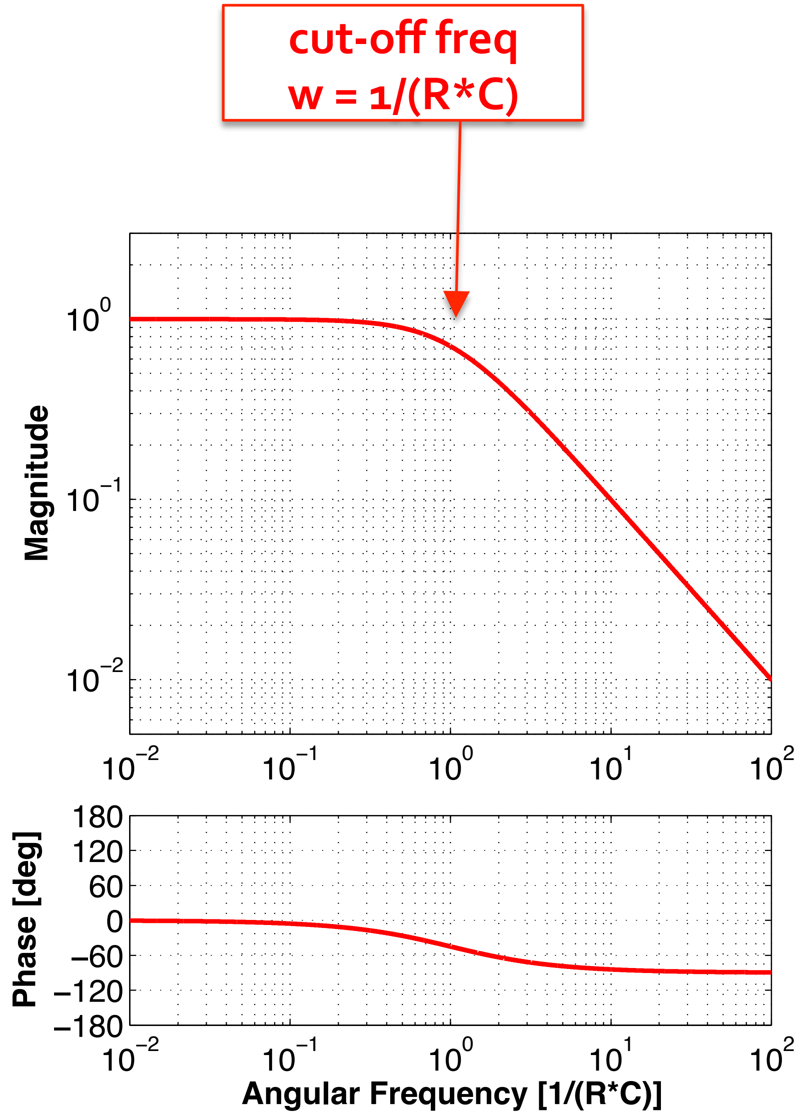
Bode diagram

Linear systems and their stability

- **e.g. RC filter**



$$V_{\text{out}} = q/C$$
$$\dot{q} = (V_{\text{in}} - V_{\text{out}})/R$$
$$\Longrightarrow i\omega C V_{\text{out}}(\omega) = (V_{\text{in}}(\omega) - V_{\text{out}}(\omega))/R$$
$$\Longrightarrow \frac{V_{\text{out}}(\omega)}{V_{\text{in}}} = \frac{1}{1 + i\omega RC}$$



Linear systems and their stability

- In most cases, a system TF can be expressed as:

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + ... + b_m s^m}{a_0 + a_1 s + a_2 s^2 + ... + a_n s^n}$$

- The roots of the numerator are called as "zeros" and the roots of the denominator are called as "poles"

$$H(s) = \frac{b_m \prod_{i=1}^{m} (s - s_{zi})}{a_n \prod_{j=1}^{n} (s - s_{pj})}$$

- Zeros (s_{zi}) and poles (s_{pi}) are

  real numbers (single zeros/poles)

  or

  pairs of complex conjugates (complex zeros/poles)

Linear systems and their stability

- In most cases, a system TF can be expressed as:

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  real numbers (single zeros/poles)

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  pairs of complex conjugates (complex zeros/poles)

Linear systems and their stability

- **Poles rule the stability of the system!**

  H(s) can be rewritten as

$$H(s) = \sum_{j=1}^{n} \frac{K_j}{(s - s_{pj})}$$

- **Each term imposes exponential time impulse response**

$$\text{T.F.: } H_j(s) = \frac{1}{s + s_{pj}} \Longleftrightarrow \text{I.R.: } h_j(t) = e^{s_{pj}t}$$

- **Therefore, if there is ANY pole with** $\mathrm{Re}(s_{pj}) > 0$

  the response of the system diverges

Linear systems and their stability

- **Poles rule the stability of the system!**

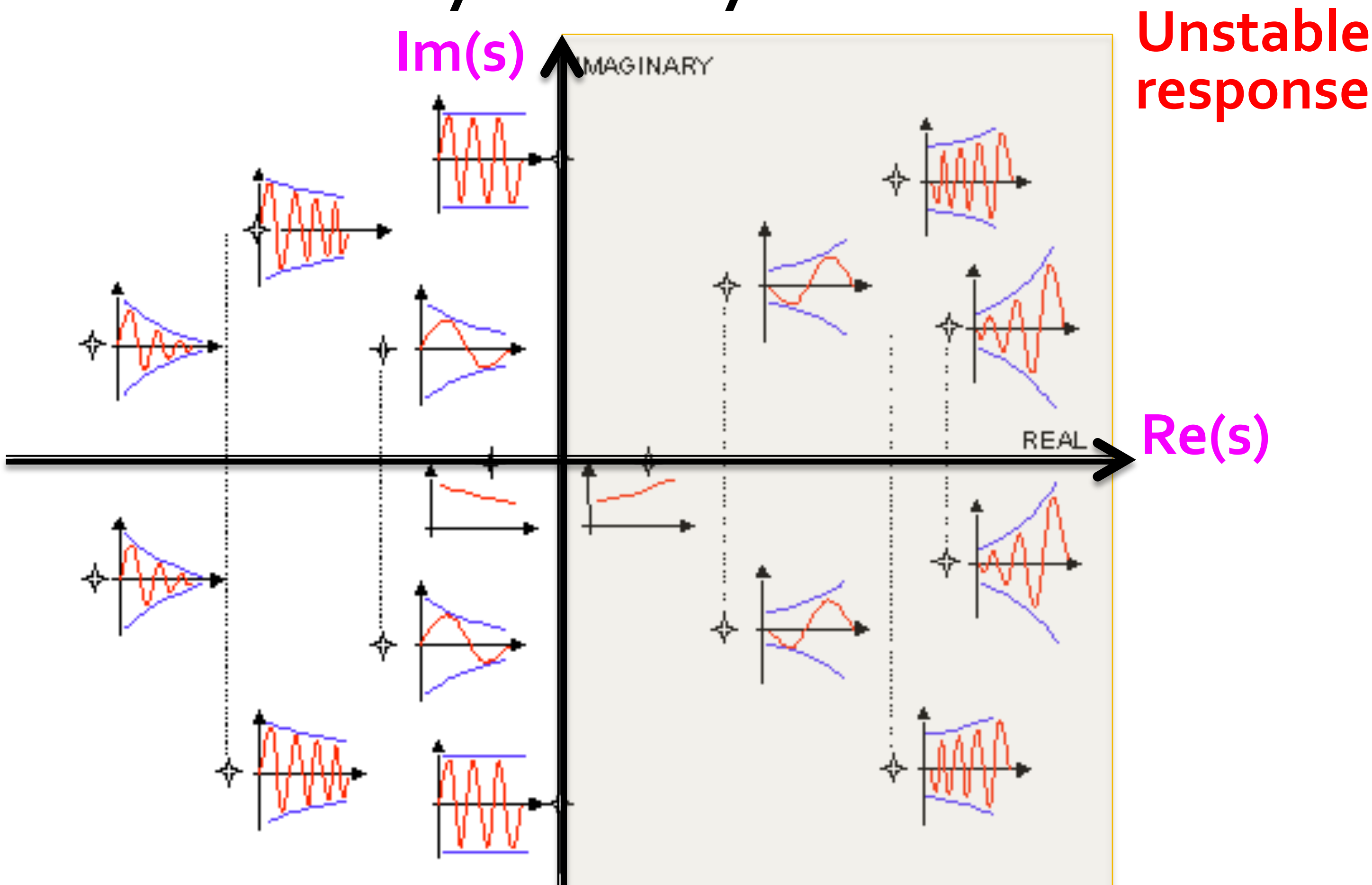
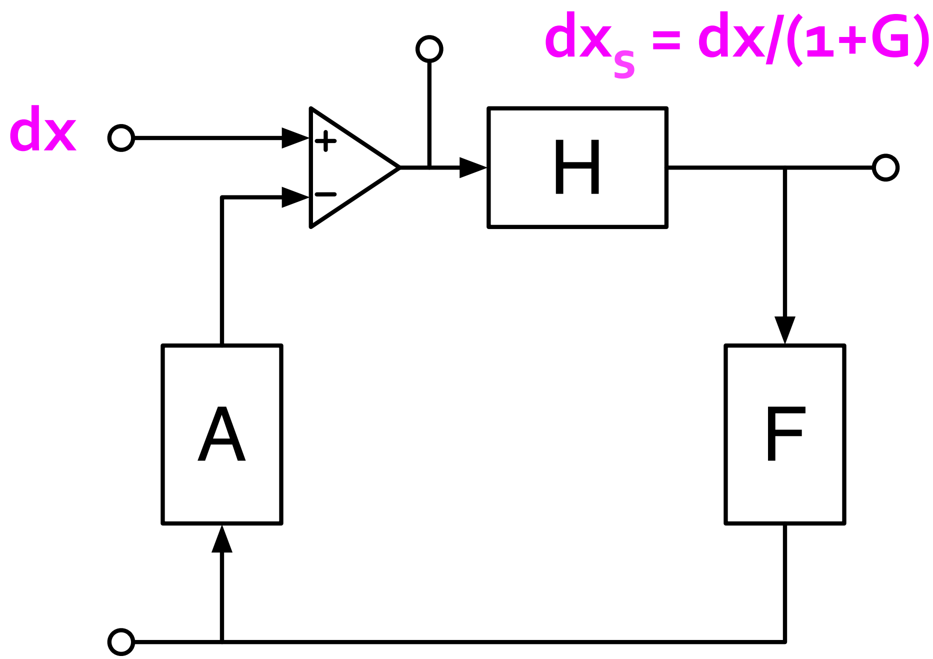


Figure 12: Root locus for different arrangements of the eigen values

http://nupet.daelt.ct.utfpr.edu.br/_ontomos/paginas/AMESim4.2.0/demo/Misc/la/SecondOrder/SecondOrder.htm

Linear systems and their stability

- Now we eventually came back to this diagram



Open loop TF:
$G = H\,F\,A$

Closed loop TF:
$G_{CL} = 1/(1+G)$

(!)

<u>Requirement:</u>

All the roots for 1+G should be in the left hand side of Laplace plane

Linear systems and their stability

- Remarks

Requirement:

All the roots for 1+G should be in the left hand side of Laplace plane

  - This does not mean all H, F, A needs to be stable.
    e.g. Unstable mechanical system A can be stabilized by
    a control loop. (cf. An inverted Rod)

  - We usually play with F to tune the result.
    It is awkward to evaluate the stability
    of 1/(1+G) every time.
    There is a way to tell the stability only from G

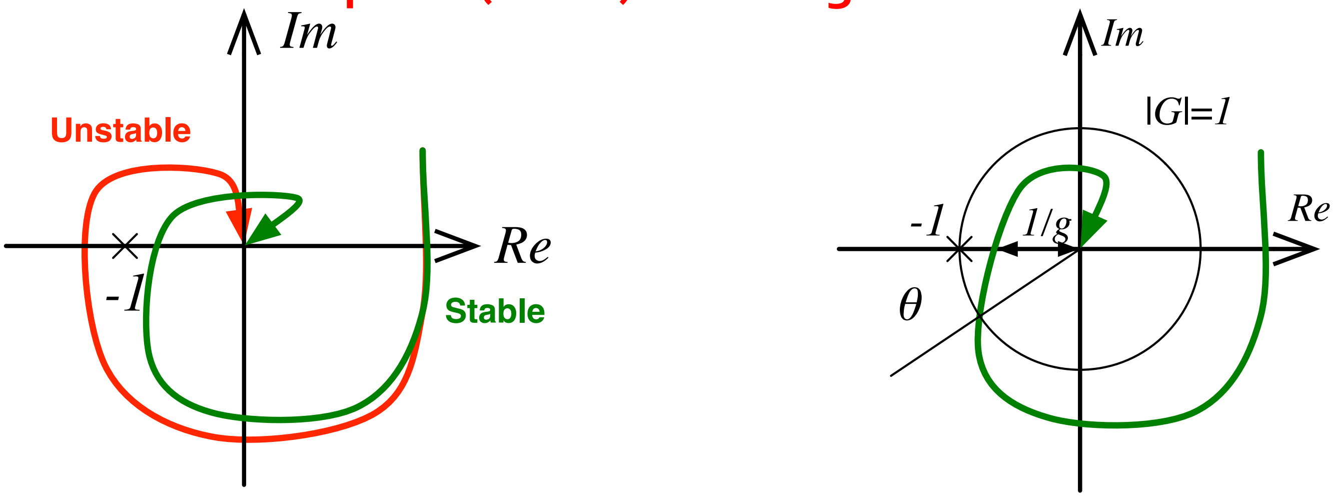
Open loop TF:
$G = H\,F\,A$

Closed loop TF:
$G_{CL} = 1/(1+G)$

Nyquist's stability criterion

Linear systems and their stability

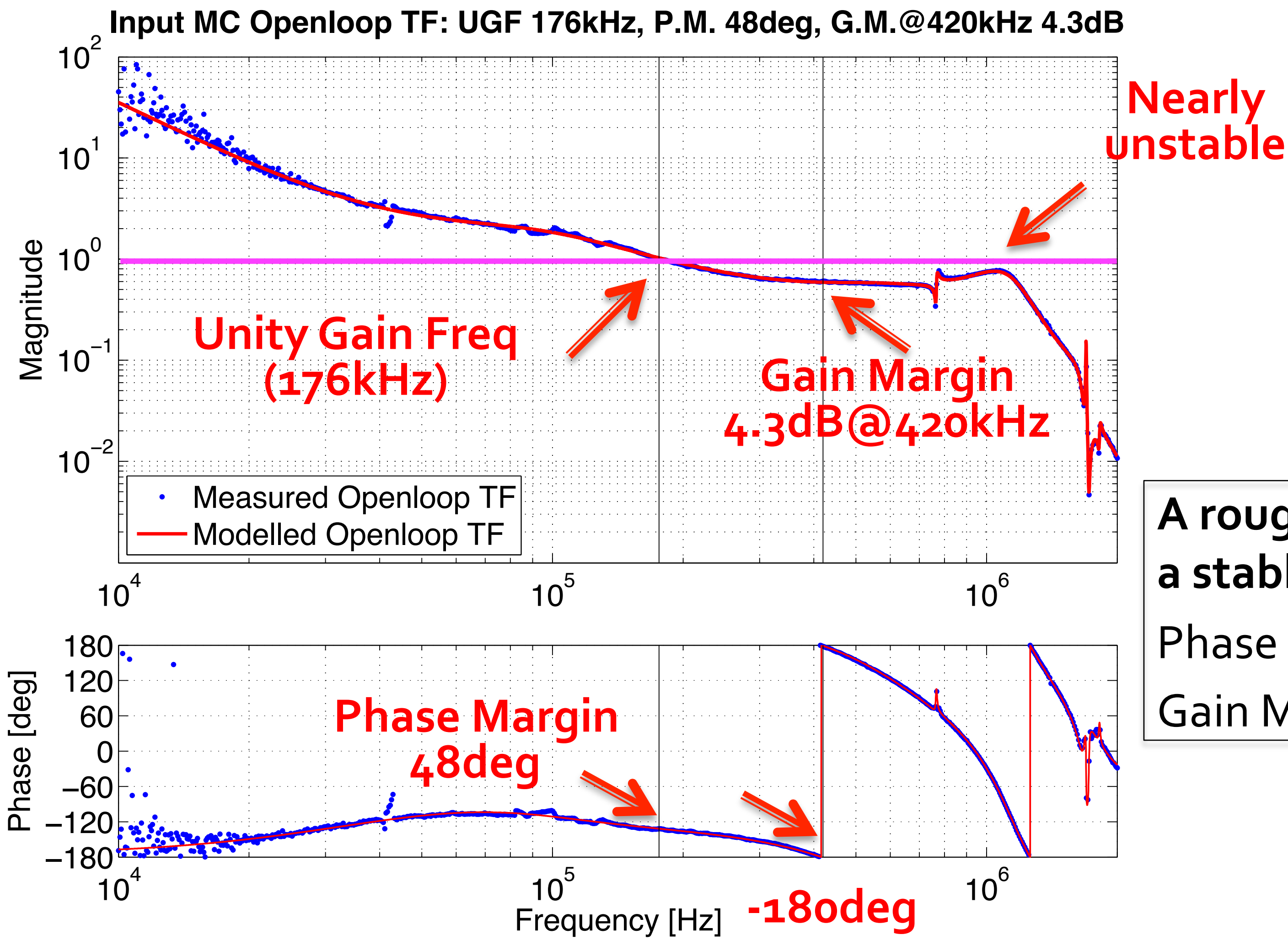
- **Nyquist stability criterion**
  - Plot openloop gain G in a complex plane (i.e. Nyquist diagram)
  - If the locus of $G(f)$ from f=0 to ∞, goes to 0 looking at the point (-1 + 0 i) at the left side => Stable
  - If the locus sees the point (-1+0 i) at the right side => Unstable



  - Unity gain frequency f_{UGF} : for $|G(f_{UGF})| = 1$
  - Phase margin ϑ : $\vartheta = \mathrm{Arg}(G(f_{UGF}))$
  - Gain margin g: $g = 1/|G(f_o)|$ where $\mathrm{Arg}(G(f_o)) = -\pi$

Linear systems and their stability

- **Phase Margin / Gain Margin in Bode diagram**
  - Most of the case, a bode diagram of G is enough to see the stability



A rough standard of a stable servo loop:

Phase Margin > 40deg

Gain Margin > 10dB

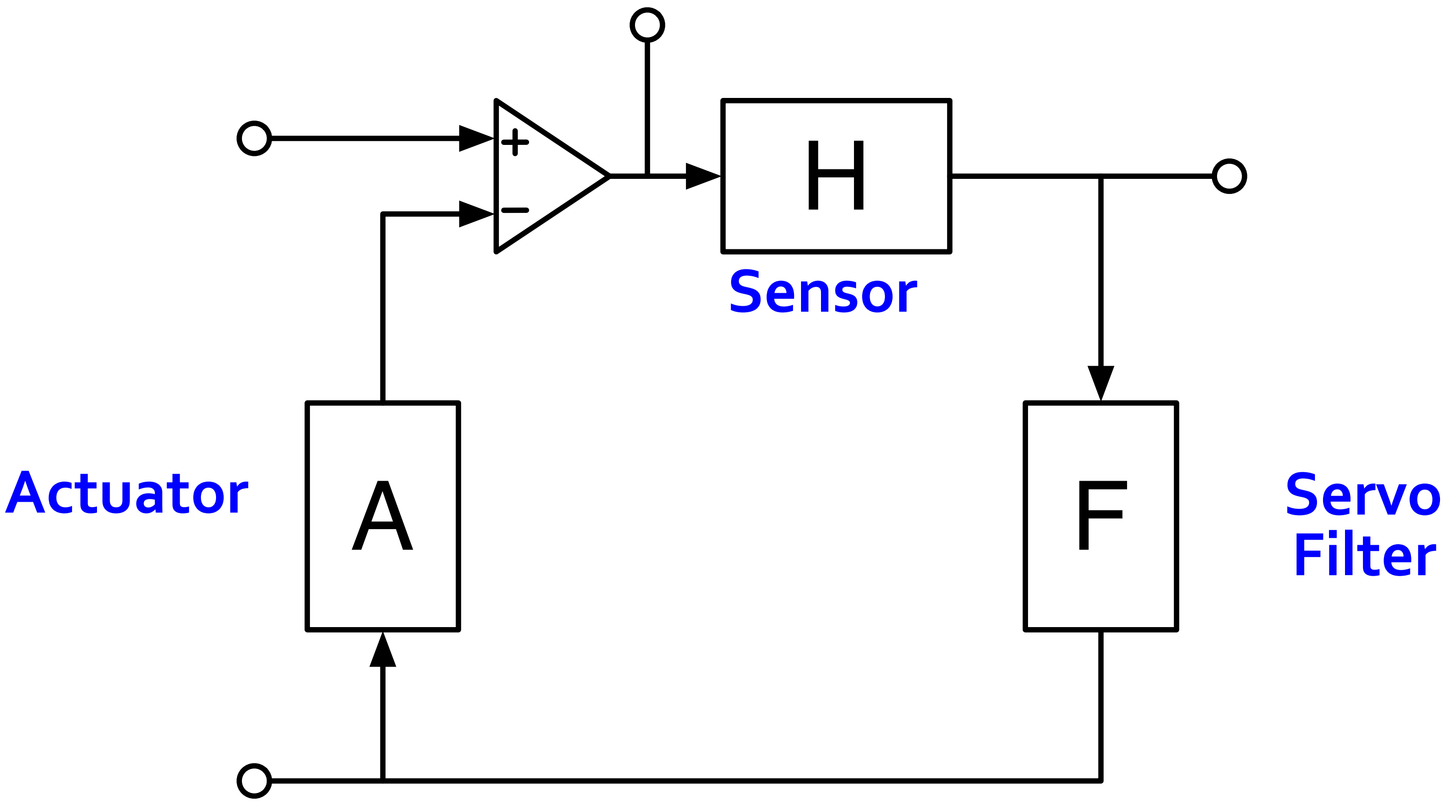
Linear systems and their stability

- Summary
  - Classical control theory
  - Design locations of poles and zeros
  - Stability: tuning of open loop transfer function is important

Control system components in GW detectors

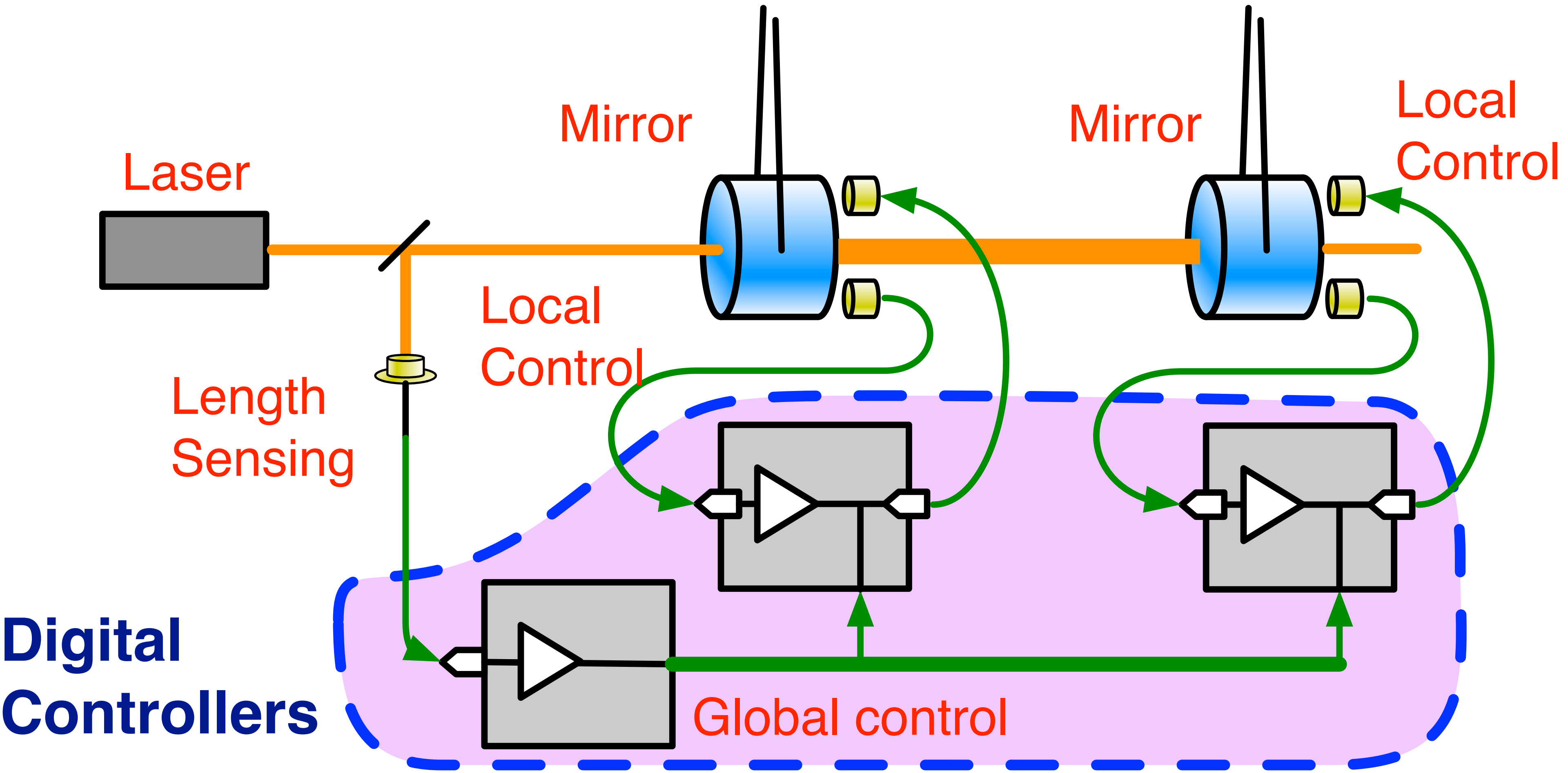
Control systems

- **Elements of a feedback loop (again)**



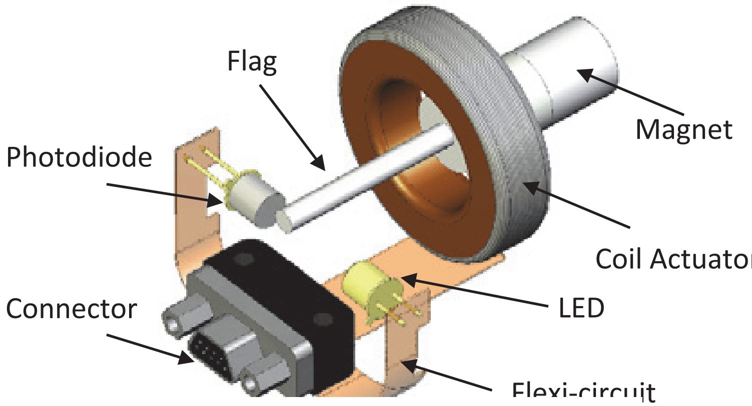
Interferometer control system

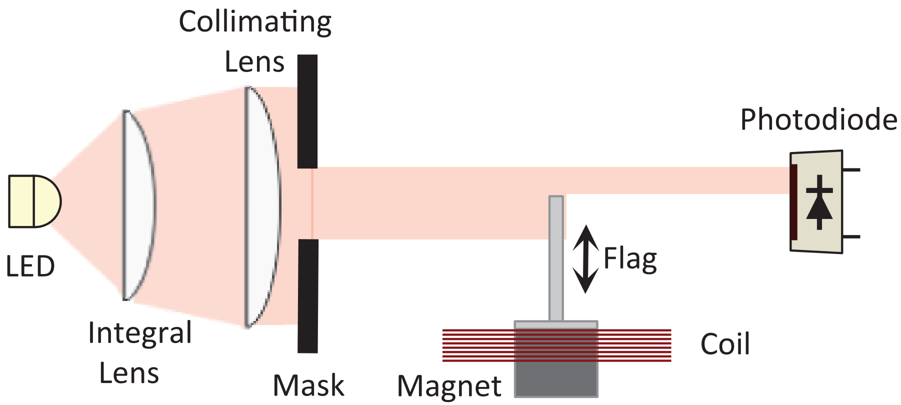
- **Local control vs global control**

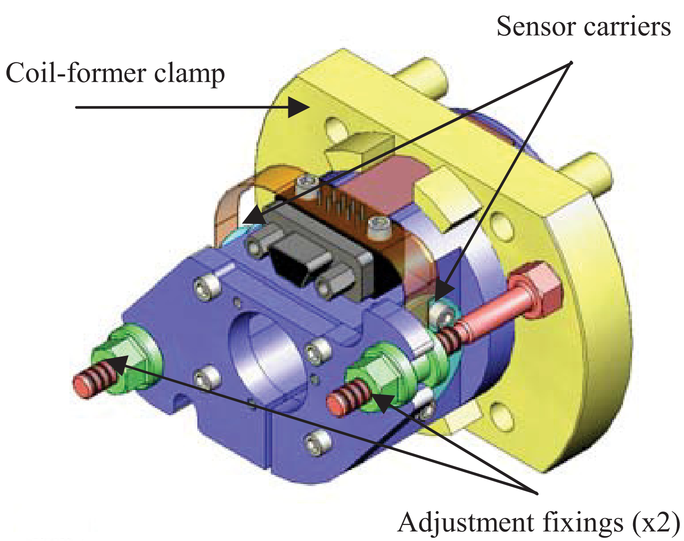


Local Sensors

- **Shadow sensor (relative displacement sensor)**
  - For suspension damping control, mirror attitude monitor
  - Typical linear range ~1mm for 0-10V => dV/dx = 10 kV/m
    Typical noise level: ~ 100 pm/sqrtHz
  - aLIGO: Birmingham Optical Sensor
    and Electro-Magnetic actuator (BOSEM)



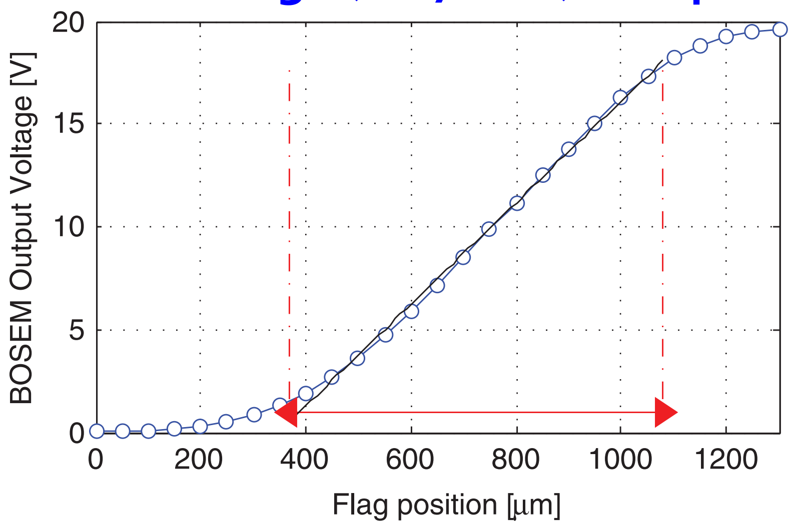


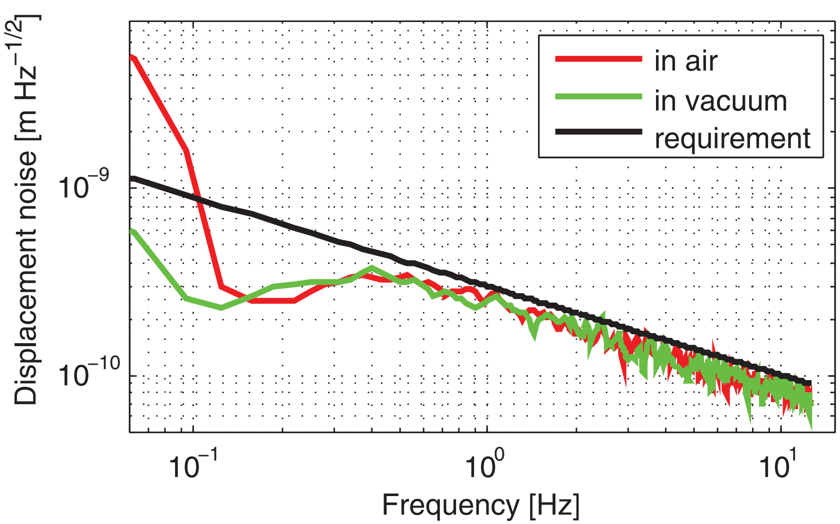


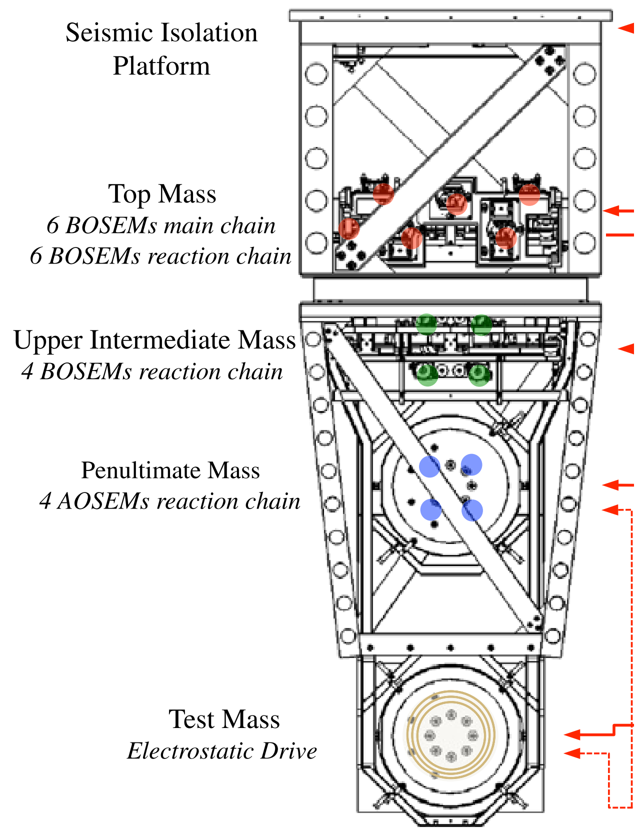
L Carbone, Class. Quantum Grav. 29 (2012) 115005

Local Sensors

- **Shadow sensor (relative displacement sensor)**
  - Linear range (~0.7 mm) / displacement noise sensor locations



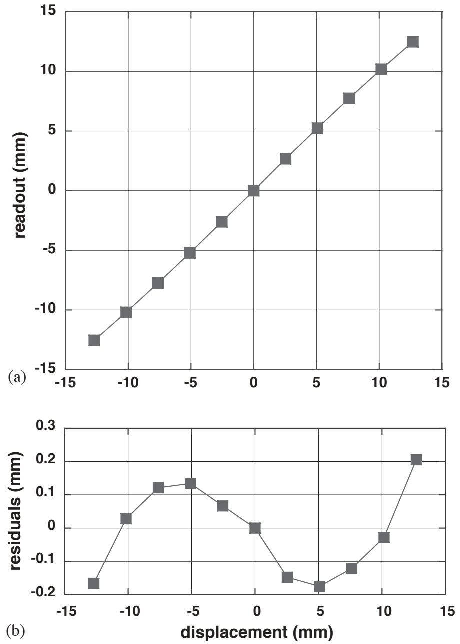


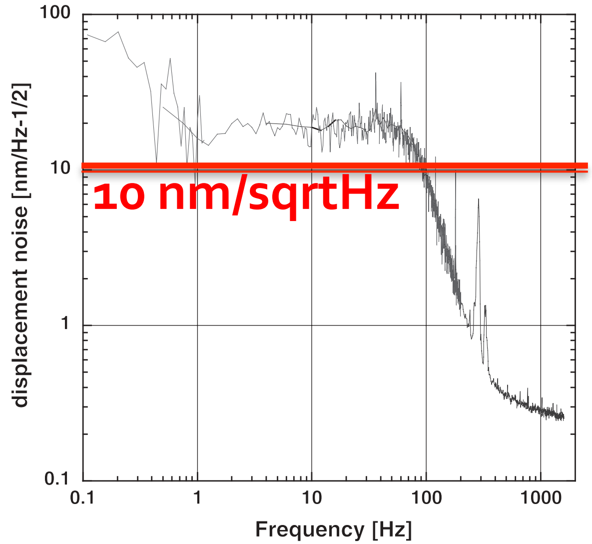


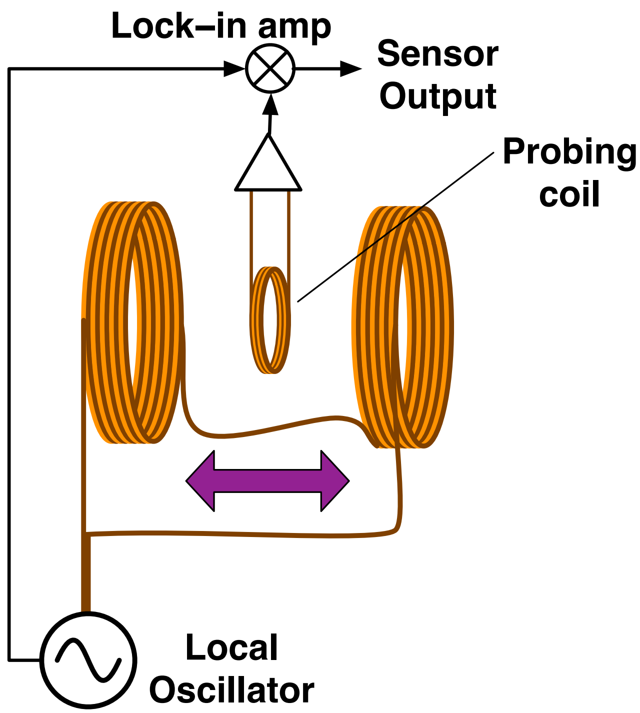
L Carbone, Class. Quantum Grav. 29 (2012) 115005

Local Sensors

- **Linear Variable Differential Transducer (relative disp. sensor)**
  - For low freq pendulum control (inverted pendulum), larger range
    VIRGO Super attenuator, KAGRA Seismic Attenuation System
  - Typical linear range ~10mm for 0-10V => dV/dx = 1 kV/m
    Typical noise level: 10~100 nm/sqrtHz





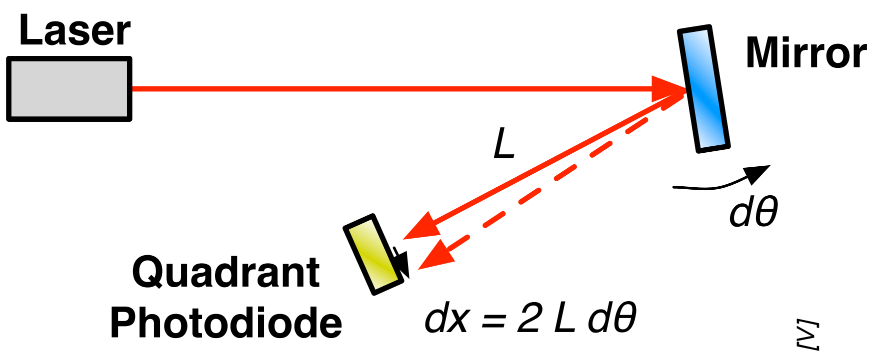


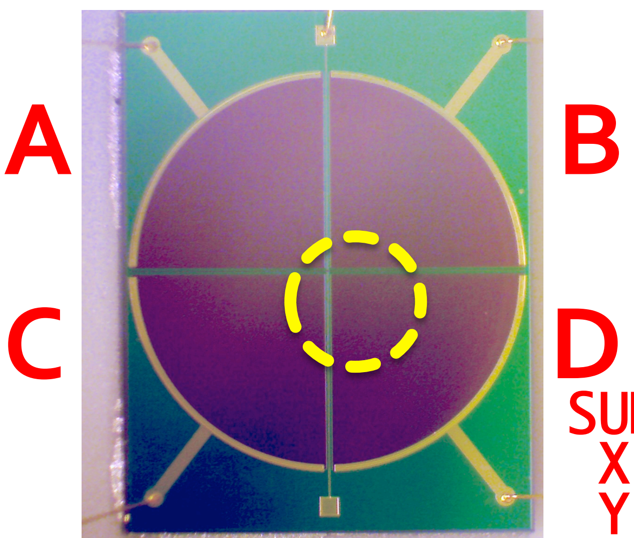
H. Tariq, Nuclear Instruments and Methods in Physics Research A 489 (2002) 570–576

Local Sensors

- Optical Lever (relative angular sensor)
  - Angle local control
  - Typical linear range ~beam side (0.1~1 mm) => $dV/d\theta$ = 1 ~10 kV/rad

    Typical noise level: 0.01~1 nrad/sqrtHz

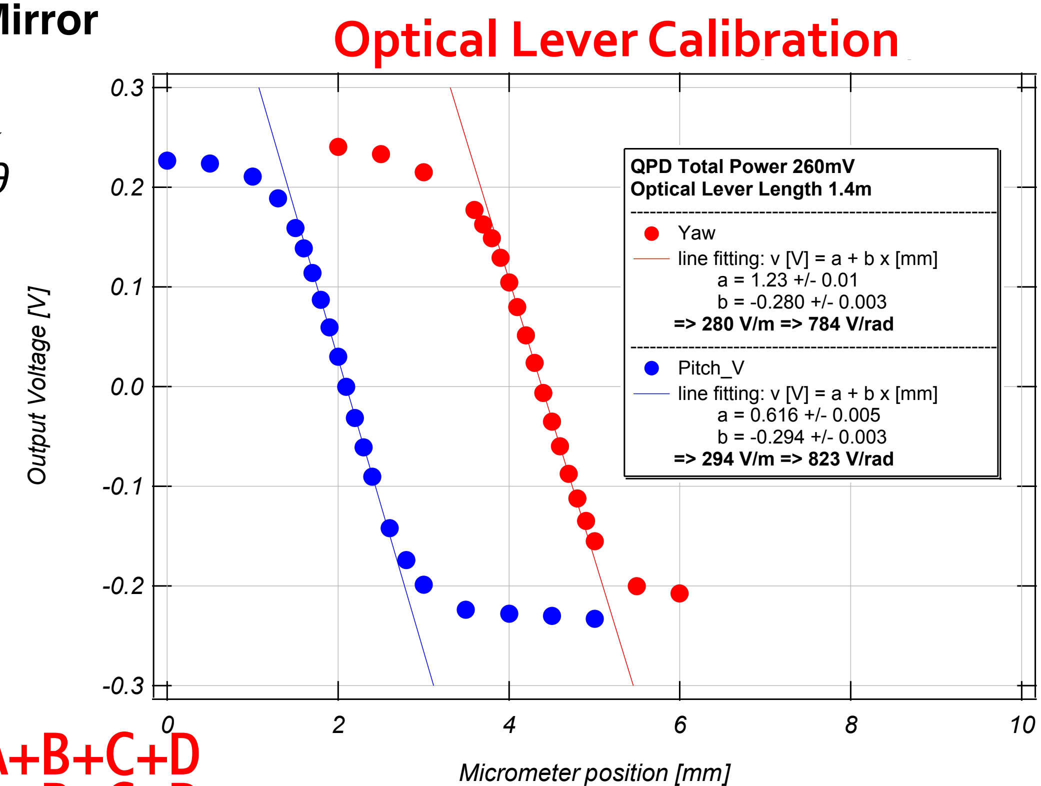




SUM = A+B+C+D

X = A−B+C−D

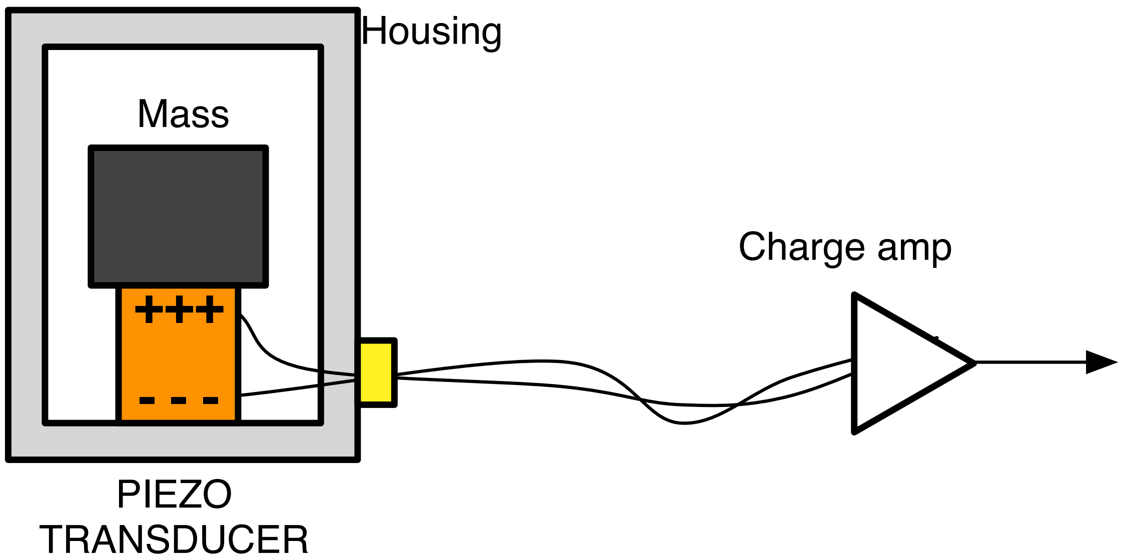
Y = A+B−C−D



Local Sensors

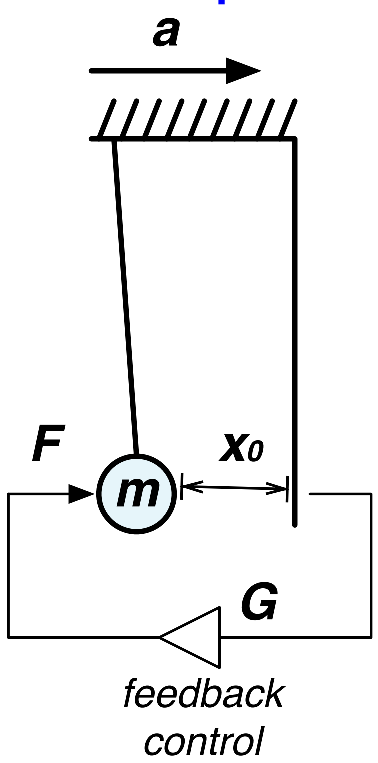
- **Piezo Accelerometer (<u>Inertial sensor</u>)**
  - Vibration measurement
  - Typical linear range ~ 100~1000 m/s^2

    Typical noise level: 0.5 ~ 50 (μm/s^2)/sqrtHz



Local Sensors

- **Servo Accelerometer (Inertial sensor)**
  - Seismic platform control (f>0.1Hz), Vibration measurement



Apply force to the suspended mass

=> Keep the distance from a reference

When the control gain G>>1

=> $a = F / m$

Local Sensors

- **Servo Accelerometer (<u>Inertial sensor</u>)**
  - Above the resonant freq: Limited by the sensor noise
  - Below the resonant freq:

    Steep rise of the noise as the mass does not move in relative to the ground

    => Low resonant freq is beneficial

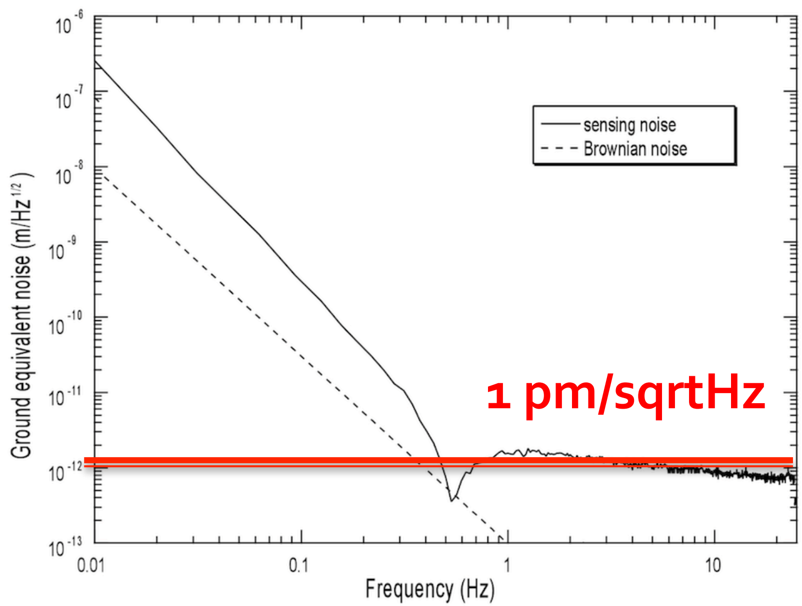
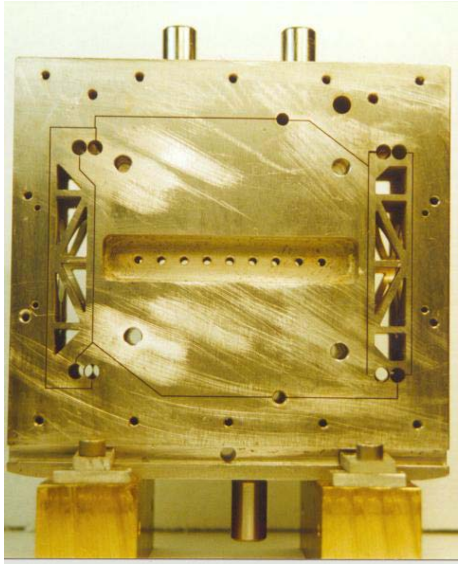
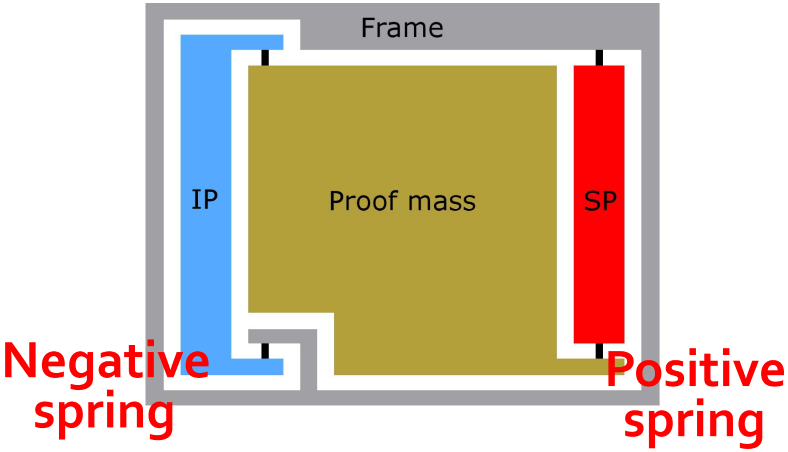


Fig.7. Equivalent frame displacement noise.

A. Bertolini et al, Nuclear Instruments and Methods in Physics Research A 564 (2006) 579–586





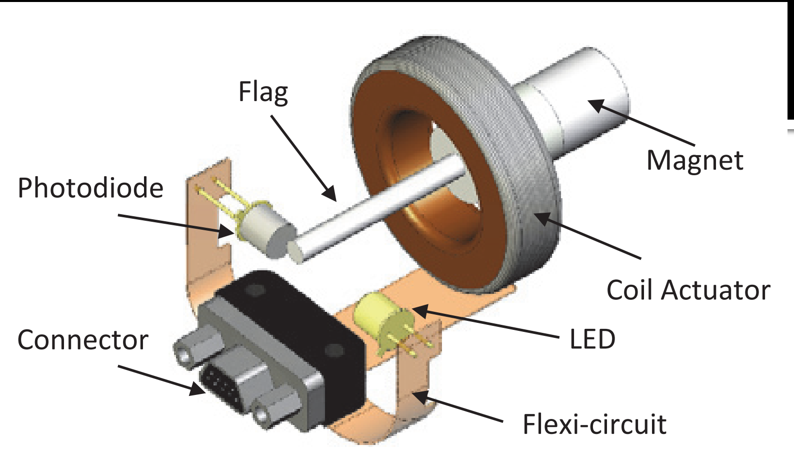
Acutuators

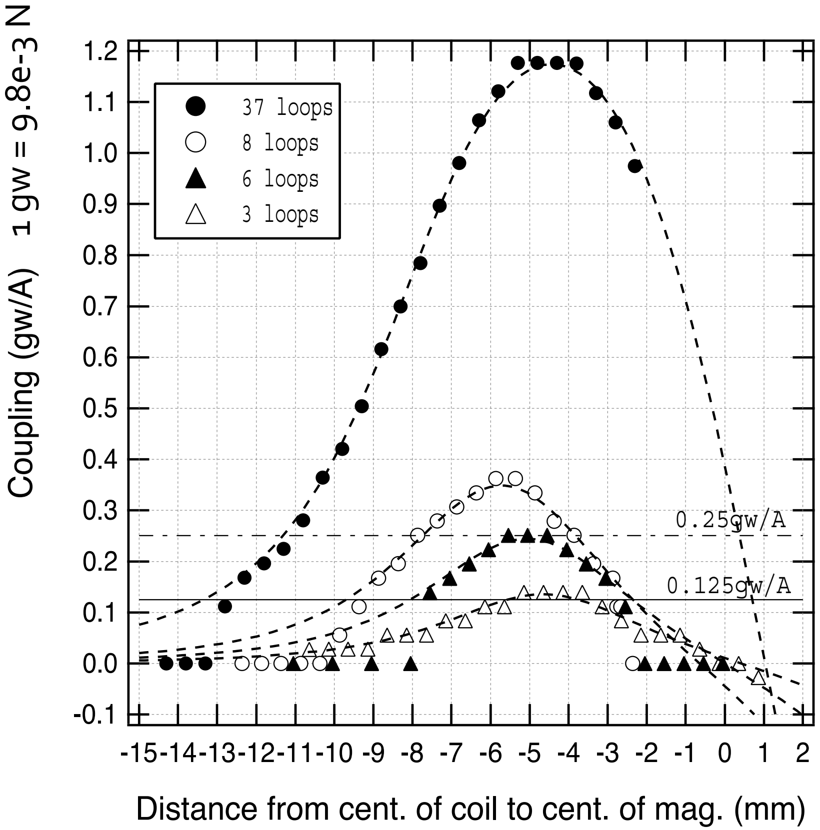
- Mechanical actuators
  - Coil Magnet actuator
  - Electro Static Driver (ESD)
  - Piezo (PZT) actuator

- Optical actuators
  - Acousto-Optic Modulator
  - Electro-Optic Modulator
  - Laser Frequency

Acutuators (Mechanical)

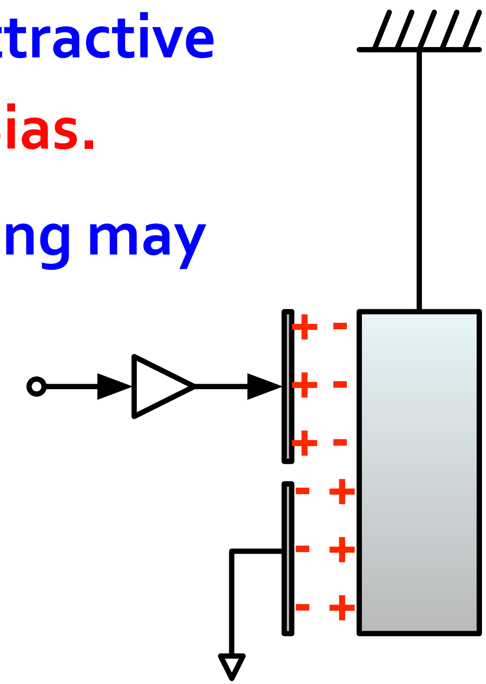
- Coil-magnet actuator
  - Coil current induces force
    on a magnet attached to a mass
  - Contactless
  - aLIGO coil-magnet actuator
    is integrated in BOSEM
  - Actuator response (coupling)
    has position dependence.
    Preferable to use it at its maximum
    in order to avoid vibration coupling

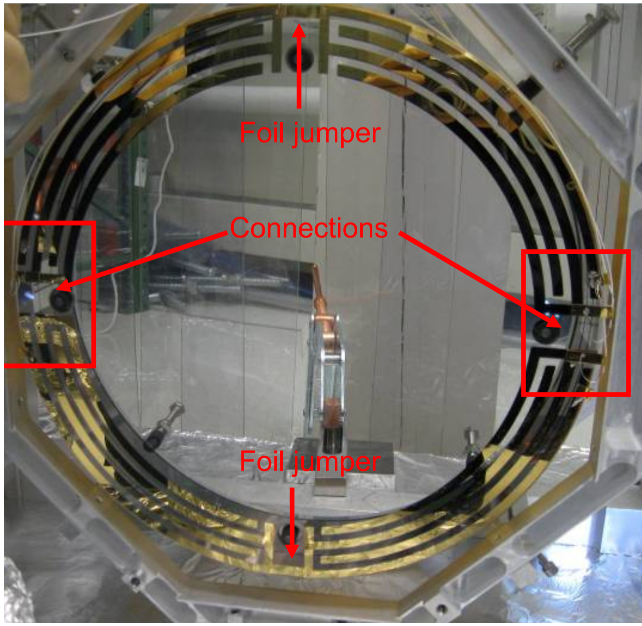




Acutuators (Mechanical)

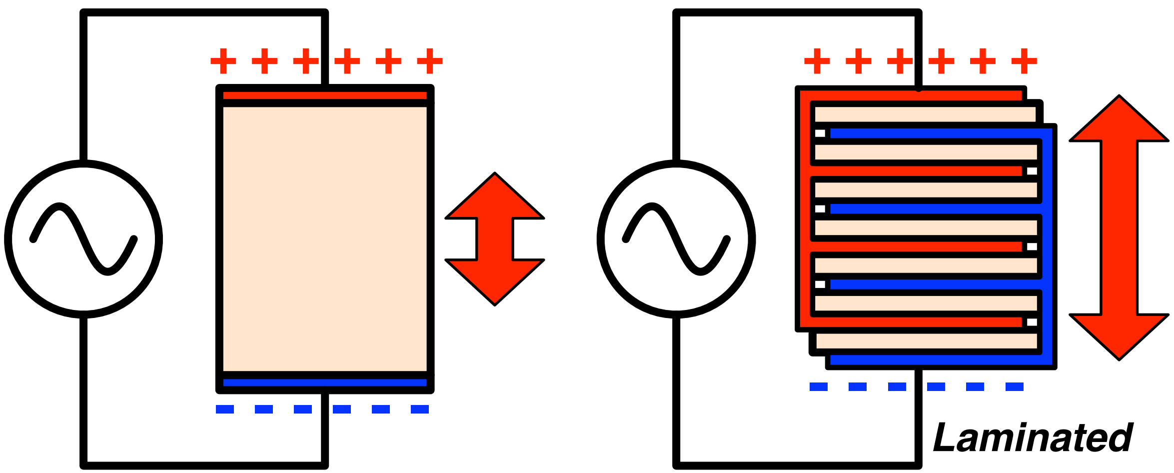
- Electro Static Driver (ESD)
  - Apply potential close to the mirror
    => induces surface charge (or polarization) and attractive force
  - In practice, comb patterns are used
    => strengthen the electric field, but less force range
  - Can produce only attractive force. => Need DC Bias.
  - Stray surface charging may cause problems.



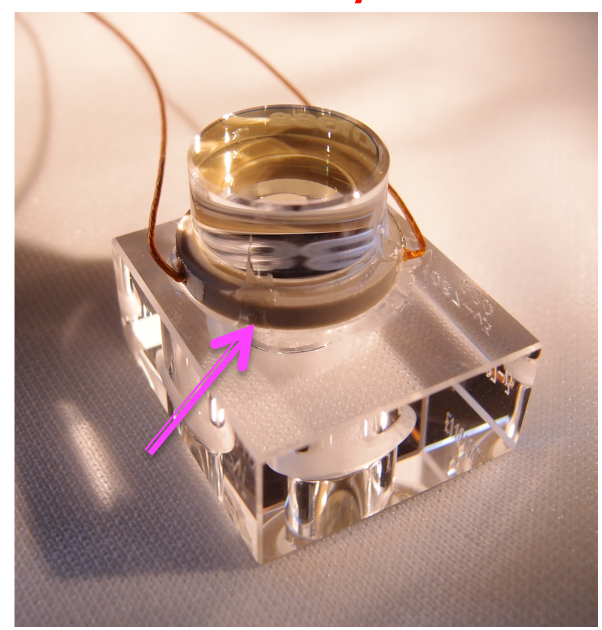


Acutuators (Mechanical)

- **Piezo (PZT) actuator**
  - Apply potential to a feroelectric material
    => cause internal polarization and induces strain
  - To increase displacement, laminated piezo is often used
    => displacement 3~10 μm
  - Requires a bias voltage and HV amplifier,
    but has wide applications

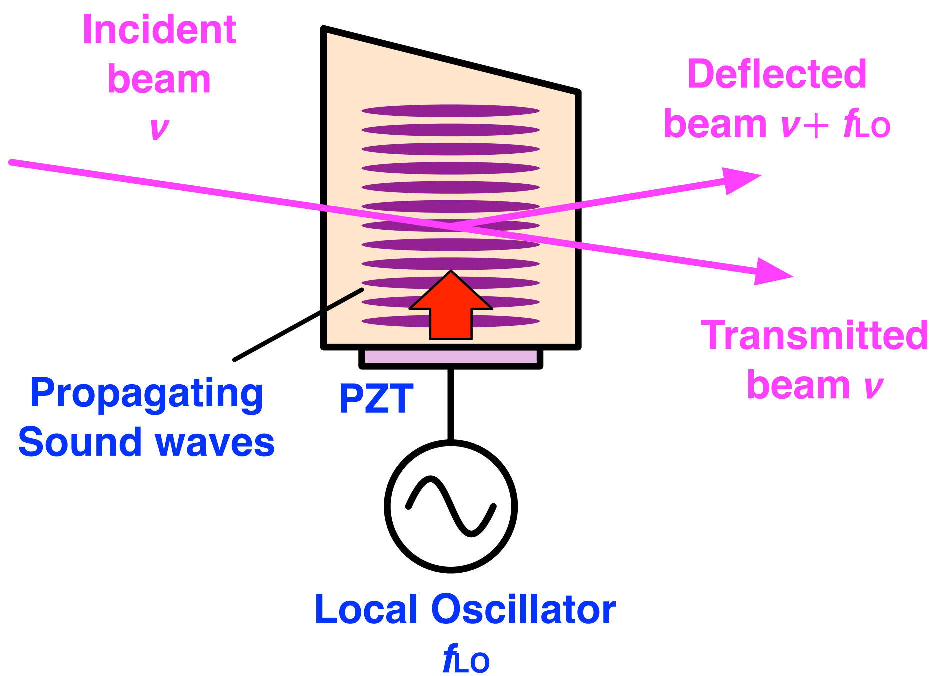


OMC cavity mirror



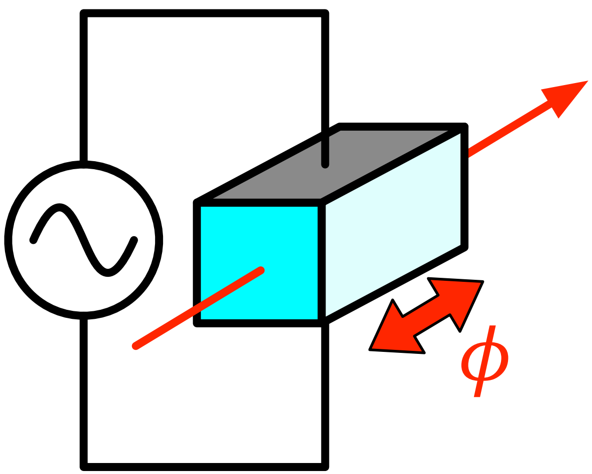
Acutuators (Optical)

- Acousto-Optic Modulator
  - Phonon-Photon scattering (or bragg diffraction) in AOM crystal
  - Effect: Beam deflection / Frequency shift
  - Application: Laser frequency actuator, Laser intensity actuator Beam angle scanner

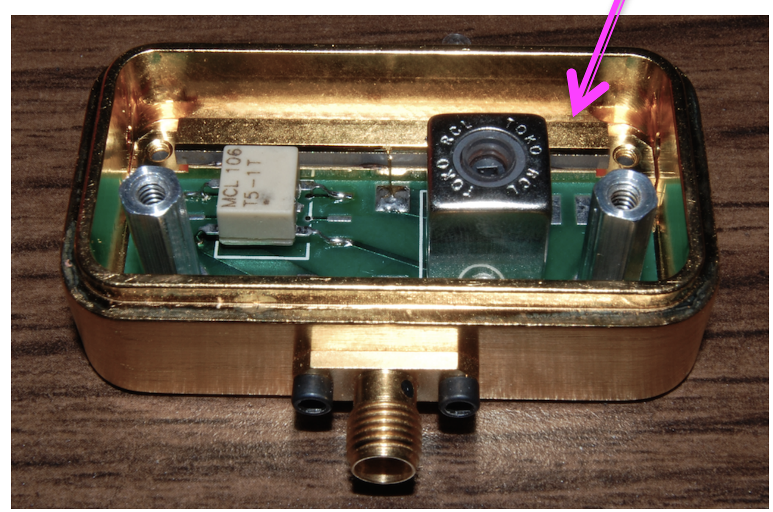


Acutuators (Optical)

- **Electro-Optic Modulator**
  - Pockels Cell effect:
    Refractive index changes linearly to the applied E-field
  - Application:
    Laser phase modulation
    Phase actuation (= frequency actuation)



LiNbO3 crystal



Acutuators (Optical)

- Laser frequency actuation (YAG NPRO laser)
  - We often control laser frequency with multiple actuators
  - 1) Thermal actuator

    Thermo-Electric Cooler attached to the laser crystal.

    Huge response (1GHz/K or 1GHz/V) but slow (f<0.1Hz)
  - 2) Fast piezo actuator

    A piezo attached on the laser crystal induces stress induced refractive index change.

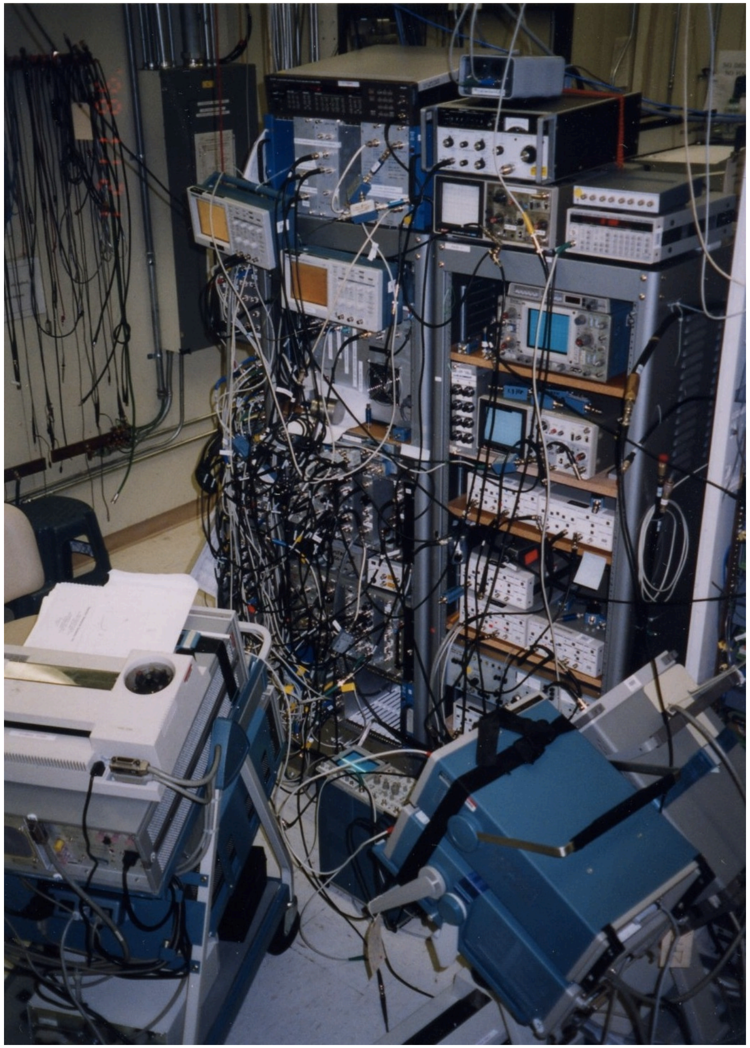
    Response (~1MHz/V). Bandwidth 10~100kHz
  - 3) External EOM

    Response (~10 mrad/V), Bandwidth ~1MHz

Servo Controller

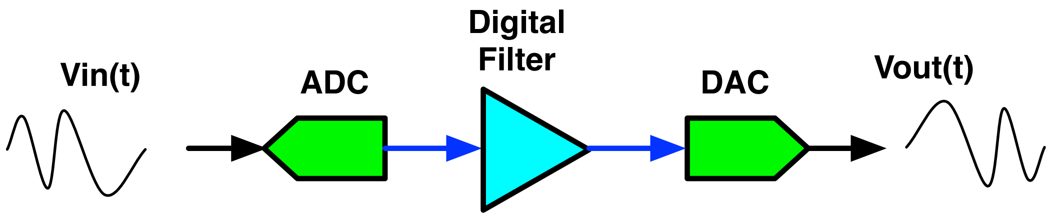
- Analog servo filters
  - High dynamic range (~1nV/sqrtHz, +/-10V), High bandwidth
  - Pole/zero placement with active op-amp filters
  - Until the end of the 20th century, analog filters have been commonly used for servo filters in our field
  - Analog servos are still in action for the feedback loops with bandwidth >1kHz. (cf. frequency stabilization, intensity stabilization)

LIGO 40m prototype (1998)



Servo Controller

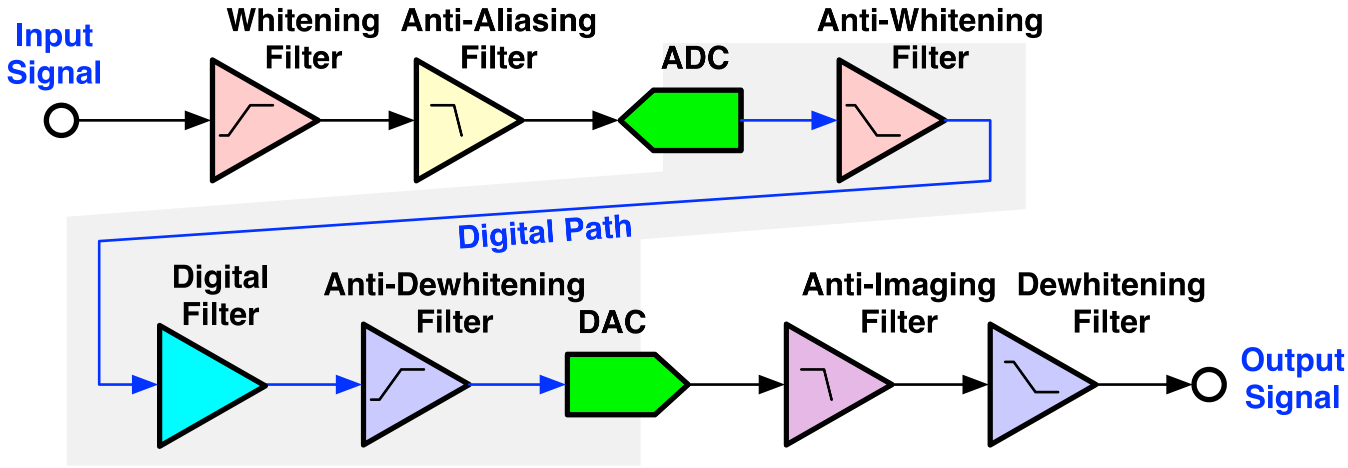
- Digital servo filters
  - Process digitized signals in a computer



  - Large flexibility
    High compatibility with detector automation and management
  - Limited dynamic range (~0.1mV/sqrtHz, +/-10V for 16bit)
  - Limited bandwidth
    - Each sample needs to be processed before the next sampled data comes
    - Inevitable sampling delay
    - Additional phase delays due to analog filters for analog-digital interface
    - e.g. 16kHz sampling, control bandwidth ~200Hz

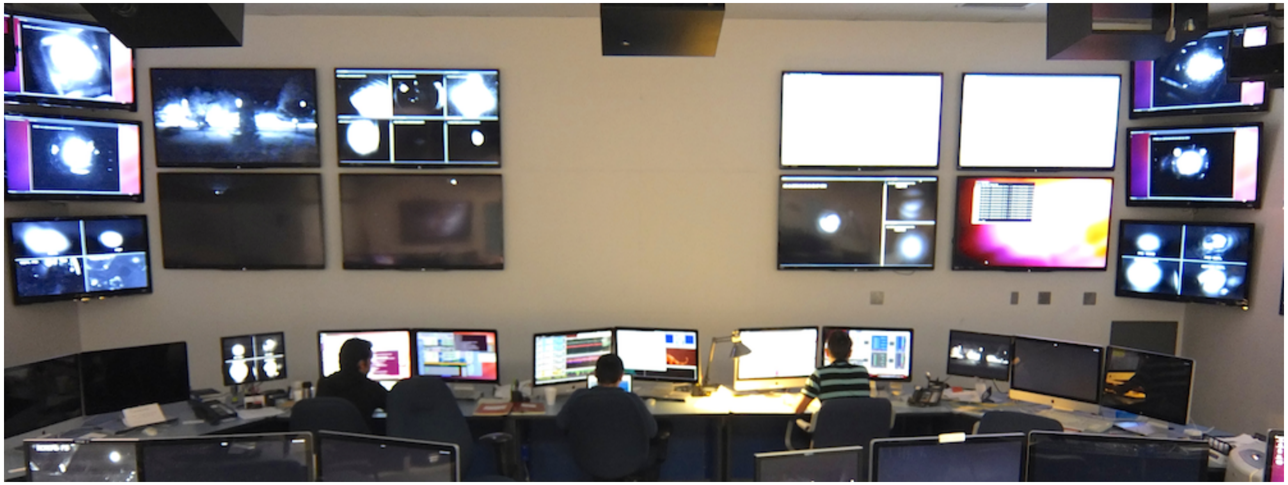
Analog/Digital interface

- Restriction of signal digitization
  - Voltage quantization: quantization noise
    => limited dynamic range
    => Requires whitening/dewhitening filters
  - Temporally discrete sampling: aliasing problem
    => limited signal bandwidth
    => Requires anti-aliasing (AA) / anti-imaging (AI) filters
  - Typical signal chain



Control room

- **Comparison of the control room in the analog and digital eras**



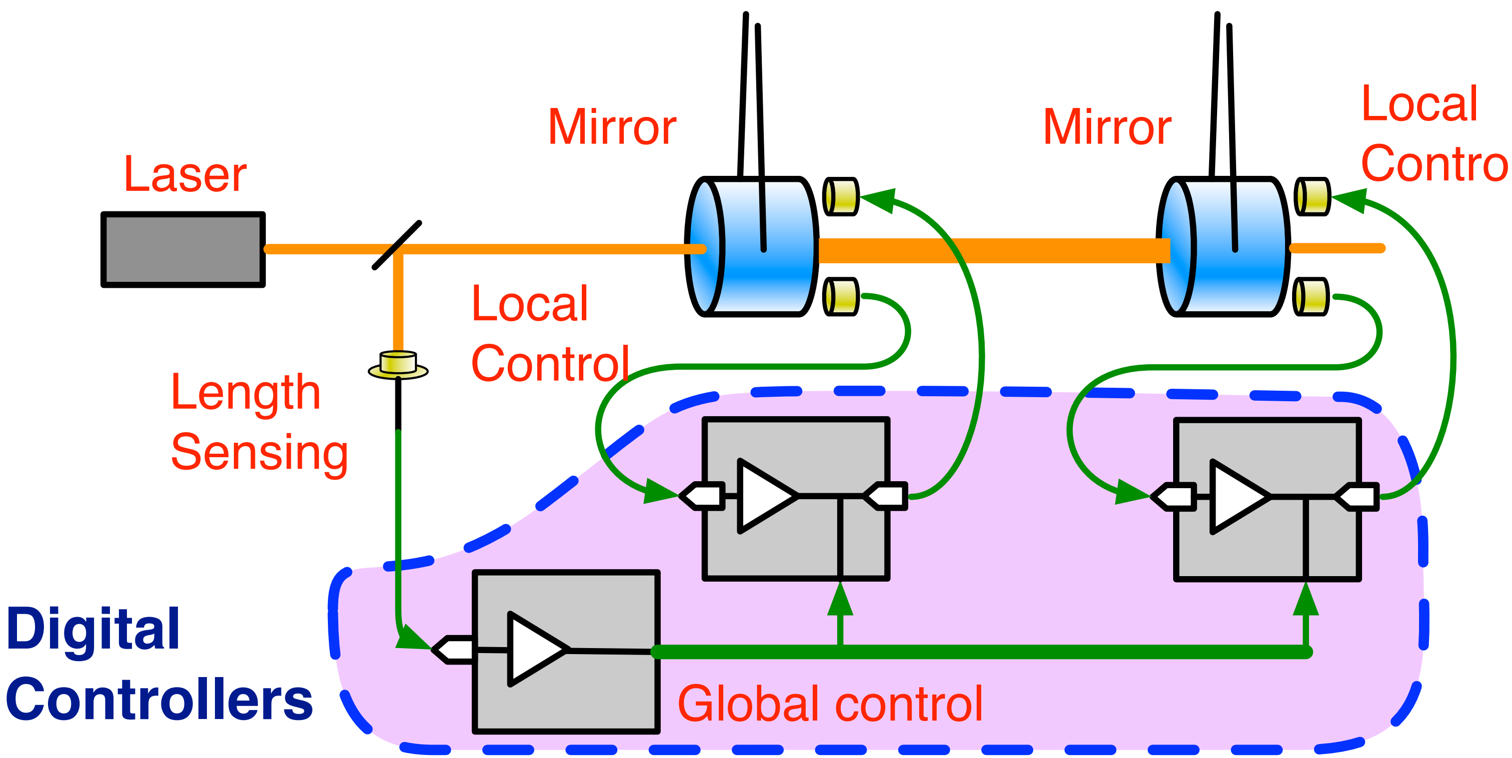
aLIGO (2014)

TAMA300 (2001)

Interferometer sensing and control

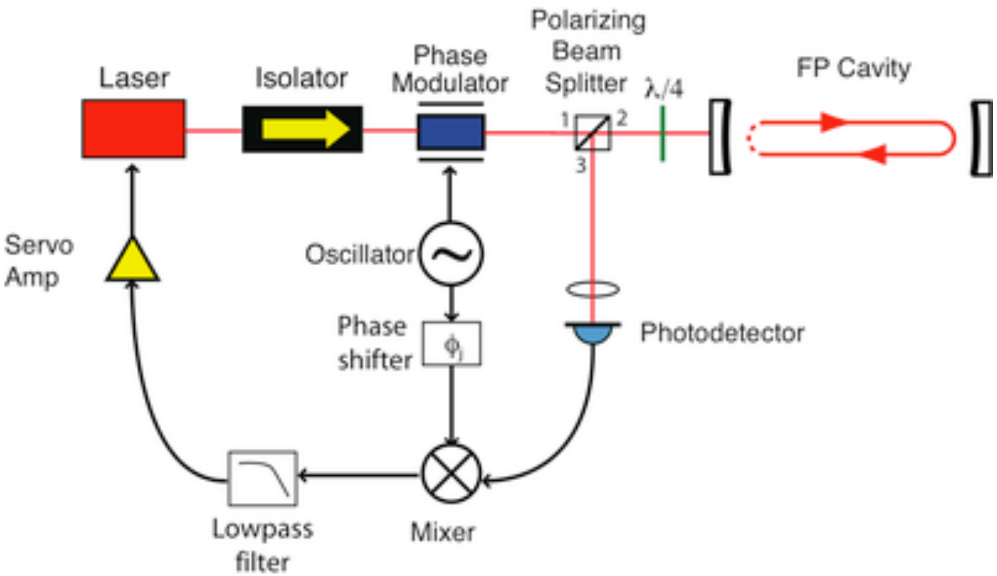
Global control

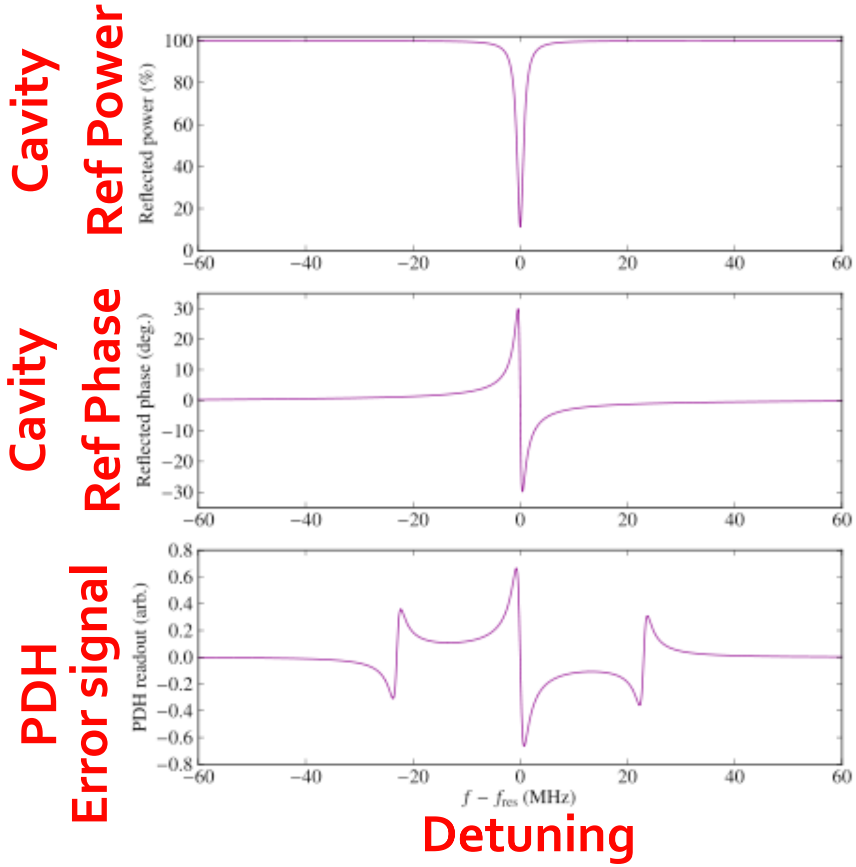
- **Interferometer control using the main laser beam**
  - **On the top of the local control,**
    optical path lengths and the mirror alignment
    need to be kept at the most sensitive state of the interferometer



Cavity length control

- Pound-Drever-Hall (PDH) technique
  - We want to keep the cavity at the TOP of the resonance
  - Phase of the cavity reflection is linear to the cavity detuning
  - Use modulation / demodulation





Drever, R. W. P., et al Appl Phys B **31** 97-105 (1983)

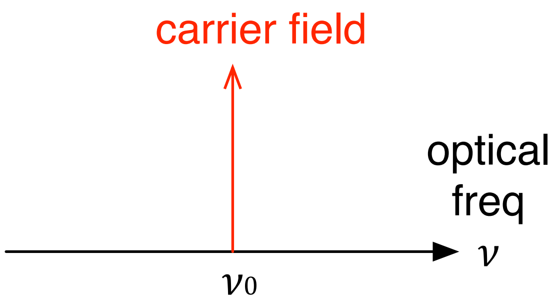
http://en.wikipedia.org/wiki/Pound%E2%80%93Drever%E2%80%93Hall_technique

http://www.sjsu.edu/faculty/beyersdorf/Archive/Phys208F07/Sideband%20generation%20in%20LIGO.pdf

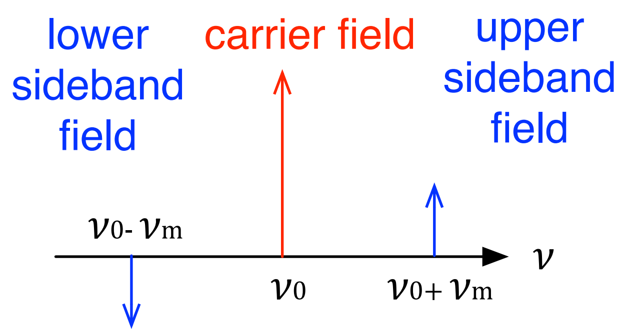
PDH control signal

- **Modulation**

$$E = E_{\rm in} e^{i2\pi\nu_0 t}$$

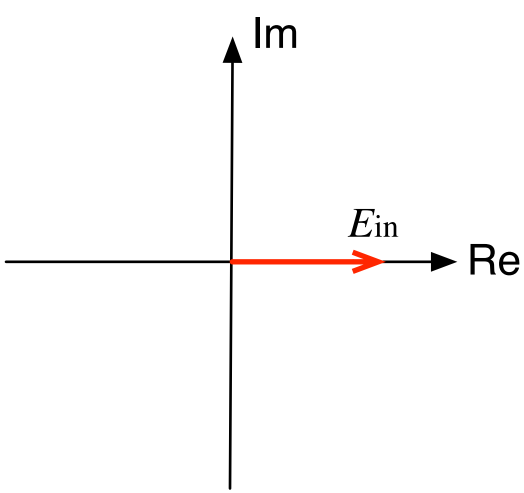


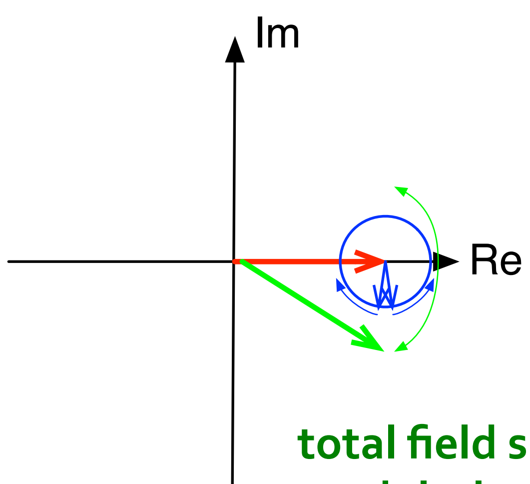
$$E = E_0 e^{i2\pi\nu_0 t} + E_1 e^{i2\pi(\nu_0-\nu_m)t} + E_1 e^{i2\pi(\nu_0+\nu_m)t}$$



- **Phaser diagram**

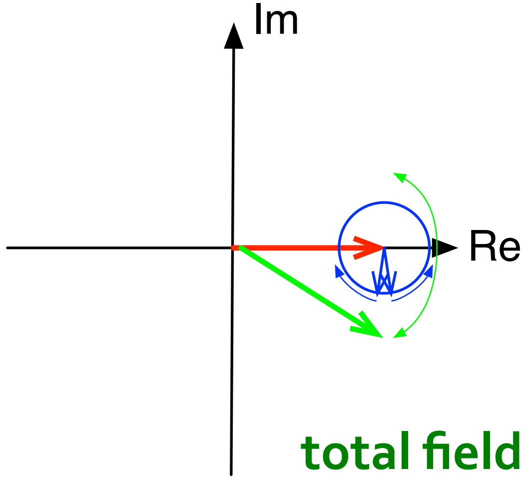
$$E \times e^{-i2\pi\nu_0 t}$$

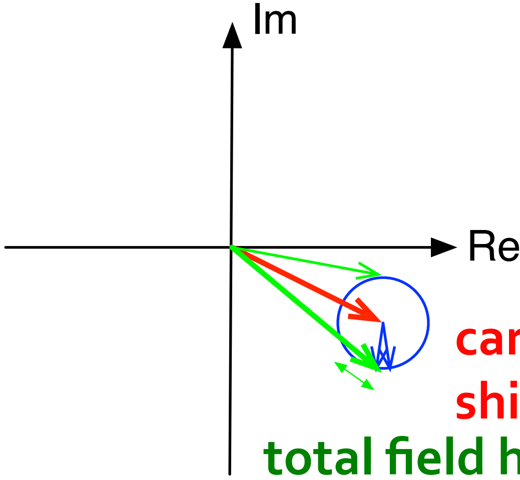




PDH control signal

- **PDH technique**
  - **Reflected field on Resonance** **Detuned**





carrier phase shift

total field only has phase modulation at ν_m

total field has amplitude modulation at ν_m

No power modulation

Power modulation at ν_m

=> Photocurrent demodulated at ν_m shows the linear signal to the detuning!

Michelson length control

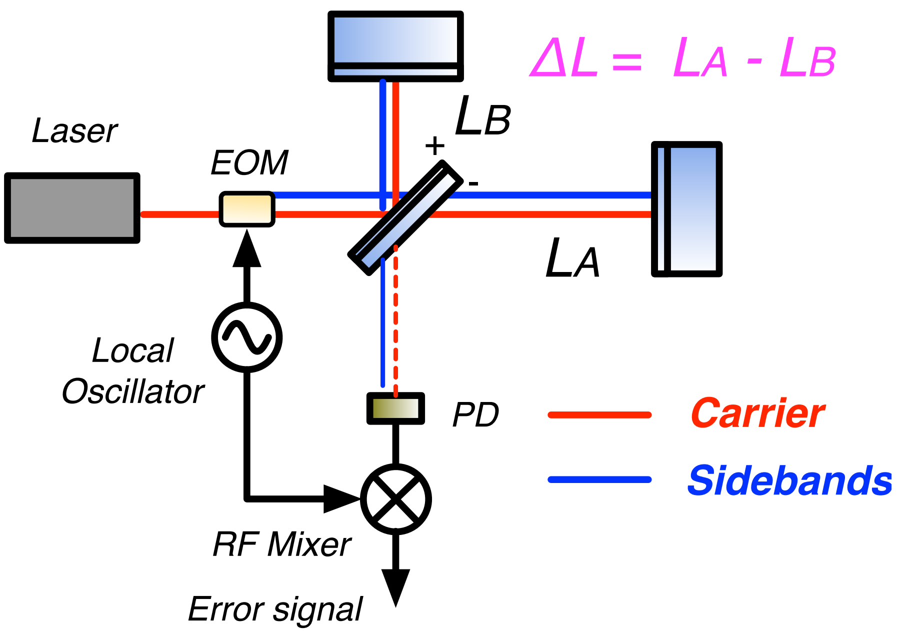
- Michelson is operated at the dark fringe

  for the shot noise and the power recycling

  - At the dark fringe, DC signals can't be a good error signal
  - Schunupp asymmetry:

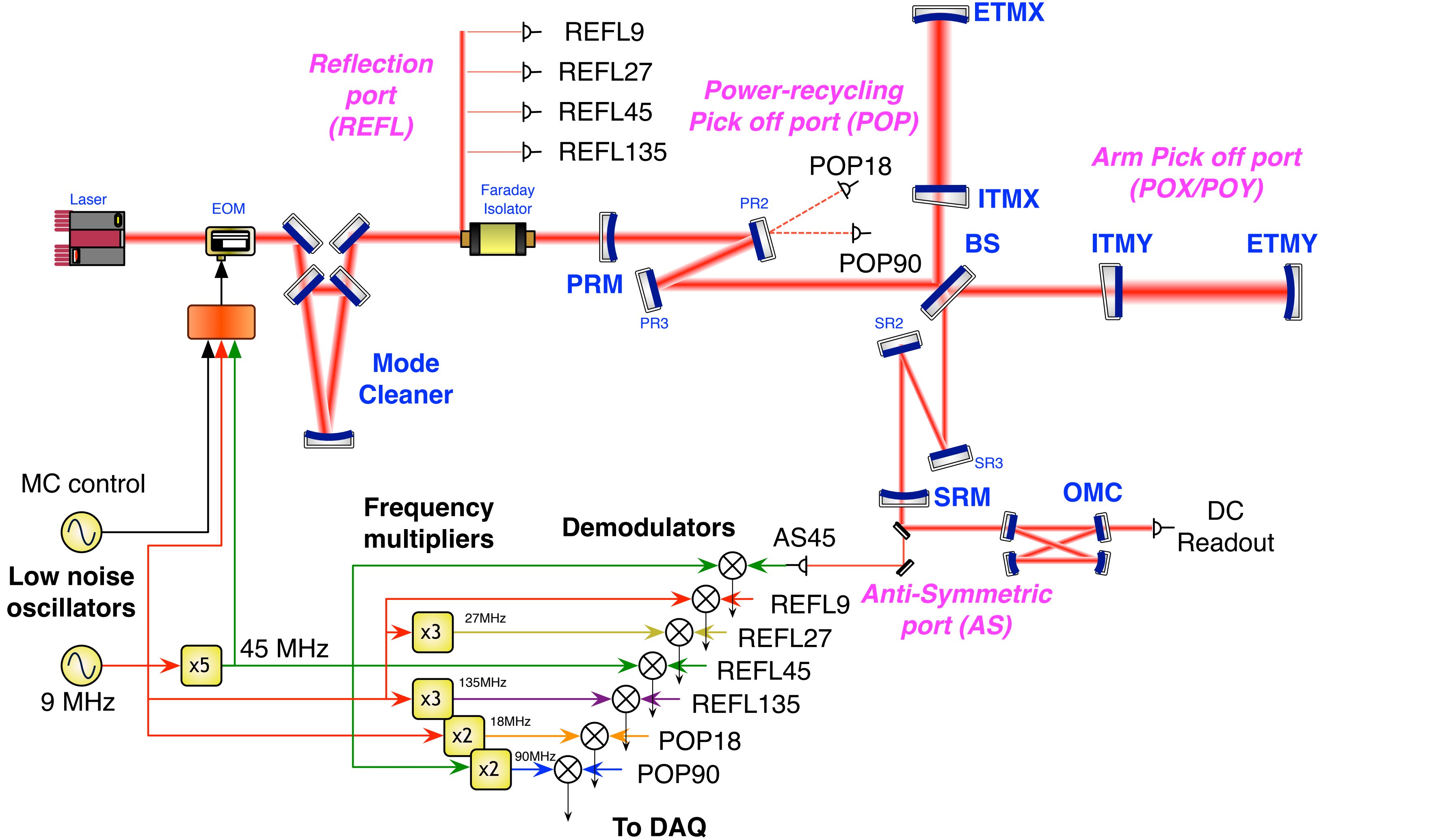
    Introduce small arm length asymmetry

    => RF sidebands leaks to the dark port



aLIGO length control

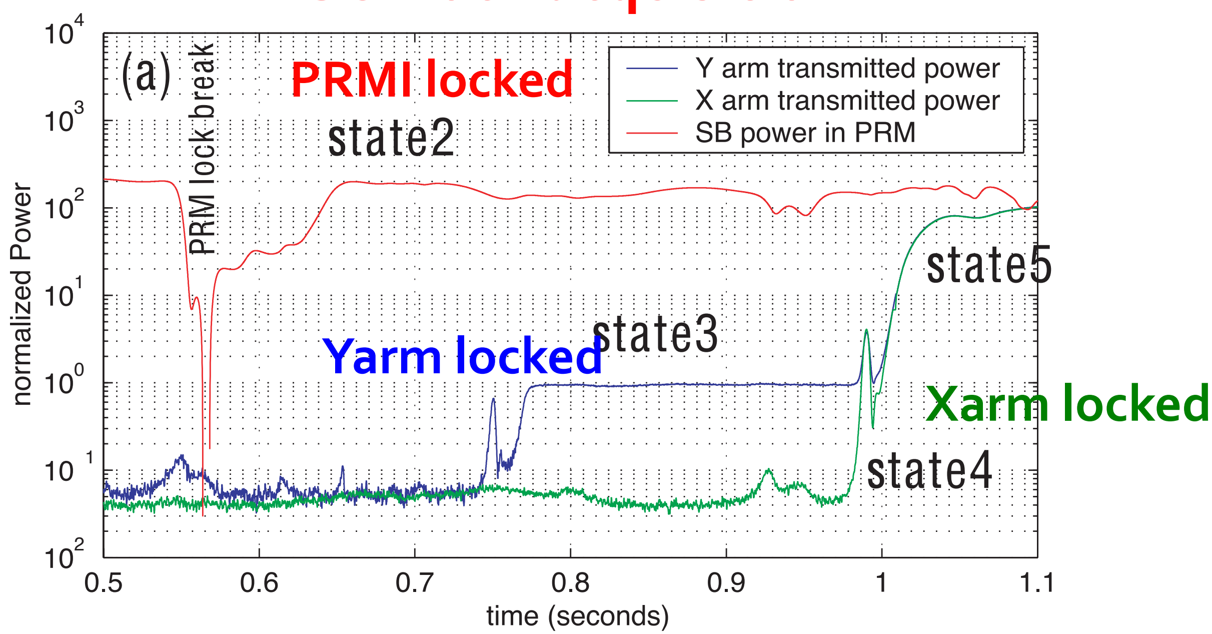
- ...In the end, combining these techniques with multiple modulations, the sensing system looks like this



Lock Acquisition: Real and Simulated

- Transition from non-operational state to the final linear state
  - Nonlinear process ~ Lock Acquisition

iLIGO Lock acquisition



Arm powers are normalized by the power when one arm is locked.
SB power is normalized by the input SB power.

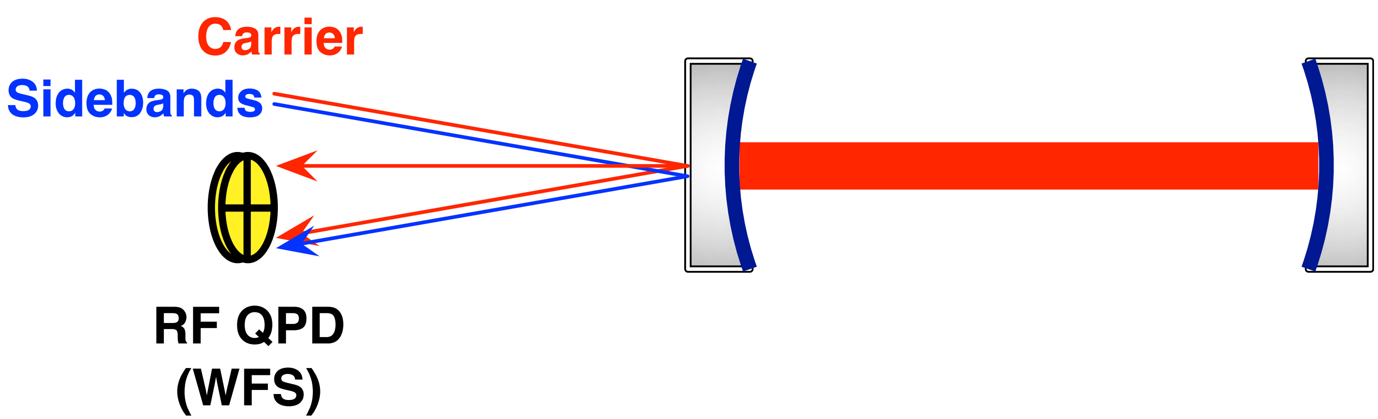
- Before the lock, we can't make a diagnosis of the control loops Without diagnosis, the lock is difficult. (Chicken & egg problem)

Angular global control

- Wave Front Sensing
  - Misalignment between the incident beam and the cavity axis
  - The carrier is resonant in the cavity
  - The reflection port has
    - Prompt reflection of the modulation sidebands
    - Prompt reflection of the carrier
    - Leakage field from the cavity internal mode

no signal

spatially distributed amplitude modulation

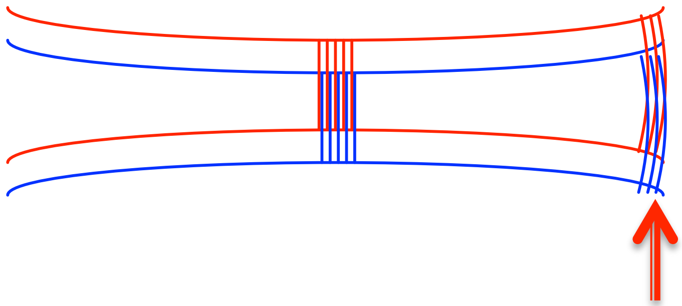


E Morrison et al Appl Optics **33** 5041-5049 (1994)

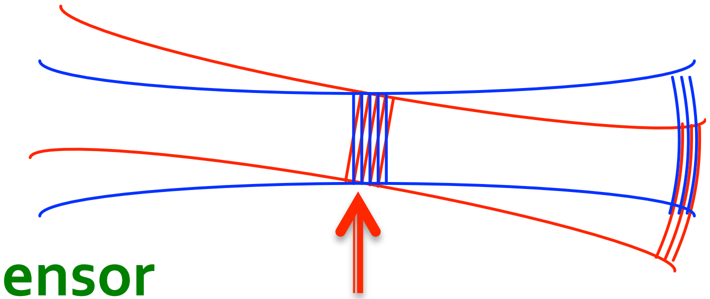
Angular global control

- **Wave Front Sensing**
  - **WFS becomes sensitive when there is an angle between the wave fronts of the CA and SB**
  - **Can detect rotation and translation of the beam separately, depending on the "location" of the sensor**
  - **Use lens systems to adjust the "location" of the sensors. i.e. Gouy phase telescope**

Sensitive at the far field

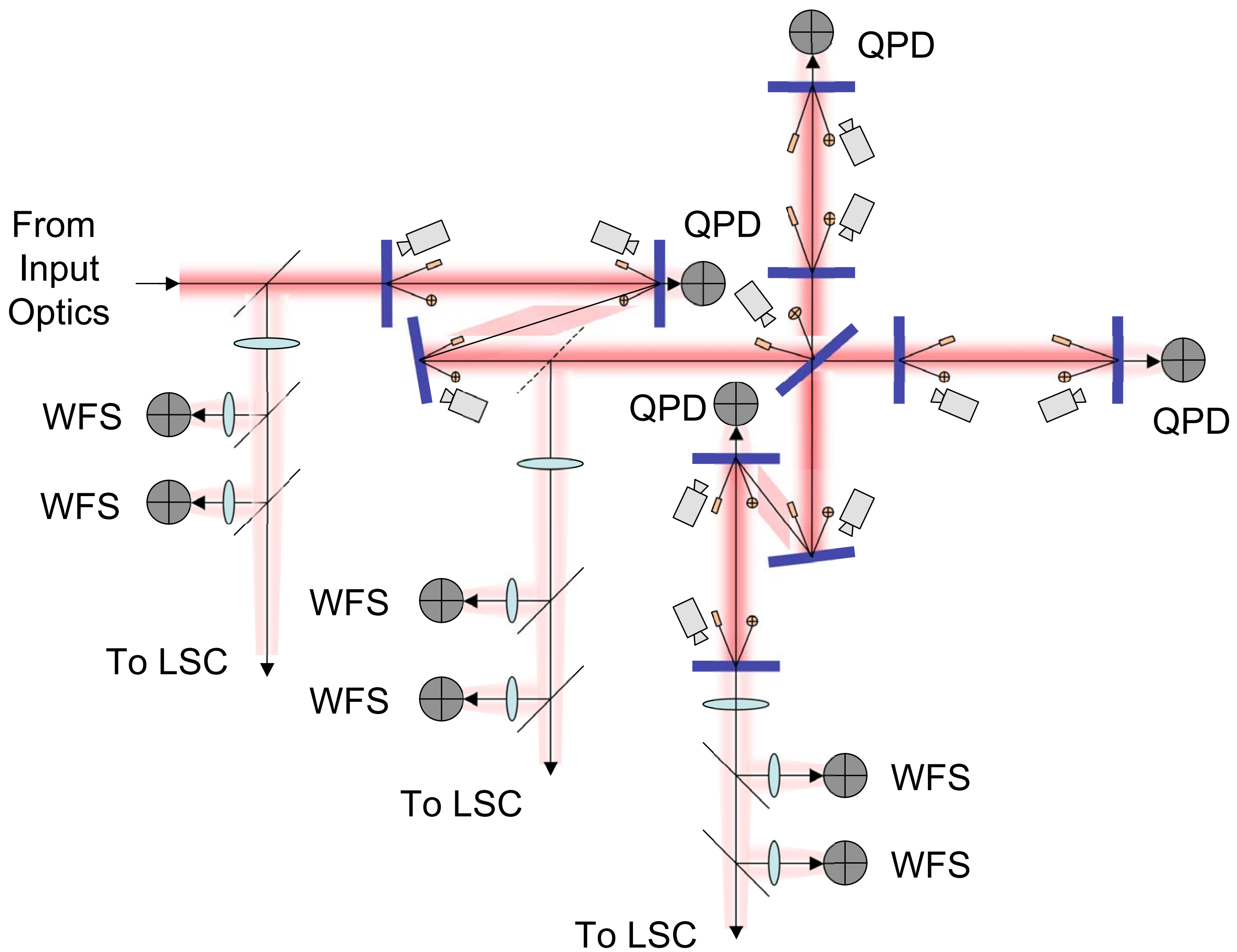


Sensitive at the near field



aLIGO angular control

- **Combine WFS, DC QPD, digital CCD cameras**



Summary

- Feedback control is an indispensable system for laser interferometer GW detectors
- It involves sensors, actuators, and filters, namely everything! (i.e. Optics, Mechanics, Electronics)