Quantum noise in Gravitational Wave Detectors

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**SHOT NOISE:**
Photon counting noise

\[ h_{\text{shot}} \propto \frac{1}{L} \frac{1}{\sqrt{P}} \]

**RADIATION PRESSURE NOISE:**
Back-action noise caused by random motion of the mirrors

\[ h_{\text{rad}} \propto \frac{1}{f^2L} \frac{\sqrt{P}}{m} \]

\[ h_{\text{quantum}} = \sqrt{h_{\text{rad}}^2 + h_{\text{shot}}^2} \]

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“Standard Quantum Limit”

\[ h_{\text{Quantum}} = \sqrt{\frac{4\hbar}{m\Omega^2 L^2}} \sqrt{\frac{1}{2} \left( K + \frac{1}{K} \right)}, \quad K = \frac{4P\omega_0}{c^2 m\Omega^2} \]

\[ h_{\text{Quantum}} \geq \sqrt{\frac{4\hbar}{m\Omega^2 L^2}} = h_{\text{SQL}} \]

\( h_{\text{SQL}} \) doesn’t depend on the optical parameters of the interferometer, just on the quantum mechanics of a harmonic oscillator mass

Slide courtesy of L. Barsotti
**Optical noises**

- **Quantum noises**
  - **Standard Quantum Limit (SQL)**
  - Trade-off Between Shot Noise and Radiation-Pressure Noise
  - Uncertainty of the test mass position due to observation
In this section, we outline a program that identifies the constraints applicable to the 3G and 4G detectors. Appendix A summarizes the status of Advanced LIGO circa mid-2012. Figure 2 presents our current best understanding of the noise sources that will limit Advanced LIGO in the high power, broadband tuning configuration. The sensitivity is determined by shot noise at high frequencies, by mirror thermal noise in the middle frequencies, and by a combination of thermal, seismic and quantum radiation pressure noises at low frequencies.

The 3G and 4G detectors will study novel astrophysics and gravitational phenomena employing technology beyond the current state of the art. These sources are well described in the ET Design Study [1] as well as a technical note being prepared within the LSC [2]. The predicted gravitational wave sources emit in a variety frequency bands with a variety of durations, which places a corresponding requirement on the detector sensitivity. Ideally, the design of the 3G and 4G detectors will match the gravitational wave sources and the noise performance will be optimized accordingly. The required sensitivity then dictates the detector technology and design. This design methodology is illustrated schematically in Figure 3.

This methodology fails in the current situation without gravitational wave detection – the input astrophysics remains speculative. In the following discussion, the process is reversed.

Figure 2: Baseline aLIGO Noise Budget (GWINC v2.0). 125 W input power; broadband RSE tuning.
Quantum noise reduction

\[ h_{\text{Quantum}} = \sqrt{\frac{4 \hbar}{m\Omega^2 L^2}} \sqrt{\frac{1}{2} \left( K + \frac{1}{K} \right)}, \quad K = \frac{4 P\omega_0}{c^2 m\Omega^2} \]

- Make the interferometer longer
  => Needs new facility

- Heavier test masses & more optical power
  => Stored power of aLIGO will be 800kW
  Have to deal with thermal effects / instabilities

- More complex optical configuration to shape optical response

Injection of squeezed states of vacuum
What is “squeezing”?  
Quantized Electromagnetic Fields  
Quadrature Field Amplitudes

\[ \hat{E} = \hat{X}_1 \cos \omega t + i \hat{X}_2 \sin \omega t \]

Classical

Quantum

Coherent State

Heisenberg’s uncertainty principle

\[ \Delta X_1 \Delta X_2 \geq 1 \]
Even when average amplitude is zero, the variance remains

=> Zero-point “vacuum” fluctuation

Vacuum fluctuations are everywhere

=> Comes into the interferometer from the open optical port and cause shot and radiation pressure noises
Squeezing

- The noise can be redistributed while keeping the minimum uncertainty product $\Delta X_1 \Delta X_2 = 1$
  
  = Squeezed light

- Squeezed light is characterized by
  - Squeezing factor $r$
    How much the noise is squeezed
    \[ \Delta X_1 = e^{-r}, \Delta X_2 = e^r \]
  - Squeezing angle $\phi_{sqz}$
    Which quadrature is squeezed
Squeezing

- Particularly useful two states

Amplitude squeezing
(Phase anti-squeezing)

Phase squeezing
(Amplitude anti-squeezing)
In practice, we inject squeezed “vacuum” from the dark port

- Squeezing angle needs to be fixed by a feedback control loop with regard to the field in the interferometer
Squeezing in action

- Actual squeezer (LIGO H1 squeezer)

- Many auxiliary phase lock loops are necessary to fix the squeezing angle

Squeezing in action

- Shot noise reduction in GW detectors has already been realized since 2007
- Squeezed light injection experiment at the LIGO 40m

**Figure 3**

Displacement (m Hz$^{-1/2}$)

**Theoretically predicted shot noise**

**Squeezed shot noise**

**Squeezing in action**

- **Squeezing in GEO600 and LIGO H1 to reduce shot noise**

![Graph showing strain sensitivity vs frequency for GEO600 and LIGO H1 with and without squeezing.]

- GEO600: 3.5 dB (1/1.5)
- LIGO H1: 2.1 dB (1/1.27)

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Slide courtesy of L. Barsotti
GEO data are courtesy of H. Grote

LSC, Nature Physics 7, 962 (2011)
LSC, Nature Photonics 7, 613–619 (2013)
Quantum noise in an interferometer

- Ponderomotive effect
  - Vacuum fluctuations from the dark port produce amplitude and phase fluctuations in the arm cavities.

- Radiation pressure
  - The test mass mechanical system work as a converter from the amplitude fluctuation to the phase fluctuation.
Quantum noise in an interferometer

- Ponderomotive effect
  - Radiation pressure:
    The test mass mechanical system work as a converter from the amplitude fluctuation to phase fluctuation

\[
\begin{align*}
E_{1\text{out}}^{\text{out}} &= E_{1\text{in}}^{\text{in}} \\
E_{2\text{out}}^{\text{out}} &= E_{2\text{in}}^{\text{in}} - \frac{I}{M\Omega^2} E_{1\text{in}}^{\text{in}} + \sqrt{IG}
\end{align*}
\]
Quantum noise in an interferometer

- **Ponderomotive effect**

\[ \frac{P}{(M\Omega^2)} \sim 1 \]

- **Input Vacuum Fluctuation**

- **Vacuum Fluctuation and GW signal (in the arm)**

- **Low frequency**
  \[ \frac{P}{(M\Omega^2)} >> 1 \]

- **SQL frequency**
  \[ \frac{P}{(M\Omega^2)} \sim 1 \]

- **High frequency**
  \[ \frac{P}{(M\Omega^2)} << 1 \]
Homodyne detection

- In order to detect the signal (and noise) with a photodetector, the output field needs to be mixed with a local oscillator field. cf. RF (or heterodyne) detection using RF sidebands

- Homodyne angle:
  Changes the projection of the GW signal field and output noise fields into the detection signal

- DC Readout
  A small (1~10pm) offset from the dark fringe is applied
  => Useful: The IFO beam itself becomes the LO field
  $\Phi_H$ is fixed at zero
Squeezed vacuum injection

- Frequency dependent squeezing
  - Rotate squeezing angle to optimize the output noise field

Figure 35: Examples of the sum quantum noise power (double-sided) spectral densities of the resonance-tuned interferometers with frequency-dependent squeezing and/or homodyne angles. Left: no optical losses, right: with optical losses, $\beta_d = 0$.

95. ‘Ordinary’: no squeezing, $\Delta_\text{LO} = \pi/2$. ‘Squeezed’: 10 dB squeezing, $\theta = 0$, $\Delta_\text{LO} = \pi/2$ (these two plots are provided for comparison). Dots [pre-filtering, Eq. (403)]: 10 dB squeezing, $\Delta_\text{LO} = \pi/2$, frequency-dependent squeezing angle. Dashes [post-filtering, Eq. (408)]: 10 dB squeezing, $\theta = 0$, frequency-dependent homodyne angle. Dash-dots [pre- and post-filtering, Eq. (410)]: 10 dB squeezing, frequency-dependent squeeze and homodyne angles. For all plots, $J = J_{\text{aLIGO}}$, and minimize the resulting sum noise spectral density:

$$S_h(\Delta) = 2ML^2 \Delta^2 \cosh 2r + \sinh^2 r \cos 2\Delta_\text{LO} + \Delta^2 K(\Delta) \sin^2 \Delta_\text{LO} + K(\Delta) e^{2r \cot \Delta_\text{LO}} + K(\Delta).$$

(406)

The sum quantum noise spectral density (408) is plotted in Figure 35 for the ideal lossless case and for $\beta_d = 0$.

Compare this spectral density with the one for the frequency-dependent squeezing angle (pre-filtering) case, see Eq. (403). The shot noise components in both cases are exactly equal to each other. Concerning the residual back-action noise, in the pre-filtering case it is limited by the available squeezing, while in the post-filtering case – by the optical losses. In the latter case, were there no optical losses, the back-action noise could be removed completely, as shown in Figure 35 (left). For the parameters of the noise curves presented in Figure 35 (right), the post-filtering still has some advantage of about 40% in the back-action noise amplitude $p_S$.

Note that the required frequency dependences (404) and (407) in both cases are similar to each other (and become exactly equal to each other in the lossless case $\beta_d = 0$). Therefore, similar...
Squeezed vacuum injection

- Frequency dependent squeezing

**Graph:**
- Vertical axis: Strain Sensitivity [1/√Hz]
- Horizontal axis: Frequency [Hz]
- Three curves:
  - Purple: Quantum Noise
  - Red: Thermal Noise
  - Black: Total Noise

**Diagram:**
- GW Signal
- ~30Hz
- Quantum Noise
- Interferometer
- Laser
- Squeezer
- Filter cavity
- Detection

**Text:**
- High finesse detuned "filter cavity" which rotates the squeezing angle as a function of frequency
- Quantum Noise ~30Hz
- Thermal Noise ~30Hz
- Frequency dependent squeezing

**Slide courtesy of L. Barsotti**
Squeezed vacuum injection: technical issues

- The enemies of the squeezing
  - Optical Loss
    Optical losses work as beamsplitters to introduce normal vacuum fluctuation
  - Phase noise
    Wobbling of the squeezing angle causes leakage of the other quadrature into the squeezed quadrature
Optical loss & Squeezing phase noise

Slide courtesy of L. Barsotti

eLIGO

Target: -10dB noise reduction

We need less than 20% total losses

aLIGO readout now has ~30% (w/o squeezing) => Need to reduce!
Phase noise mitigation

- Invac OPO for aLIGO

Let’s move the OPO into the vacuum envelope on seismic isolated tables!

E. Oelker et al., Optics Express, Vol. 22, Issue 17, pp. 21106-21121 (2014) (P1400064)
Quantum noise in GW detectors
- Shot noise & Radiation pressure noise

Squeezed vacuum injection
- Shot noise reduction already demonstrated.

Radiation pressure
- Will eventually limit the sensitivity
- Frequency Dependent squeezed vacuum injection will mitigate the radiation pressure noise

Technical Issues: Optical loss & phase fluctuation
- R&D on going