Noises in Gravitational Wave Detectors
GW detection
Data stream of differential arm strain

Once recorded:
Signals and noises are indistinguishable
What we can do is to catch “likely” features

Reduce any kind of noises!
Introduction ~ Noise?

- Time domain vs frequency domain

  **Time domain:** transient noises
  **Frequency domain:** stationary noises
Introduction ~ Noise?

- **Power Spectral Density (PSD)**
  Double sided PSD (-Infinity < f < Infinity)

\[
S_{DS}(f) = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t)e^{-2\pi ift} \, dt \right|^2
\]

- **Single sided PSD (0 <= f < Infinity)**

\[
S_x(f) = 2S_{DS}(f) \quad [x_{unit}^2 / \text{Hz}]
\]

- **Linearized PSD:**

\[
G_x(f) = \sqrt{S_x(f)} \quad [x_{unit}/ \text{sqrtHz}]
\]
Parseval’s Theorem for signal RMS and PSD

\[ x^2(t) = \int_0^\infty S_x(f) \, df \]

\[ \equiv x_{\text{RMS}}^2 \]

Root Mean of \( x(t) \):
average signal power density (per sec)
(cf. variance, std deviation)

PSD \( S_x(f) \):
power density per frequency (per sec)
Example

PSD [fm/sqrtHz] in log-log scale,  \( \text{RMS [fm] } \sim 50\text{fm} = 0.05\text{pm} \)

PSD [fm/sqrtHz] in log-lin scale

RMS [fm]

Time series [pm]
Components of the interferometer

- 3 fundamentals of the GW detector
  - Mechanics
  - Optics
  - Electronics
3 fundamentals of the GW detector

Mechanics
Optics
Electronics

Components of the interferometer

- **Optics**
  - Low optical loss mirror
  - Low optical loss coating
  - Mirror presicise polishing
  - Long baseline optics
  - Optical recycling

- **Mechanics**
  - Low mech. loss substrate
  - Low mech. loss coating
  - High rigidity optics supports
  - Multiple pendulum suspension
  - Monolithic suspensions
  - Vibration isolation
  - High vacuum environment

- **Electronics**
  - RF modulation
  - Analog high speed ctrl
  - Analog front end
  - Real time digital cont
  - User interface
  - Data acquisition
  - Data archive
  - Computing
  - Actuators
  - Low noise position sensors
  - Low noise accelerometers
  - Active vibration isolation
3 fundamentals of the GW detector

- **Mechanics** -> Displacement noises
- **Optics**    -> Optical noises
- **Electronics** -> Electrical noises
Noise budget

- Laser shot noise
- Laser radiation pressure noise
- Thermal noise
- Seismic noise
- Laser intensity/frequency noise
- Electronics noise
- Digitization noise
- Angular control noise
Displacement noises
Displacement noise

- Mechanical displacement sensed by a laser interferometer
- The longer the arm length, the smaller the strain noise
  - Seismic noise
  - Thermal noise
  - Newtonian Gravity noise
Displacement noise

- Seismic noise
  - Even when there is no noticeable earthquake...

Target disp. noise $10^{-20} \text{m/rtHz}$

**Displacement noise**

- **Vibration isolation ~ utilize a harmonic oscillator**
  - A harmonic oscillator provides vibration isolation above its resonant frequency

\[ v_0 = 1\text{Hz} \]

\[ |x/X| \approx 1/1000 @ 30\text{Hz} \]

\[ m\ddot{x} = -k(x - X) - \gamma(\dot{x} - \dot{X}) \]

\[ \left( \omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2 \right) \tilde{x} = \left( \omega_0^2 + i\frac{\gamma}{m}\omega \right) \tilde{X} \]

\[ \frac{\tilde{x}}{\tilde{X}} = \frac{\omega_0^2 + i\frac{\gamma}{m}\omega}{\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2} \]

\[ \omega_0 = \sqrt{\frac{k}{m}} \]
Displacement noise

- How to get more isolation?

**Damping**
Lower the peak height
- Worse isolation

**Multi stage**
Steeper isolation curve
- More peaks

**Lower resonant freq**
Better isolation
- Complex to realize

- In practice: employ combination of these measures
Displacement noise

- Initial LIGO vibration isolation
- Hydraulic active isolation / Isolation stack / Single Pendulum
Displacement noise

- Advanced LIGO vibration isolation
- Hydraulic active isolation / In-vacuum Active Isolation Platforms / Multiple Pendulum
Virgo: super attenuator
- 8m high
- 9 stages in horizontal
- 6 stages in vertical
Displacement noise

![Graph showing magnitude vs. frequency for different ground to mirror transfer functions.](http://link.aps.org/doi/10.1103/RevModPhys.86.121)

Question:

- \( n \)-stage multiple pendulum with fixed height of \( H \)
- How many stages \( n \) do we need to realize the vibration isolation of \( A \) at frequency of \( f \)?
- For a given \( A \) what is the minimum \( f \), we can realize by increasing \( n \)?

(Mass distribution)
- For equal \( m \) for each stage
  - or
- For arbitrary mass \( m_i \) and length \( h_i \)
Displacement noise

- Thermal noise:
  - System in thermal equilibrium
    - the system can dissipate its energy to the heat bath
    - the system is thermally excited by thermal fluctuation

- Mechanical thermal noises
  - suspension thermal noise
  - mirror substrate thermal noise
  - mirror coating thermal noise
Displacement noise

- Fluctuation Dissipation Theorem
- Friction: interaction with “bath” = huge number of degrees of freedom
- Fluctuation force: produced by huge number of d.o.f.
- Dissipation and fluctuation have certain relationship

System description (Langevin equation)

\[ m\ddot{q} + R\dot{q} = F + F'(t) \]

q: generalized coordinate  m: generalized mass
R: friction (dissipation)
F: internal force (restoring force, etc)
F'(t): fluctuating force from heat bath

Power spectrum density (PSD) of the fluctuation force

\[ S_{F'}(\omega) = 4k_B TR \]
Displacement noise

- **Transfer function approach**

  Equivalently, the fluctuation of the system can be obtained from the response of the system

\[
S_q(\omega) = \frac{4k_B T \text{Re}[1/Z(\omega)]}{\omega^2} = -\frac{4k_B T \text{Im}[H(\omega)]}{\omega}
\]

- where \(Z(\omega)\) and \(H(\omega)\) are the impedance and force-to-displacement transfer function of the system

\[
Z(\omega) = \frac{F(\omega)}{\dot{q}(\omega)}, \quad H(\omega) = \frac{q(\omega)}{F(\omega)}
\]
Displacement noise

Question:
- Velocity damping of a pendulum
  \[ m\ddot{x} + \Gamma \dot{x} + m\omega_0^2 x = f \]

- Structural damping
  loss angle: \( 0 < \phi \ll 1 \)
  \[ m\ddot{x} + m\omega_0^2 (1 + i\phi) x = f \]

- How does anti-spring change the thermal noise spectrum?
  anti-spring parameter: \( 0 < \alpha < 1 \)
  \[ m\ddot{x} + m\omega_0^2 (1 - \alpha + i\phi) x = f \]
Displacement noise

- In some cases, calculating the system response is complicated (e.g. deformation of an elastic body)
- Systems response (impedance) at a certain freq:

\[ Z(\omega) = \frac{F(\omega)}{\dot{q}(\omega)} \]

- Average rate of energy dissipation

\[ W_{\text{diss}} = \langle \text{Re}(F)\text{Re}(\dot{q}) \rangle \]

\[ = \frac{1}{2} \text{Re}[1/Z(\omega)]F_0^2 = \frac{1}{2} \frac{\text{Re}[Z(\omega)]}{|Z(\omega)|^2} F_0^2 \]

\[ S_q(\omega) = \frac{4k_B T}{\omega^2} \text{Re}[1/Z(\omega)] \]

\[ S_q(\omega) = \frac{8k_B T W_{\text{diss}}}{F_0^2 \omega^2} \]
Sensing of the mirror surface deformation with a laser beam (with intensity profile of $f(r)$)

Apply periodic pressure with profile of $f(r)$

This induces deformation of $x(r)$ which is different from our sensing profile of $f(r)$, but that’s OK

Calculate the rate of dissipation $W_{\text{diss}}$ analytically, using FEA, or etc

Put this into the formula

$$S_x(\omega) = \frac{8k_B T W_{\text{diss}}}{F_0^2 \omega^2}$$

Y. Levin PRD 57, 659-663 (1998)
Mirror substrate thermal noise

- Brownian motion
  Mechanical loss associated with the internal friction
  \[\Leftrightarrow\text{Thermally excited body modes}\]
  Optical coating (high mechanical loss) will be limiting noise source in aLIGO

- Thermo-elastic noise
  Elastic strain & thermal expansion coefficient
  \[\Rightarrow\text{cause heat distribution & flow in the substrate}\]
  \[\Leftrightarrow\text{Temperature fluctuation causes mirror displacement}\]

- Thermo-refractive noise
  \[\Leftrightarrow\text{Temp. fluctuation causes fluctuation of refractive index}\]
Displacement noise

- Suspension thermal noise
  - Brownian motion
    Mechanical loss of the suspension fiber
    ⇔ Thermally excited pendulum modes
  - Thermo elastic noise
    Elastic strain of the fiber & thermal expansion coefficient
    ⇒ cause heat distribution & flow in the fiber
    ⇔ Temperature fluctuation causes mirror motion

<- Monolithic suspension for high pendulum Q
**Displacement noise**

**Question**

- **Induced current damping (electro-mechanical system)**
  1. How does the Q factor of the system depend on R?
  2. How much is the thermal noise displacement of the mass?
  3. How does the thermal noise of the resistor shakes the mass?
  4. How are the above questions with a capacitive coupling instead of the coil?

- **Cold damping**
  1. If the resister is cooled, how does the thermal noise motion change?
  2. Is the pendulum actually cooled? Down to what temperature?
  3. How fast the pendulum recovers the original temperature once R is returned to the room temp.?
Newtonian Gravity noise

- Mass density fluctuations around the test masses
  => test mass motion via gravitational coupling
- Dominant source of Newtonian noise
  = Seismic surface wave

Mitigation
1) Going to quiet place (underground)
2) Feedforward subtraction
3) Passive reduction by shaping local topography

J Harms, et al, Class. Quantum Grav. 31 185011 (2014)
Displacement noise

- Mechanical upconversion noise
  - Large low frequency ($f < 1\text{Hz}$) motion
    => upconverted to $10\sim 100\text{Hz}$ motion via nonlinear processes
  - Barkhausen noise
    => low freq mirror actuation cause BH noise and upconversion
    Select better magnet materials (e.g. SmCo)

https://www.nde-ed.org/
Optical noises
Noises that contaminate the readout signal

- Quantum noises (shot noise, radiation pressure noise)
- Laser technical noises (frequency/intensity noise)
- Modulation noises
- Scattered light noise
Quantum noises: **Shot noise**

- Noise due to photon counting statistics
- $N$ detected photon $\Rightarrow$ standard deviation $\sqrt{N}$

- Increasing the incident power $P_{\text{in}}$,
  $\Rightarrow$ The shot noise is increased by $\sqrt{P_{\text{in}}}$
  $\Rightarrow$ The signal amplitude is increased by $P_{\text{in}}$

- In total, the signal-to-noise ratio is improved by

$$\text{SNR} \propto \sqrt{P_{\text{in}}}$$
Optical noises

- **Quantum noises: Shot noise**
  - Photon shot noise associated with photodetection
    \[ i_{\text{shot}} = \sqrt{2e} i_{\text{DC}} \frac{[A]}{\sqrt{\text{Hz}}} \]
  - **Michelson interferometer**
    \[ i_{\text{DC}} = \frac{\eta P_{\text{in}}}{h\nu} \frac{1 - \cos \delta \phi}{2} \frac{[A]}{\text{rad}/\sqrt{\text{Hz}}} \]
    \[ i_{\text{shot}} / \frac{di_{\text{DC}}}{d\phi} = \sqrt{\frac{2h\nu}{\eta P_{\text{in}}}} \frac{[\text{rad}/\sqrt{\text{Hz}}]}{\text{rad}/\sqrt{\text{Hz}}} \]

  Shot-noise limit of the Michelson phase sensitivity

- **Michelson response (@DC)**
  \[ \frac{\delta \phi}{h_{\text{GW}}} = \frac{4\pi L \nu}{c} \frac{[\text{rad}/\text{strain}]}{} \]

  \( i_{\text{DC}} \): DC Photocurrent
  \( \eta \): PD Quantum Efficiency
  \( \nu \): Optical Frequency

  at the limit of \( d\phi \rightarrow 0 \)

  Michelson Strain Sensitivity

  \( 1.3 \times 10^{-20} \ 1/\sqrt{\text{Hz}} \) at \( 1 \text{W} \)
Optical noises

- Supplemental slide ~ Shot noise derivation

- Take an average of Current $I(t)$ for a period of $T$, and sample it every $T$.
- Number of photons in this period $T$ is $N = \bar{I}T/e$.
- Fluctuation of photon number in $T$ is $\sigma_N = \sqrt{N}$. cf Poisson statistics.
- Thus, the standard deviation (RMS) of $\bar{I}$ is $\sigma_I = e\sqrt{\bar{N}}/T = \sqrt{e\bar{I}/T}$.
- Think about the transfer function of this box car average filter. It is $H(f) = \text{sinc}(\pi fT)$.
- Parsevals theorem: $\sigma_I = \int_0^\infty H(f)^2 i_s^2 df$, where $i_s$ is the linear power spectrum density of the current (white spectrum).
- According to the above integration, $i_s = \sigma_I \sqrt{2T}$.
- Therefore we obtain $i_s = \sqrt{2e\bar{I}}$. 

Optical noises

- Quantum noises ~ Radiation pressure noise
  - Photon number fluctuation in the arm cavity
    => Fluctuation of the back action force
  - Quantum noise of the input laser
    => Common noise for two arms
    => cancelled and does not appear in the signal
  - Vacuum fluctuation injected from the dark port
    => Differentially power fluctuation
    => Cause the noise in the GW signal

\[
\delta P = \sqrt{2\hbar \nu \bar{P}}
\]

\[
f_{\text{backaction}} = \frac{2\delta P}{c}
\]

\[
\tilde{x} = \frac{f_{\text{backaction}}}{M\omega^2}
\]
- **Quantum noises**
  - **Standard Quantum Limit (SQL)**
    - Trade-off Between Shot Noise and Radiation-Pressure Noise
    - Uncertainty of the test mass position due to observation

\[ \sqrt{S_{h}^{\text{SQL}}} = \sqrt{\frac{8\hbar}{M(2\pi f)^2 L^2}} \]
Laser frequency noise

- Laser wavelength \( \lambda = \frac{c}{\nu} \)
  = reference for the displacement measurement
- Optical phase \( \phi = 2\pi \nu L / c \)
  \[ d\phi = \frac{2\pi}{c} \left( L \, d\nu + \nu \, dL \right) \leq \text{indistinguishable} \]

\[
\frac{dL}{L} = \frac{d\nu}{\nu}
\]

- \( dL/L \text{ target } 10^{-24} \)
  
  \[ \Rightarrow \, d\nu = 10^{-24} \times 3 \text{ THz (1064nm YAG laser)} \]
  \[ = 3 \times 10^{-10} \text{ Hz/rtHz} \]
Optical noises

- **Laser frequency noise**
  - **Target:** $dν_{eff} = 3 \times 10^{-10}$ Hz/rtHz
  - **Laser stability**
    $dν = 10\sim100$ Hz/rtHz @100Hz

Michelson’s differential sensitivity provides Frequency noise cancellation of $1/100\sim1/1000$
“Common Mode Rejection”
Optical noises

- Laser intensity noise
  - Relative Intensity Noise (RIN): \( \frac{dP}{P} \)
  - Sensor output \( V = P \times x \)
    \[ \Rightarrow dV = P \, dx + x \, dP \]
    \( \leq \) indistinguishable

\[ \frac{dx}{x_{\text{offset}}} = \frac{dP}{P} \]

- Requirement: \( \text{RIN} = 10^{-9} \, 1/\sqrt{\text{Hz}} \)
Optical noises

- Laser intensity noise ~ intensity stabilization
  - Requirement: $\text{RIN} = 10^{-9} \, 1/\sqrt{\text{Hz}}$
  - 2-stage cascaded intensity stabilization control
  - Challenge: requires 300mA of photodetection

Shot noise limited RIN

$$\frac{i_{\text{shot}}}{i_{\text{DC}}} = \sqrt{\frac{2e i_{\text{DC}}}{i_{\text{DC}}}} = \sqrt{\frac{2e}{i_{\text{DC}}}}$$

- In-vacuum 8-branch Photodiode array

P. Kwee et al, Optics Express 20 10617-10634 (2012)
Optical noises

- Modulation noises
  - RF Residual Amplitude Modulation
  - Modulation Oscillator Phase Noise
  - Modulation Oscillator Amplitude Noise
- Produce noise sidebands on the modulation sidebands

- Mitigation
  - For the GW signal: Use DC readout and eliminate them by an “output mode cleaner cavity”

Optical noises

- Scattered light noise
  - Scattered light recouples to the interferometer beam with an arbitrary phase
    => causes amplitude and phase fluctuation
  - Two effects:
    1. Small motion regime: linear coupling of the phase fluctuation
    2. Large motion regime: low freq large motion of the scattering object => upconversion via fringe wrapping

- Mitigation
  - Reduce scattered light
  - Vibration isolation of the scattering object
Electrical noises
Electrical noises

- General rules for electrical noises
- Electrical noise in photo detection
- Digitization noise (ADC/DAC) / Aliasing
- Control noise
- Actuator noise
General rules for electrical noises

- Low noise amplification at the beginning
- Give necessary gain as early as possible
- Don’t attenuate (and amplify again)

\[
\text{input noise: } v_1 \quad \text{input noise: } v_2 \quad \text{output noise } V_{\text{out}}: \\
\text{Sqrt}[(v_1 G_1 G_2)^2 + (V_2 G_2)^2]
\]

\[
\text{Input equivalent noise} \\
V_{\text{out}} / (G_1 G_2) = \text{Sqrt}[v_1^2 + (v_2/G_1)^2]
\]

Lessons

- The input referred noise is determined by \( v_1 \)
- It won’t become better by the later stages
- If \( G_1 \) is big enough, we can ignore the noise of later stages
Photodiodes

- PIN photodiodes
  - (InGaAs for near IR, Si for visible)
  - Good linearity
  - Low noise
  - High Quantum Efficiency (>90%)

“Photodiode Amplifiers”, J. Graeme (McGrawHill 1995)
Noise in photodetectors

- Photodetectors are the first electrical block of the control chains
  - It is important to have low input-referred current noise

- Photo detection
  - AF (Audio Frequency 0~100kHz)
    - Plenty of light (photocurrent ~mA)
      - Not a big electrical issue
  - RF (Radio Frequency 10~200MHz)
    - Large diode aperture -> high RF noise
      - Need careful consideration
**Noise in photodetectors**

- **Photodiode equivalent circuit**
  - Shunt Resistance $R_D (~100\text{M}Ω)$ Usually not a problem
  - Junction Capacitance $C_D (1\text{pF}~1\text{nF})$
  - Series Resistance $R_S (1Ω~100Ω)$

The diode aperture size needs to be $\sim \text{mm}$ => $C_d$ tends to be big.

2mm InGaAs PD: $R_s \sim 10Ω$, $C_d \sim 100\text{pF}$

$\Rightarrow i_{Rs} = 20 \text{ pA/sqrtHz} @100\text{MHz}$

(equivalent to the shot noise of 1mA light $\sim 1.3\text{mW}@1064\text{nm}$)

\[ i_{Rs} \sim \omega C_d \sqrt{4k_B T R_S} \]
Restriction of signal digitization

- **Voltage quantization:** quantization noise
  - => limited dynamic range
  - => Requires whitening/dewhitenning filters

- **Temporally discrete sampling:** aliasing problem
  - => limited signal bandwidth
  - => Requires anti-aliasing (AA) / anti-imaging (AI) filters

Typical signal chain
Digitization (Quantization) noise

- Analog signals (~+/-10V) -> Digital signal
  - Digitized to a discrete N bit integer number
  - Quantization causes a white noise
    
    \[ V_n = \frac{\Delta}{\sqrt{12}} \quad [V/\sqrt{\text{Hz}}] \]
    
    e.g. +/-10V 16bit => \( \Delta = 0.3\text{mV} \) => \( V_n \sim 100 \text{ \mu V/sqrtHz} \)
    
    cf. Input noise of a typical analog circuit 10nV/sqrtHz

Digitization (Quantization) noise

- **Whitening**
  - Amplify a signal in the freq band where the signal is weak

- **Dewhitening**
  - Amplify a signal in the freq band where the signal is weak
Noise couplings from auxiliary loops

- e.g. Angle control feedback
  - noise injection to the GW channel

Mitigation
1) Make the coupling smaller
2) Make the noise itself smaller
3) Limit the control bandwidth of the aux loop
Actuator noise appears in the GW signal as an external disturbance

Mitigation
1) Make the noise itself smaller
2) Make the actuator response smaller

We need to keep sufficient actuator strength for lock acquisition
=> Transition to a low-noise mode after achieving lock
Summary
Summary

- There are such large number of noises
- They are quite omnidisciplinary
- Even only one noise can ruin our GW detection

GW detection will be achieved by
  - Careful design / knowledge / experience
  - Logical, but inspirational trouble shooting

- Noise “hunting”