Control system in Gravitational Wave Detectors

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Gravitational wave detection
Laser displacement sensor
Requires linear displacement detection

Control for measurement
Laser interferometer = nonlinear device
Feedback control => linearization
What is the feedback control?

- A scheme to monitor and modify output(s) of a system by changing the input(s) depending on the output(s)

Examples

- Shower temperature
- Car driving
- Tight rope walking
- Air conditioning
- Bike riding
- Inverted bar on a hand

Imagine what happens

- If the response is too slow?
- If the response is too fast?
Elements of a feedback loop

- Disturbance
- Sensor
- Actuator
- Feedback signal
- Error signal
- Transducer
- Filter

Stabilized motion

m or m/Hz^{1/2}

V or V/Hz^{1/2}

m or m/Hz^{1/2}

V or V/Hz^{1/2}

V/m

m/V

m/V

V/m
Introduction ~ Control?

- Sensor:
  Transducer for displacement-to-voltage conversion

- If the sensor is completely linear (and has or no frequency dependence)

  \[ V = H x \]

  We don’t need feedback control!
In reality: Sensors, laser interferometers in particular, are nonlinear!

Enclose the operating point in the linear region

=> The system recovers linearity

Was the displacement modified by the feedback?

=> Precise knowledge of the control system for signal reconstruction
Introduction ~ Control?

- Elements of a feedback loop

Disturbance $d_x$, Stabilized disturbance $d_x_s$, Actuator $A$, Sensor $F$, Servo filter $H$, Error signal $V_{err}$, Feedback signal $V_{fb}$.

**Open loop transfer function**

$\frac{d x_s}{d x} = G = H F A$

- $d x_s = d x - G d x_s$;
- $\Rightarrow d x_s = d x / (1+G)$;
- $\Rightarrow d x = V_{err} (1+G) / H$;
- $d x = V_{fb} A (1+G) / G$. 
When $G$ is small:

- Disturbance
- Stabilized disturbance
- Error signal
- Actuator
- Sensor
- Feedback signal
- Servo filter

Open loop transfer function

\[
\frac{dx_s}{dx} = 1 - G \frac{dx_s}{dx} \\
\Rightarrow \frac{dx_s}{dx} = \frac{1}{1+G} \\
\Rightarrow \frac{dx}{dx} = \frac{V_{err}}{H} (1+G) \\
\Rightarrow dx = V_{fb} A (1+G) / G
\]
When $G$ is big: e.g. $G = 10$, $100$, or $1000$

$dx_s = dx/(1+G) = 0.09 \; dx, \; 0.01 \; dx, \; 0.001 \; dx$

$V_{err} = dx \; H/(1+G)$

$V_{fb} = G/(1+G) \; /A \; dx$

$= 0.91 \; dx/A, \; 0.99 \; dx/A, \; 0.999 \; dx/A$
When the openloop gain $G$ is $\gg 1$, the error signal gets suppressed.

"Wow! our sensor signal became smaller!"

Is our system more sensitive now?

- No. We are just moving the actuator so that the error signal looks smaller. The signal and noise are equally suppressed in the error signal. Thus the SNR does not change.

OK... So can we still measure gravitational waves even if the error signal is almost zero?

- Yes. We should be able to recover the original signal by compensating the effect of the control i.e. $(1+G)$
- And we can also use the feedback signal in order to reconstruct the original signal with appropriate compensation i.e. $(1+G)/G$
Important difference between

- “Feedback control for stabilization” and “Feedback control for measurement”
- Feedback control changes the stabilized motion but reconstructed Disturbance is not modified by the loop\(^*\)
  (*if everything is linear)

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**Feedback control**

H

\[ \text{Disturbance} \rightarrow + \rightarrow \text{Sensor} \rightarrow \text{Error signal} \]

\[ \text{Actuator} \]

\[ \text{Servo Filter} \]

\[ \text{Feedback signal} \]
Linear systems and their stability
Linear systems and their stability

- A deterministic and time-invariant system: \( H \)

- The system \( H \) is LTI (linear & time-invariant) when
  \[
  y_1(t) = H \{x_1(t)\} \\
  y_2(t) = H \{x_2(t)\} \\
  \Rightarrow \alpha y_1(t) + \beta y_2(t) = H \{\alpha x_1(t) + \beta x_2(t)\}
  \]

- We can deal with such a system using Laplace transform (or almost equivalently Fourier Transform)
Linear systems and their stability

- Time domain vs Laplace (or Fourier) frequency domain

Time domain

\[ x(t) \xrightarrow{\text{Laplace}} X(s) \]

\[ h(t) \]

\[ y(t) = h(t) \ast x(t) \]

Frequency domain

\[ X(s) \xrightarrow{\text{Laplace}} H(s) \]

\[ Y(s) = H(s) \cdot X(s) \]

Impulse response

Transfer function

http://en.wikipedia.org/wiki/Linear_system
http://en.wikipedia.org/wiki/LTI_system_theory
Linear systems and their stability

- It is easy to convert from an ordinary differential equation to a transfer function

\[
\frac{d}{dt} \implies s
\]

\[
\Rightarrow i\omega = i2\pi f
\]

- e.g. Damped oscillator

\[
m\ddot{x}(t) = -kx(t) - \gamma \dot{x}(t) + F(t)
\]

\[
ms^2 X(s) = -kX(s) - \gamma sX(s) + F(s)
\]

\[
H(s) \equiv \frac{X(s)}{F(s)} = \frac{1}{ms^2 + \gamma s + k}
\]
Linear systems and their stability

- e.g. Damped oscillator

\[
H(s) = \frac{1}{ms^2 + \gamma s + k}
\]
\[
H(s) = \frac{1}{m \frac{s^2 + \omega_0^2}{Q} s + \omega_0^2}
\]
\[
H(\omega) = \frac{1}{m - \omega^2 + i \frac{\omega_0}{Q} \omega + \omega_0^2}
\]

\[
\omega_0 = \sqrt{\frac{k}{m}}, \quad \gamma = m\omega_0/Q
\]

Bode diagram

Magnitude

Phase [deg]

Angular Frequency [$\omega_0$]

log-log

log-lin
Linear systems and their stability

- e.g. RC filter

\[
\begin{align*}
V_{\text{out}} &= \frac{q}{C} \\
\dot{q} &= \frac{(V_{\text{in}} - V_{\text{out}})}{R} \\
\implies i\omega CV_{\text{out}}(\omega) &= \frac{(V_{\text{in}}(\omega) - V_{\text{out}}(\omega))}{R} \\
\implies \frac{V_{\text{out}}(\omega)}{V_{\text{in}}} &= \frac{1}{1 + i\omega RC}
\end{align*}
\]

Cut-off freq 
\( w = \frac{1}{R*C} \)
In most cases, a system TF can be expressed as:

\[ H(s) = \frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \ldots + a_n s^n} \]

The roots of the numerator are called as “zeros” and the roots of the denominator are called as “poles”

\[ H(s) = \frac{b_m \prod_{i=1}^{m} (s - s_{zi})}{a_n \prod_{j=1}^{n} (s - s_{pj})} \]

Zeros \((s_{zi})\) and poles \((s_{pj})\) are real numbers (single zeros/poles)

or pairs of complex conjugates (complex zeros/poles)
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real numbers (single zeros/poles)
or

pairs of complex conjugates (complex zeros/poles)
Linear systems and their stability

- Poles rule the stability of the system!

H(s) can be rewritten as

\[ H(s) = \sum_{j=1}^{n} \frac{K_j}{(s - s_{pj})} \]

- Each term imposes exponential time impulse response

  \[ T.F.: H_j(s) = \frac{1}{s + s_{pj}} \iff I.R.: h_j(t) = e^{s_{pj}t} \]

- Therefore, if there is ANY pole with \( \text{Re}(s_{pj}) > 0 \)

  the response of the system diverges
Linear systems and their stability

- Poles rule the stability of the system!

http://nupet.daelt.ct.utfpr.edu.br/_ontomos/paginas/AMESim4.2.0/demo/Misc/la/SecondOrder/SecondOrder.htm
Now we eventually came back to this diagram

**Requirement:**
All the roots for $1 + G$ should be in the left hand side of Laplace plane.
Remarks

Requirement:
All the roots for $1+G$ should be in the left hand side of Laplace plane

- This does not mean all $H$, $F$, $A$ needs to be stable.
  e.g. Unstable mechanical system $A$ can be stabilized by a control loop. (cf. An inverted Rod)

- We usually play with $F$ to tune the result.
  It is awkward to evaluate the stability of $1/(1+G)$ every time.
  There is a way to tell the stability only from $G$

Open loop TF: $G = H \cdot F \cdot A$
Closed loop TF: $G_{CL} = 1/(1+G)$

Nyquist’s stability criterion
Linear systems and their stability

- Nyquist stability criterion
  - Plot openloop gain $G$ in a complex plane (i.e. Nyquist diagram)
  - If the locus of $G(f)$ from $f=0$ to $\infty$, goes to $0$ looking at the point $(-1 + 0i)$ at the left side => Stable
  - If the locus sees the point $(-1+0i)$ at the right side => Unstable

- Unity gain frequency $f_{UGF}$:
  \[ |G(f_{UGF})| = 1 \]

- Phase margin $\theta$:
  \[ \theta = \arg(G(f_{UGF})) \]

- Gain margin $g$:
  \[ g = \frac{1}{|G(f_o)|} \text{ where } \arg(G(f_o)) = -\pi \]
Linear systems and their stability

- Phase Margin / Gain Margin in Bode diagram
  - Most of the case, a bode diagram of G is enough to see the stability

Input MC Openloop TF: UGF 176kHz, P.M. 48deg, G.M.@420kHz 4.3dB

Nearly unstable

Unity Gain Freq (176kHz)
Gain Margin 4.3dB@420kHz
Phase Margin 48deg

A rough standard of a stable servo loop:
Phase Margin > 40deg
Gain Margin > 10dB
Summary

- Classical control theory
- Design locations of poles and zeros
- Stability: tuning of open loop transfer function is important
Control system components in GW detectors
Control systems

- Elements of a feedback loop (again)
Interferometer control system

- Local control vs global control

Laser
Length Sensing

Digital Controllers

Global control
Local Control

Mirror

Local Control

Mirror
Local Sensors

- Shadow sensor (relative displacement sensor)
  - For suspension damping control, mirror attitude monitor
  - Typical linear range \(~1\text{mm}\) for \(0-10\text{V}\) \(\Rightarrow dV/dx = 10 \text{kV/m}\)
  - Typical noise level: \(~100 \text{pm/sqrtHz}\)
  - aLIGO: Birmingham Optical Sensor and Electro-Magnetic actuator (BOSEM)

L Carbone, Class. Quantum Grav. 29 (2012) 115005
Local Sensors

- Shadow sensor (relative displacement sensor)
  - Linear range (~0.7 mm) / displacement noise
  - Sensor locations

![Graph showing BOSEM output voltage vs. flag position and displacement noise vs. frequency.]

- Seismic Isolation Platform
- Top Mass: 6 BOSEMs main chain, 6 BOSEMs reaction chain
- Upper Intermediate Mass: 4 BOSEMs reaction chain
- Penultimate Mass: 4 AOSEMs reaction chain
- Test Mass: Electrostatic Drive

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Local Sensors

- Linear Variable Differential Transducer (relative disp. sensor)
  - For low freq pendulum control (inverted pendulum), larger range
    VIRGO Super attenuator, KAGRA Seismic Attenuation System
  - Typical linear range ~10mm for 0-10V => dV/dx = 1 kV/m
    Typical noise level: 10~100 nm/sqrtHz

![Graphs and diagrams illustrating the linear variable differential transducer and its performance metrics.](image_url)
**Local Sensors**

- **Optical Lever (relative angular sensor)**
  - Angle local control
  - Typical linear range ~beam side (0.1~1 mm) => \( \frac{dV}{d\theta} = 1 \sim 10 \text{ kV/rad} \)
  - Typical noise level: 0.01~1 nrad/sqrtHz

![Optical Lever Diagram]

\[
L = \frac{dx}{2L \, d\theta}
\]

\[\begin{align*}
\text{SUM} &= A+B+C+D \\
X &= A-B+C-D \\
Y &= A+B-C-D
\end{align*}\]
Local Sensors

- **Piezo Accelerometer** *(Inertial sensor)*
  - Vibration measurement
  - Typical linear range ~ 100~1000 m/s²
  - Typical noise level: 0.5 ~ 50 (μm/s²)/sqrtHz

![Piezo Transducer Diagram](image)
**Local Sensors**

- **Servo Accelerometer** (*Inertial sensor*)
  - Seismic platform control \((f>0.1\text{Hz})\), Vibration measurement

![Diagram](https://via.placeholder.com/150)

Apply force to the suspended mass

=> Keep the distance from a reference

When the control gain \(G>>1\)

=> \(a = \frac{F}{m}\)
Local Sensors

- **Servo Accelerometer** (*Inertial sensor*)
  - Above the resonant freq: Limited by the sensor noise
  - Below the resonant freq: Steep rise of the noise as the mass does not move in relative to the ground
  - => Low resonant freq is beneficial

![Diagram](image)

**Acutuators**

- **Mechanical actuators**
  - Coil Magnet actuator
  - Electro Static Driver (ESD)
  - Piezo (PZT) actuator

- **Optical actuators**
  - Acousto-Optic Modulator
  - Electro-Optic Modulator
  - Laser Frequency
**Acutuators (Mechanical)**

- **Coil-magnet actuator**
  - Coil current induces force on a magnet attached to a mass
  - Contactless
  - aLIGO coil-magnet actuator is integrated in BOSEM
  - Actuator response (coupling) has position dependence. Preferable to use it at its maximum in order to avoid vibration coupling

![Diagram of a coil-magnet actuator](image)

![Graph showing coupling vs. distance](image)
Electro Static Driver (ESD)

- Apply potential close to the mirror
  => induces surface charge (or polarization) and attractive force

- In practice, comb patterns are used
  => strengthen the electric field, but less force range

- Can produce only attractive force. => Need DC Bias.

- Stray surface charging may cause problems.
**Acutuators (Mechanical)**

- **Piezo (PZT) actuator**
  - Apply potential to a ferroelectric material
    - => cause internal polarization and induces strain
  - To increase displacement, laminated piezo is often used
    - => displacement $3 \sim 10 \, \mu m$
  - Requires a bias voltage and HV amplifier, but has wide applications
Acutuators (Optical)

- Acousto-Optic Modulator
  - Phonon-Photon scattering (or bragg diffraction) in AOM crystal
  - Effect: Beam deflection / Frequency shift
  - Application: Laser frequency actuator, Laser intensity actuator
  - Beam angle scanner

[Diagram showing incident beam, propagating sound waves, deflected beam, and transmitted beam]
Electro-Optic Modulator

- **Pockels Cell effect:**
  Refractive index changes linearly to the applied E-field

- **Application:**
  Laser phase modulation
  Phase actuation (= frequency actuation)

LiNbO₃ crystal
**Acutuators (Optical)**

- Laser frequency actuation (YAG NPRO laser)
  - We often control laser frequency with multiple actuators
  - 1) Thermal actuator
    Thermo-Electric Cooler attached to the laser crystal.
    Huge response (1GHz/K or 1GHz/V) but slow (f<0.1Hz)
  - 2) Fast piezo actuator
    A piezo attached on the laser crystal induces stress induced refractive index change.
    Response (~1MHz/V). Bandwidth 10~100kHz
  - 3) External EOM
    Response (~10 mrad/V), Bandwidth ~1MHz
Analog servo filters

- High dynamic range (~1nV/sqrtHz, +/-10V), High bandwidth
- Pole/zero placement with active op-amp filters

- Until the end of the 20th century, analog filters have been commonly used for servo filters in our field

- Analog servos are still in action for the feedback loops with bandwidth >1kHz. (cf. frequency stabilization, intensity stabilization)
Digital servo filters

- Process digitized signals in a computer

- Large flexibility
  - High compatibility with detector automation and management

- Limited dynamic range (~0.1mV/sqrtHz, +/-10V for 16bit)

- Limited bandwidth
  - Each sample needs to be processed before the next sampled data comes
  - Inevitable sampling delay
  - Additional phase delays due to analog filters for analog-digital interface

- e.g. 16kHz sampling, control bandwidth ~200Hz
Restriction of signal digitization

- **Voltage quantization:** quantization noise
  => limited dynamic range
  => Requires whitening/dewhitenening filters

- **Temporally discrete sampling:** aliasing problem
  => limited signal bandwidth
  => Requires anti-aliasing (AA) / anti-imaging (AI) filters

**Typical signal chain**

```
Input Signal → Whitening Filter → Anti-Aliasing Filter → ADC → Anti-Whitening Filter
```

```
Digital Filter → Anti-Dewhitenening Filter → DAC → Anti-Imaging Filter → Dewhitenening Filter → Output Signal
```
Control room

Comparison of the control room in the analog and digital eras

aLIGO (2014)

TAMA300 (2001)
Interferometer sensing and control
**Global control**

- Interferometer control using the main laser beam
  - On the top of the local control, optical path lengths and the mirror alignment need to be kept at the most sensitive state of the interferometer

Pound-Drever-Hall (PDH) technique

- We want to keep the cavity at the TOP of the resonance
- Phase of the cavity reflection is linear to the cavity detuning
- Use modulation / demodulation


http://en.wikipedia.org/wiki/Pound%E2%80%93Drever%E2%80%93Hall_technique
http://www.sjsu.edu/faculty/beyersdorf/Archive/Phys208F07/Sideband%20generation%2oin%20LIGO.pdf
**PDH control signal**

- **Modulation**
  \[ E = E_{in} e^{i2\pi \nu_0 t} \]
  \[ E = E_0 e^{i2\pi \nu_0 t} + E_1 e^{i2\pi (\nu_0 - \nu_m) t} + E_1 e^{i2\pi (\nu_0 + \nu_m) t} \]

- **Phaser diagram**
  \[ E \times e^{-i2\pi \nu_0 t} \]
  Sidebands rotate at the rate of \( \nu_m \)
  Total field shows phase modulation at \( \nu_m \)
PDH control signal

- PDH technique
  - Reflected field on Resonance
  - Detuned

\[\text{Im} \quad \text{Re}\]

**Reflected field on Resonance**
- Total field only has phase modulation at \(\nu_m\)
- No power modulation

**Detuned**
- Total field has amplitude modulation at \(\nu_m\)
- Power modulation at \(\nu_m\)

\[\text{Im} \quad \text{Re}\]

=> Photocurrent demodulated at \(\nu_m\) shows the linear signal to the detuning!
Michelson length control

- Michelson is operated at the dark fringe for the shot noise and the power recycling
  - At the dark fringe, DC signals can’t be a good error signal
  - Schunupp asymmetry:
    Introduce small arm length asymmetry
    => RF sidebands leaks to the dark port

\[ \Delta L = L_A - L_B \]
...In the end, combining these techniques with multiple modulations, the sensing system looks like this.
Lock Acquisition: Real and Simulated

- Transition from non-operational state to the final linear state
  - Nonlinear process ~ Lock Acquisition

Arm powers are normalized by the power when one arm is locked.
SB power is normalized by the input SB power.

- Before the lock, we can’t make a diagnosis of the control loops
  - Without diagnosis, the lock is difficult. (Chicken & egg problem)
Angular global control

- Wave Front Sensing
  - Misalignment between the incident beam and the cavity axis
  - The carrier is resonant in the cavity
  - The reflection port has
    - Prompt reflection of the modulation sidebands
    - Prompt reflection of the carrier
    - Leakage field from the cavity internal mode

E Morrison et al Appl Optics 33 5041-5049 (1994)
Angular global control

- Wave Front Sensing
  - WFS becomes sensitive when there is an angle between the wave fronts of the CA and SB
  - Can detect rotation and translation of the beam separately, depending on the “location” of the sensor
  - Use lens systems to adjust the “location” of the sensors. i.e. Gouy phase telescope
Combine WFS, DC QPD, digital CCD cameras
Feedback control is an indispensable system for laser interferometer GW detectors.

It involves sensors, actuators, and filters, namely everything!

(i.e. Optics, Mechanics, Electronics)