Geometry and Kinematics of Parallel Manipulators

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Geometry and Kinematics of Parallel Manipulators [TEQIP Workshop, IIT Kanpur, 15-19 March, 2016]
Introduction

Position analysis

Two circles

Three circles

Conclusions
A simple serial manipulator: planar 2-R
A comparison with serial manipulators
Basic features of a parallel manipulator

- “Parallel manipulator” is a short form of “in-parallel actuated manipulator”.
- Defining features:
  - More than one point on the manipulator is fixed, i.e., the links form one or more loops between the fixed points on the fixed base platform.
  - Typically, more than one limb meet at an end-effector, which is generally termed as the moving platform.
  - The manipulator has more links than actuators; these additional links are called “passive”, and their positions are determined by solving the loop-closure equations.
Introduction to parallel manipulators

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Example: The 3-RRR planar parallel manipulator
Introduction to parallel manipulators

Kinematic model

- Fixed base: $\Delta b_1 b_2 b_3$
- Moving platform: $\Delta p_1 p_2 p_3$
- Kinematic loops:
  \[ b_1 - p_1 - p_2 - b_2 - b_1, \]
  \[ b_2 - p_2 - p_3 - b_3 - b_2, \]
  \[ b_3 - p_3 - p_1 - b_1 - b_3 \]
- Active variables:
  \[ \theta = (\theta_1, \theta_2, \theta_3)^T \]
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Advantages of parallel manipulators

- High load capacity
- High stiffness
- High accuracy
- High speed
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Disadvantages of parallel manipulators

- Small workspaces, with complicated geometry
- Singularities \textit{inside} the workspace
- Complicated kinematics
- Frequent occurrences of link interference
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Locking/gain-type singularities in parallel manipulators

- Singularities (gain-type) occur inside the workspace, locking the mechanism, and leading to a gain of uncontrollable DoF at the same time!
Forward kinematics of parallel manipulators

- Usually requires the solution of non-linear (i.e., trigonometric/algebraic) equations.
- Generally takes a significant amount of effort to reach at the final result – a single univariate polynomial equation.
- The polynomial can be of moderate to high degree:
  - 5-bar planar manipulator: 2 degrees
  - 3-RRR planar manipulator: 6 degrees
  - 3-RPS spatial manipulator: $2 \times 8$ degrees
  - MaPaMan spatial manipulator: 16 degrees
  - Stewart platform manipulator: 40 degrees
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Additional issues: singularities

- Singularities occur whenever one or more pair(s) of the solutions merge.
- Analysis of singularities is much harder – leads to polynomials of much higher degrees!
  - E.g., 3-RRR, singularities occur on a degree 42 curve in the plane of the manipulator!
- Even if the results are obtained, they cannot always be interpreted geometrically.
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Geometric approach to position analysis

► Offers a visual interpretation of the solutions
► Often leads to an intuitive understanding of the singularities
► Leads to a mathematical formulation in terms of algebraic/trigonometric equations
► Needs a bit of geometric imagination
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The circle in the plane

A circle can be described as *all* of the following:

- Locus of a point moving in a plane, maintaining a *constant* distance from a *fixed* point – e.g., points on a R-jointed rigid link
- An ellipse with zero eccentricity
- In the $XY$ plane, it is a special case of the general quadratic curve/conic section:

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

$$\Downarrow \quad a = b = 1, h = 0$$

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$
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Two circles in the same plane

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Specialties of circles: the “missing” pair of intersections

- Two (non-circular) ellipses can intersect at the most at *four distinct, isolated* points:

- Two circles can intersect only at *two distinct, isolated* points, at the most.
- Question: where did the “missing pair” go to?
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Brief digression: Bézout’s limit

Consider two algebraic curves in the $XY$ plane: $f(x, y) = 0$, and $g(x, y) = 0$. If the total degree of $f, g$ in $x, y$ are $m$ and $n$ respectively, then the maximum number of (isolated) points of intersection of $f, g$ is given by $m \times n$. 
In search of the missing pair

Consider the pair of circles in the real plane, $XY$:

$$C_1 : x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$$
$$C_2 : x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$$

Take them to the projective plane, $XYW$, i.e.,

$$(x, y) \mapsto (X : Y : w), \ w \neq 0, \ x = X/w, \ y = Y/w$$

$$C_1 : X^2 + Y^2 + 2g_1 Xw + 2f_1 Yw + c_1 w^2 = 0$$
$$C_2 : X^2 + Y^2 + 2g_2 Xw + 2f_2 Yw + c_2 w^2 = 0$$
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  \[ C_2 : X^2 + Y^2 + 2g_2Xw + 2f_2Yw + c_2w^2 = 0 \]
In search of the missing pair (contd.)

- If \( w = 0, x = X/w = \infty, y = Y/w = \infty \). Also:

\[
C_1 : X^2 + Y^2 = 0,
\]

\[
C_2 : X^2 + Y^2 = 0.
\]

Therefore, **any two circles in the plane intersect at infinity**, at the pair of points \( Y = \pm iX \), i.e., at \((1 : \pm i : 0)\).

- These two points at infinity account for the missing pair, and take the number of intersection points to the Bézout limit.
Two circles in the same plane

In search of the missing pair (contd.)

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- These two points at infinity account for the missing pair, and take the number of intersection points to the \textit{Bézout limit}.
Finding the real pair of intersections

- Consider $w \neq 0$, i.e., $w = 1 \Rightarrow x = X, y = Y$:

$$C_1 : x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$$
$$C_2 : x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$$

- Perform $C_1 - C_2$:

$$L_{12} : 2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$$
Two circles in the same plane

Finding the real pair of intersections

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- Perform \( C_1 - C_2 \):

\[
L_{12} : 2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0
\]
Finding the real pair of intersections (cont’d.)

- Solving for $y$ from $L_{12} = 0$:

$$y = -(2(g_1 - g_2)x + (c_1 - c_2))/(f_1 - f_2), \text{ assuming } f_1 \neq f_2;$$

- Substituting $y$ in either $C_1$ or $C_2$:

$$Ax^2 + Bx + C = 0.$$
Finding the real pair of intersections (cont’d.)

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Finding the real pair of intersections (cont’d.)
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Finding the real pair of intersections (cont’d.)

\[ C_2 \]

\[ L_{12} \]
Equivalence of *ideals*

\[ C_2 \equiv \langle C_1, C_2 \rangle = \langle C_2, L_{12} \rangle \]
Characterisation of two circles in the plane

Nature of the roots depends on the discriminant:

\[ \Delta = B^2 - 4AC. \]

\( \Delta < 0: \)

\( \Delta = 0: \)

\( \Delta > 0: \)
Two circles in the same plane

Summary of the required results

- Two circles can intersect only at a pair of isolated (real) points.
- Finding this point is equivalent of intersecting a line with one of the circles.
- Any two circles share a pair of points at infinity.
Two circles in the same plane

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Application: planar 5-bar manipulator

Position analysis of the planar 5-bar manipulator

\[ C_1 : (p - a_1)^\top (p - a_1) - r^2 = 0, \]
\[ C_2 : (p - a_2)^\top (p - a_2) - r^2 = 0. \]
Two circles of the planar 5-bar manipulator
Intersection of three circles in the plane

\[ C_i : x^2 + y^2 + 2g_i x + 2f_i y + c_i = 0, \quad i = 1, 2, 3. \]

- Find the common chords, \( L_{ij} = 0: \)
  \[ L_{ij} : C_i - C_j, \quad i \neq j, \quad i, j = 1, 2, 3. \]

- Find the condition for intersection of three chords.
Intersection of three circles in the plane

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Intersection of three circles in the plane (cont’d.)

Three circles in a plane meet at a point if their pair-wise common chords meet at a (i.e., at the same) point:

\[ \langle C_1, C_2, C_3 \rangle = \langle L_{12}, L_{23}, L_{31} \rangle. \]
Recap: condition for three lines in the plane to concur

\[ L_i : a_i x + b_i y + c_i = 0, \; i = 1, 2, 3. \]

- The three lines meet at a point, iff:

\[
\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \det \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = 0.
\]
Position analysis of the 3-RRR manipulator

Three circles: $(p_i - a_i)^	op (p_i - a_i) - r^2 = 0, \ i = 1, 2, 3.$
Three circles of the 3-RRR manipulator
Geometry of the 3-RPS manipulator

- 3-DoF: roll, pitch and heave
- Moving platform: $\Delta p_1 p_2 p_3$
- Base platform: $\Delta b_1 b_2 b_3$
- Active variables: $\theta = (l_1, l_2, l_3)^\top$
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Geometry of the 3-RPS manipulator

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Three circles in space: 3-RPS manipulator

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Circles of constraint on each leg

Spherical constraint: \((p_i - b_i)^\top(p_i - b_i) = l_i^2, \ i = 1, 2, 3.\)
Operation modes and singularities
Position analysis of the 3-RPS manipulator: 3 circles again!
Conclusions

- Position analysis of parallel manipulators can be reduced to simple geometric problems.
- In many cases, it demands the knowledge of geometry not above the level of high-school!
- It is not just about forming and solving equations – it is interesting, and beautiful!
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Further readings

- Robotics: Fundamental Concepts and Analysis, by Ashitava Ghosal
- Parallel Robots, by J.P. Merlet
- NPTEL course on Robotics: http://nptel.iitm.ac.in/courses/112108093/
- A website dedicated to parallel manipulators: www.paralleemic.org