Gas Flow in Complex Microchannels

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Motivation

• Devices for continual monitoring the health of a patient is the need of the hour
• Microdevices are required which can perform various tests on a person
• Microdevice based tests can yield quick results, with smaller amount of reagents, and lower cost

From internet
Motivation

*Slide taken from Dr. A. Bhattacharya, Intel Corp*
Motivation: Electronics cooling

- Heat Flux > 100 W/cm²*: IC density, operating frequency, multi-level interconnects
- Impact on Performance, Reliability, Lifetime
- Hot spots (static/dynamic) and thermal stress can lead to failure
- Strategy to deal with Very High Heat Flux (VHHF) Transients required

*2002 AMD analyst conference report @ AMD.com

<table>
<thead>
<tr>
<th>Channel length</th>
<th>130 nm</th>
<th>90 nm</th>
<th>65 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die Size</td>
<td>0.80 cm²</td>
<td>0.64 cm²</td>
<td>0.40 cm²</td>
</tr>
<tr>
<td>Heat flux</td>
<td>63 W/cm²</td>
<td>78 W/cm²</td>
<td>125 W/cm²</td>
</tr>
</tbody>
</table>
Applications

• Several potential applications of microdevices
  – Example: Micro air sampler to check for contaminants
  – Breath analyzer, Micro-thruster, Fuel cells
  – Other innovative devices

• Relevant to space-crafts, vehicle re-entry, vacuum appliances, etc
# Introduction

Knudsen number \[ Kn = \frac{\lambda}{L} \]

- \( \lambda \) is mean free path of gas
- \( L \) is characteristic dimension
- \(~ 70 \text{ nm for N}_2 \text{ at STP}\)

\( Kn \sim 7 \times 10^{-3} \) for 10 micron channel

<table>
<thead>
<tr>
<th>Flow regimes</th>
<th>( Kn ) Range</th>
<th>Flow Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuum flow</td>
<td>( Kn &lt; 10^{-3} )</td>
<td>Navier-stokes equation with no slip B. C.</td>
</tr>
<tr>
<td>Slip flow</td>
<td>( 10^{-3} \leq Kn &lt; 10^{-1} )</td>
<td>Navier-stokes equation with slip B. C.</td>
</tr>
<tr>
<td>Transition flow</td>
<td>( 10^{-1} \leq Kn &lt; 10 )</td>
<td>Navier-stokes equation fails but intermolecular collisions are not negligible</td>
</tr>
<tr>
<td>Free molecular flow</td>
<td>( 10 \leq Kn )</td>
<td>Intermolecular collisions are negligible as compared to molecular collision to the wall</td>
</tr>
</tbody>
</table>

Gas flow mostly in slip regime – regime of our interest
Introduction

How to determine the slip velocity at the wall?

Maxwell’s model (1869)

\[ u_{\text{slip}} = \left( \frac{2 - \sigma}{\sigma} \right) \lambda \frac{du}{dy} \bigg|_{\text{wall}} \]

\( \sigma \) is tangential momentum accommodation coefficient (TMAC) = fraction of molecules reflected diffusely from the wall

Sreekanth’s model (1969)

\[ u_{\text{slip}} = -C_1 \lambda \frac{du}{dy} \bigg|_{\text{wall}} - C_2 \lambda^2 \frac{d^2u}{dy^2} \bigg|_{\text{wall}} \]

\( C_1, C_2 \): slip coefficients
Maxwell’s Slip Model

\[ u_{\text{slip}} = \left( \frac{2 - \sigma}{\sigma} \right) \lambda \frac{du}{dy} \bigg|_{\text{wall}} \]

\[ \sigma = \frac{\tau_i - \tau_r}{\tau_i} \]

\[ \tau_i = \text{tangential momentum of incoming molecule} \]

\[ \tau_r = \text{tangential momentum of reflected molecule} \]

In practice, fraction of collisions specular, others diffuse

\[ \sigma \text{ is fraction of diffuse collisions to total number of collisions} \]

Specular reflection \((\tau_r = \tau_i; \sigma = 0)\)

Diffuse reflection \((\tau_r = 0; \sigma = 1)\)
Introduction

• New effects with gases
  – Slip, rarefaction, compressibility

• Continuum assumptions valid with liquids
  – Conventional wisdom should apply
  – Similar behavior possible at nano-scales
Outline and Scope

• Flow in straight microchannel
  – Analytical solution
  – Experimental data

• Applicability of Navier-Stokes to high Knudsen number flow regime?

• Flow in complex microchannels (sudden expansion / contraction, bend)
  – Lattice Boltzmann simulation
  – Experimental data

• Conclusions
Solution for gas flow in microchannel

Assumptions:
Flow is steady, two-dimensional and locally fully developed
Flow is isothermal

Governing Equations:

\[ -A \frac{dp}{dx} - \tau_w dx = d \left( \int \rho u^2 dA \right) \]

\[ p = \rho RT \]

\[ p.Kn = \text{const} \]

Integral momentum equation
Ideal gas law

From,
\[ \lambda = \frac{\mu}{p} \sqrt{\frac{\pi RT}{2}} \]
Solution for gas flow in microchannel
(contd.)

Boundary conditions:

\[ u = u_{slip} \] at \( y = 0, H \)

\[ p = p_0 \}

\[ Kn = Kn_0 \} \text{ specified at some reference location, } x = x_0 \]

Solution procedure:

1. Assume a velocity profile (parabolic in our case)
2. Assume a slip model (Sreekanth’s model in our case)
3. Integrate the momentum equation to obtain \( p \)
4. Obtain the streamwise variation of \( u \)
5. Obtain \( v \) from continuity
Solution for gas flow in microchannel  
(\textit{contd.})

**Velocity profile:**

\[
u(x, y) = u(x) \frac{y / H - (y / H)^2 + 2C_1Kn + 8C_2Kn^2}{1/6 + 2C_1Kn + 8C_2Kn^2}
\]

**Pressure can be obtained from:**

\[
\left( \frac{p}{p_0} \right)^2 - 1 + 24C_1Kn_0 \left( \frac{p}{p_0} - 1 \right) + 96C_2Kn_0^2 \log \left( \frac{p}{p_0} \right) +
2 \Re^2 \beta \chi \left\{ 12 C_1Kn_0 \left( \frac{p}{p_0} - 1 \right) + 24C_2Kn_0^2 \left[ \left( \frac{p}{p_0} \right) - 1 \right]^2 - \log \left( \frac{p}{p_0} \right) \right\} = -48 \Re \beta \frac{x - x_0}{H}
\]

where

\[
\Re = \frac{2 \rho u H}{\mu}; \quad \beta = \frac{2}{\pi} Kn_0^2;
\]

\[
\chi = \frac{1}{A} \int \left( \frac{u}{u} \right)^2 dA = \frac{1/30 + 2 / 3C_1Kn + 8 / 3C_2Kn^2 + 4C_1^2Kn^2 + 32C_1C_2Kn^3 + 64C_2^2Kn^4}{(1/6 + 2C_1Kn + 8C_2Kn^2)^2}
\]
Solution for Velocity

Streamwise velocity continuously increases – flow accelerates

Pressure drop non-linear

Lateral velocity is non-zero (but small in magnitude)
Extension of Navier-Stokes to high Knudsen number regime

First-order slip-model fails to predict the Knudsen minima

Navier-Stokes equations seems to be applicable to high Knudsen number (Kn) with modification in slip-coefficients

Questions?

• Can we extend this approach to other cases?

• What are the values of slip coefficients?
  – or tangential momentum accommodation coefficient (TMAC)
    \[ C_1 = \frac{2 - \sigma}{\sigma} \]

• Why does this approach work at all?
Extension of Navier-Stokes to high Knudsen number regime (contd.)

Idea extended to

- Gas flow in capillary (Agrawal & Dongari, IJMNTFTP, 2012)

- Cylindrical Couette flow (Agrawal et al., ETFS, 32, 991, 2008)

Conductance versus Kn

Experimental Data and Curve Fit to Data by Knudsen (1950)
Extension of Navier-Stokes to high Knudsen number regime (contd.)

\[ \mu^e(y) = \frac{\mu \lambda^e(y)}{\lambda} \]

Normalized volume flux versus inverse Knudsen number

Normalized Velocity Profile

Determination of Tangential Momentum Accommodation Coefficient (TMAC)

What does TMAC depend upon?
- Surface material
  - For same pressure drop, will flow rate in glass & copper tubes be different?
- Gas
  - Will nitrogen and helium behave differently, in the same material tube?
- Surface roughness and cleanliness
- Temperature of gas
- Knudsen number

Three main methods to determine TMAC:
- Rotating cylinder method
- Spinning rotor gauge method
- Flow through microchannels

Recall

Specular reflection
($\tau_r = \tau_i; \sigma = 0$)

Diffuse reflection
($\tau_r = 0; \sigma = 1$)
TMAC versus Knudsen number for monatomic & diatomic gases

Monatomic

Diatomic: $\sigma = 1 - \log(1 + Kn^{0.7})$
Summary of Tangential Momentum Accommodation Coefficient

- **TMAC = 0.80 – 1.02** for **monatomic** gases irrespective of Kn and surface material
  - Exception is platinum where $\sigma_{\text{measured}} = 0.55$ and $\sigma_{\text{analytical}} = 0.19$
  - TMAC = 0.926 for all monatomic gases
- **TMAC = 0.86 – 1.04** for **non-monatomic** gases
  - Correlation between TMAC and Kn $\sigma = 1 - \log \left(1 + Kn^{0.7}\right)$
  - Different behavior for di-atomic versus poly-atomic gases?
- **TMAC depends on surface roughness & cleanliness**
  - Effect (increase/ decrease) is inconclusive
- **Temperature affects TMAC**
  - Effect is small for $T >$ room temperature

Boltzmann Equation

- Boltzmann Equation
  \[ \frac{\partial f}{\partial t} + c \cdot \nabla_r f + F \cdot \nabla_c f = J(f, f) \]

- Chapman and Cowling Series
  \[ f = f^{(0)} + Kn f^{(1)} + Kn^2 f^{(2)} \ldots \ldots \ldots \ldots \ldots \]
  - \[ \rho(r, t) = m \int f(r, c, t) \, dc \]
  - \[ u(r, t) = \frac{1}{n} \int cf(r, c, t) \, dc \]
  - \[ e(r, t) = \frac{1}{n} \int \frac{m}{2} C^2 f(r, c, t) \, dc \]

<table>
<thead>
<tr>
<th>O(Kn)</th>
<th>Resulting Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Euler Equations</td>
</tr>
<tr>
<td>1</td>
<td>Navier Stokes Equations</td>
</tr>
<tr>
<td>2</td>
<td>Burnett Equations</td>
</tr>
<tr>
<td>3</td>
<td>Super Burnett Equations</td>
</tr>
</tbody>
</table>
Generalized Equations

\[ \frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0 \]

\[ E = E_t + E_v, \]

\[ F = F_t + F_v, \]

\[ E_t = \begin{bmatrix} q^u \\ q u^2 + p \\ q u v \\ (e_t + p) u \end{bmatrix} \quad Q = \begin{bmatrix} q \\ q u \\ q v \\ e_t \end{bmatrix} \]

\[ F_t = \begin{bmatrix} q v \\ q u v \\ q v^2 + p \\ (e_t + p) v \end{bmatrix} \quad E_v = \begin{bmatrix} 0 \\ \sigma_{11} \\ \sigma_{12} \\ \sigma_{11} u + \sigma_{12} v + q_1 \end{bmatrix} \]

\[ F_v = \begin{bmatrix} 0 \\ \sigma_{21} \\ \sigma_{22} \\ \sigma_{21} u + \sigma_{22} v + q_2 \end{bmatrix} \]

\[ \sigma_{11} = \sigma_{11}^{(0)} + \sigma_{11}^{(1)} + \sigma_{11}^{(2)} \ldots \ldots \]

\[ \sigma_{22} = \sigma_{22}^{(0)} + \sigma_{22}^{(1)} + \sigma_{22}^{(2)} \ldots \ldots \]

\[ \sigma_{21} = \sigma_{12} = \sigma_{21}^{(0)} + \sigma_{21}^{(1)} + \sigma_{21}^{(2)} \ldots \ldots \]

\[ q_1 = q_1^{(0)} + q_1^{(1)} + q_1^{(2)} \ldots \ldots \]

\[ q_2 = q_2^{(0)} + q_2^{(1)} + q_2^{(2)} \ldots \ldots \]
Burnett Equations

- **Navier Stokes And Burnett Shear Stresses**
  - \((\sigma_{12})^E = 0\)
  - \((\sigma_{12})^{N-S} = (\sigma_{21})^{N-S} = -\mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)\)
  - \((\sigma_{12})^B = (\sigma_{21})^B = (\sigma_{21})^{N-S} + \frac{\mu^2}{p} (\beta_1 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \beta_2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} + \beta_2 \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \beta_1 \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \beta_3 R \frac{\partial^2 T}{\partial x \partial y} + \beta_4 \frac{RT}{\bar{q}} \frac{\partial^2 \bar{q}}{\partial x \partial y} + \beta_5 \frac{R \frac{\partial T}{\partial x}}{T} \frac{\partial \bar{T}}{\partial y} + \beta_6 \frac{RT \frac{\partial \bar{q}}{\partial x}}{\bar{q}} \frac{\partial \bar{q}}{\partial y} + \beta_7 \frac{R \frac{\partial T}{\partial y}}{\bar{q}} \frac{\partial \bar{T}}{\partial x} + \beta_7 \frac{R \frac{\partial \bar{T}}{\partial y} \frac{\partial \bar{q}}{\partial x}}{\bar{q} \frac{\partial y}{\partial x}})\)

- **Challenges**
  - Additional Boundary Conditions
  - Non linear constitutive relations
  - Complexity
### Higher Order Continuum Models

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Super Burnett Equations</td>
<td>Augmented Burnett Equations</td>
</tr>
<tr>
<td></td>
<td>(1991)</td>
</tr>
<tr>
<td>Grad’s Moment Equation</td>
<td>R13 Moments Equation</td>
</tr>
</tbody>
</table>

- Have been applied to Shock Waves
- Here, want to apply to Micro flows

Analytical Solution from Higher Order Continuum Equations

- Karniadakis & Beskok (1999)
  - Asymptotic model by introducing $\varepsilon$ as the ratio of transverse and longitudinal length scales
  - Solution is valid for isothermal flows with $Kn<1$

- Stevanovic (2007)
  - Perturbation method used in varying cross-sectional area microchannel with conventional Burnett equations
  - Solution is applicable for $Re \ll 1$ and $Kn \sim \varepsilon^{1/4}$

- Gatignol (2012)
  - Adopted Principle of Least Degeneracy
  - First-order slip model as boundary condition at the walls and
  - The resultant solution is limited to small Mach numbers and small or moderate Knudsen numbers
Numerical Solution of Higher-Order Continuum Equations

• Agarwal et al. (2001): Planar Poiseuille flow; Convergent solution till Kn = 0.2; solution diverged beyond it

• Xue et al. (2003): Couette flow; Kn < 0.18

• Bao & Lin (2007): Planar Poiseuille flow; Kn < 0.4

• Uribe & Garcia (1999); Planar Poiseuille flow; Kn < 0.1
Present Approach

• Start with the solution of the Navier-Stokes equations;
• Substitute the solution in the Burnett equations and evaluate the order of magnitude of additional terms in the Burnett equations;
• Identify the highest order term and augment the governing equation with this additional term, and re-solve the governing equations;
• Repeat above two steps, till convergence is achieved.
Solution of the Equation

- **Streamwise Velocity**
  \[ u(x, y) = \left( \frac{Re \mu RT}{pD_h} \right) \left\{ \frac{\frac{y}{H} - \left( \frac{y}{H} \right)^2 + 2C_1 Kn + 8C_2 Kn^2}{\frac{1}{6} + 2C_1 Kn + 8C_2 Kn^2} \right\} \]

- **Cross-Stream Velocity**
  \[ v(x, y) = \left( \frac{12C_1 Kn + 96C_2 Kn^2}{1 + 12C_1 Kn + 48C_2 Kn^2} \right) \left( \frac{Re \mu RT}{D_h} \right) \left( \frac{1}{p^2} \right) \left( \frac{dp}{dx} \right) \times \left\{ y \left[ 1 - \frac{3y}{H} - 2 \left( \frac{y}{H} \right)^2 + 12C_1 Kn + 48C_2 Kn^2 \right] \right\} \]

- **Pressure**
  \[ \left( \frac{p}{p_o} \right)^2 - 1 + \frac{Re^2 \omega \mu R T}{(D_h^2 p_o^2)} \left\{ 12C_1 Kn_o \left( \frac{p_o}{p} - 1 \right) + 24C_2 Kn_o^2 \left( \left( \frac{p_o}{p} \right)^2 - 1 \right) + \log \left( \frac{p_o}{p} \right) \right\} \]
  \[ + 96C_2 Kn_o^2 \log \left( \frac{p}{p_o} \right) + 24C_1 Kn_o \left( \frac{p}{p_o} - 1 \right) = 96Re \frac{\mu^2 R T}{(D_h^2 p_o^2)} \frac{(x_o - x)}{D_h} \]
Results

Error (in %) versus Knudsen number as given by Burnett equation and Navier-Stokes at the wall ($C_1 = 1.066, C_2 = 0.231$; Ewart et al. (2007))
Slip Velocity

Slip velocity comparison with DSMC data (Karniadakis et al. 2005)

Complex Microchannels

Test section for flow through sudden expansion / contraction

Test section for flow through 90° bend

- Gas – Nitrogen (Helium to a limited extent)
- Developing length is 10% greater than required from calculations
Lattice Boltzmann Simulation
Microchannel with Sudden Contraction or Expansion

1. Pressure drop in each section follows theory
2. Secondary losses are small
3. Limited transfer of information in microchannel

Possible to understand complex microchannels in terms of primary units?

Experimental Setup

- **Vacuum pump**
  - Rotary with speed of 350 lpm
  - Maximum vacuum 0.001 mbar

- **MFC**
  - 0 - 20 sccm
  - 0 - 200 sccm
  - 0 - 5000 sccm

- **APT**
  - 0 – 1 mbar
  - 0 – 100 mbar
  - 0 – 1 bar

Flow parameters range
- $Re = 0.2 – 837$
- $Kn_o = 0.0001 – 0.0605$

The leakage is ensured to be less than 2 % of the smallest mass flow rate. The absolute static pressure along the wall is measured for different mass flow rates of $N_2$ at 300 K.
11 different gas-solid combinations; \( Kn \) range 0.0004-2.77 and \( Re \) range of 0.001-227

\[
f Re = \frac{64}{1 + 14.89 Kn}
\]

\( f Re \) for different combinations of gas-surface material seems small as compared to experimental scatter.

Experimental setup and test section geometry for sudden expansion

![Schematic diagram of the experimental set up](image)

**Figure 1a.** Schematic diagram of the experimental set up

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Maximum uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow rate</td>
<td>± 2% of full scale</td>
</tr>
<tr>
<td>Absolute pressure</td>
<td>± 0.15 % of the reading</td>
</tr>
<tr>
<td>Diameter</td>
<td>± 0.1 %</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>± 2 %</td>
</tr>
<tr>
<td>Knudsen number</td>
<td>± 0.5 %</td>
</tr>
<tr>
<td>Pressure loss coefficient</td>
<td>± 6 %</td>
</tr>
<tr>
<td>Temperature</td>
<td>± 0.3 K</td>
</tr>
</tbody>
</table>

**Flow parameters range**

Re  = 0.2 – 837  
Kn₀ = 0.0001 – 0.075

![Test section geometry for sudden expansion](image)

**Figure 1b:** Test section geometry for sudden expansion
Flow through tube with sudden expansion

Oliveira and Pinho (1997) noted separation at $Re = 12.5$ (AR = 6.76) for liquid flow. Goharzadeh and Rodgers (2009) noted separation at $Re = 100$ (AR = 1.56) for liquid flow.
Radial static pressure variation just upstream and just downstream of the junction for AR = 64.
Comparison with straight tube

Static pressure variation (normalized with the outlet pressure) along the wall.

Sudden Expansion, AR = 12.43
Reₜ = 11.3, Knⱼ = 0.0112, Nitrogen

Junction

Straight
SE

Lu
Ld
Variation in $L_u$ (normalized with smaller section tube diameter $d$) versus Knudsen number at junction ($Kn_j$).

Variation in $L_d$ (normalized with smaller section tube diameter $d$) versus Knudsen number at junction ($Kn_j$).
Velocity profile

(a) Straight tube

- $Re = 4.2$
- $Kn = 0.009$

- Graph showing velocity profile with data points and lines indicating different scenarios.

(b) Sudden expansion

- $Re = 1.6$
- $Kn = 0.0181$

- Graph showing uncertainty bars with ±20% variation.

- Different lines representing various experimental conditions (e.g., $X = 0, SE$, $X = 0.050, SE$, $X = 0.077, SE$).

- Note: By Pitot tube indicates a method of pressure measurement used in fluid dynamics.
Analysis of velocity distribution

\[ (p_1 - p_2)A - \tau_w \pi Ddx = [M_2 - M_1] \]

\(p_1, p_2,\) known from experimental measurements, Need velocity distribution for calculating shear stress and momentum \(M_1\) and \(M_2\).

Assuming velocity distribution as second order parabolic velocity profile

\[ u = a + cr^2 \]

The mass flow rate can be calculated as

\[ \dot{m} = \int_0^R \rho 2\pi rdu \]

At \(r = R,\) velocity at the wall \(u = u_s\)
Streamlines near the junction for rarefied gas flow.
Streamlines near sudden expansion junction

Figure 13b: Schematic streamlines of rarefied gas near junction.

Figure 13c: Schematic streamlines of laminar, incompressible, separated flow (Oliveira and Pinho, 1997; Lee et al. 2002) near junction.
Bend Microchannel (Lattice Boltzmann Simulations)

Streamlines near the bend for $Kn = 0.202$

Presence of an eddy at the corner for $Re = 2.14!$
Mass flow rate

Mass flux in bend to mass flux in straight can be up to 2% more than that in straight under same condition.

White et al. (2013) found same result using DSMC.

Experimental data for bend

Test section for flow through 90° bend
Heat Transfer Coefficient
(Experimental setup)
Nusselt number measurements
(Circular tube with constant wall temperature BC)

\[
K_n \approx 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4
\]

\[
Re \text{ range } (0.5-124.3)
\]

\[
Gr / Re^2 < 10^{-4}
\]

---

**Nu versus Kn**

**Nu versus Re**
Comparison against theoretical values (at Kn = 0.01)

<table>
<thead>
<tr>
<th>Source</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larrode et al. [5]</td>
<td>3.6</td>
</tr>
<tr>
<td>Ameel et al. [10]</td>
<td>3.75</td>
</tr>
<tr>
<td>Tunc and Bayazitoglu [12]</td>
<td>3.55</td>
</tr>
<tr>
<td>Aydin and Avei [20]</td>
<td>3.55</td>
</tr>
<tr>
<td>Hooman [21]</td>
<td>8.58</td>
</tr>
<tr>
<td>Present (Experimental)</td>
<td>0.00166</td>
</tr>
</tbody>
</table>

Our experimental data predicts 3 orders of magnitude smaller value as compared to theoretical analysis!!
Validation & Repeatability

<table>
<thead>
<tr>
<th>Re</th>
<th>Nu from Dittus-Boelter Correlation</th>
<th>Nu from the present experiment</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>10870</td>
<td>34.3</td>
<td>38.3</td>
<td>12</td>
</tr>
<tr>
<td>8420</td>
<td>28.0</td>
<td>25.0</td>
<td>11</td>
</tr>
</tbody>
</table>
Why is heat transfer coefficient so small?

- Rarefaction reduces ability of gas to remove heat from wall
- Temperature jump at wall
  \[ T_g - T_w = \frac{2 - \sigma_T}{\sigma_T} \left[ \frac{2\gamma}{1 + \gamma} \right] Kn \left( \frac{\partial T}{\partial n} \right)_w \]
- Radial convection may be important (i.e., \( v \frac{\partial T}{\partial r} \) term comparable to \( u \frac{\partial T}{\partial x} \)), but ignored in theoretical analysis
- Reason for difference is still not fully understood
Conclusions

• Possible to derive solution for microchannel flow for higher-order slip-model
• Possible to extend the applicability of N-S equations by modification of slip coefficients
• TMAC of monatomic gas = 0.93 (independent of temperature and surface and $Kn$)
• Flow in complex microchannel exhibits non-intuitive behavior
Acknowledgements

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**Funding:** IIT Bombay, ISRO
Effect of sudden expansion on velocity distribution

Rate of increase in shear stress and momentum is more in sudden expansion than in straight tube towards the expansion junction

Varade et al. (2012) (to be submitted)
Maxwell’s Slip Model

\[ u_{slip} = \left( \frac{2 - \sigma}{\sigma} \right) \lambda \frac{du}{dy}_{wall} \]

\[ \sigma = \frac{\tau_i - \tau_r}{\tau_i} \]

\( \tau_i \) = tangential momentum of incoming molecule

\( \tau_r \) = tangential momentum of reflected molecule

In practice, fraction of collisions specular, others diffuse

\( \sigma \) is fraction of diffuse collisions to total number of collisions

Specular reflection
\( (\tau_r = \tau_i; \ \sigma = 0) \)

Diffuse reflection
\( (\tau_r = 0; \ \sigma = 1) \)
DSMC Simulations
(Couette flow)

Slip from DSMC/ Slip from Maxwell model versus Knudsen number

Slipratio

Kn(log scale base=10)
DSMC Simulations
(Couette flow)

Slip from DSMC/ Slip from Maxwell model
versus
Knudsen number
Microchannel with Sudden Contraction or Expansion
(Pressure distribution)
Microchannel with sudden Contraction or Expansion

(Velocity distribution)
What do we learn?

1. Good agreement with theory
2. Pressure drop in each section follows theory
3. Secondary losses are small
4. Compressibility and rarefaction have opposite effects
5. Limited transfer of information in microchannel

Possible to understand complex microchannels in terms of primary units?
TMAC versus Knudsen number
Figure 2. Comparison of proposed sol. with the exp. data of Ewart et al. (2007) and the sol. of linearized Boltzmann equation by Cercignani et al. (2004). (C₁ = 1.1466 and C₂ = 0.5, Chapman & Cowling (1970).
For flow to be compressible: \[ \frac{1}{\rho} \frac{D\rho}{Dt} = 0 \] (Gad-el-Hak, 1999)

i.e., both spatial and temporal variations in density are small as compared to the absolute density.

Compressibility effect is important because of large pressure drop in microchannel.
Gas versus liquid flow

• New effects with gases
  – Rarefaction, slip, compressibility

• Continuum assumptions valid with liquids
  – Conventional wisdom should apply
  – Similar behavior possible at nano-scales
Velocity measurement in rarefied gas flow through tube

For low Re the magnitude of viscous forces are comparable with inertia forces hence Bernoulli formula needs correction

\[ Cp = \frac{P_o - P_s}{\frac{1}{2} \rho u^2} \]
<table>
<thead>
<tr>
<th>Author</th>
<th>Cp (Pitot tube geometry)</th>
<th>Re range (Re is based on pitot tube ID)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homann (1952)</td>
<td>$1 + \frac{8}{(Re + 0.64Re^{1/2})}$ Cylinder</td>
<td>3.2 - 120</td>
<td>Experimental</td>
</tr>
<tr>
<td>Hurd et al. (1953)</td>
<td>$\frac{11.2}{Re}$ Circular blunt nosed</td>
<td>1.7 - 100</td>
<td>Experimental</td>
</tr>
<tr>
<td>Lester (1961)</td>
<td>$\frac{2.975}{Re^{0.43}}$ Circular</td>
<td>1 - 10</td>
<td>Numerical</td>
</tr>
<tr>
<td>Chebbi and Tavoularis (1990)</td>
<td>$\frac{4.2}{Re}$ Circular</td>
<td>&lt; 1</td>
<td>Experimental</td>
</tr>
</tbody>
</table>
Velocity measurement in rarefied gas flow through tube

Variation in $C_p$ with $Re$ (based on Pitot tube ID)

Variation in central velocity with $Re$ (based on Pitot tube ID)
Velocity measurement in rarefied gas flow through tube

% deviation in central velocity with reference to Sreekanth (1969)
Velocity measurement in rarefied gas flow through tube

Comparison of experimental velocity measurement with Sreekanth (1969), Homann’s correction
Velocity measurement in rarefied gas flow through sudden expansion using Pitot tube

Continuum flow

Sudden expansion, AR = 3.74, 
Res = 363, Kn = 0.00023
Nitrogen

Uncertainty bars with ± 20%

Study of rarefied gas flow through microchannel
Velocity measurement in rarefied gas flow through sudden expansion using Pitot tube

**Slip flow**

![Graph showing velocity measurement in rarefied gas flow](image)

- Sudden expansion, $AR = 3.74$
- $Re = 1.6$, $Kn = 0.0181$
- Nitrogen

Uncertainty bars with ± 20%

Study of rarefied gas flow through microchannel