Mechanics of Soft Materials

Tuesday and Thursday
L13, 2:00-3:30 PM
What Are Soft Materials?

- Rubber
- Skin
- Tissue
- Paper
- Gel … Modulus < 10 MPa

Deformation when subjected to load

Type of load

Geometry: micro-nano structures

Rheological properties

Interfacial properties
Soft Materials have modulus of the order of **few Pa** to few **MPa**

<table>
<thead>
<tr>
<th>Material</th>
<th>Shear Modulus (10^9 N/m², GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acrylic</td>
<td>3.2</td>
</tr>
<tr>
<td>Aluminum</td>
<td>69</td>
</tr>
<tr>
<td>Bone</td>
<td>9</td>
</tr>
<tr>
<td>Brasses</td>
<td>100 - 125</td>
</tr>
<tr>
<td>Bronzes</td>
<td>100 - 125</td>
</tr>
<tr>
<td>Reinforced Plastic</td>
<td>150</td>
</tr>
<tr>
<td>Concrete</td>
<td>30</td>
</tr>
<tr>
<td>Diamond</td>
<td>1,050 - 1,200</td>
</tr>
<tr>
<td>Glass</td>
<td>50 - 90</td>
</tr>
<tr>
<td>Magnesium (grain)</td>
<td>45</td>
</tr>
<tr>
<td>Polycarbonate</td>
<td>2.6</td>
</tr>
<tr>
<td>Polyethylene HDPE</td>
<td>0.8</td>
</tr>
<tr>
<td>Terephthalate PET</td>
<td>2 - 2.7</td>
</tr>
<tr>
<td>Polyimide</td>
<td>2.5</td>
</tr>
<tr>
<td>Polypropylene</td>
<td>1.5 - 2</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>3 - 3.5</td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>450</td>
</tr>
<tr>
<td>Titanium Alloy</td>
<td>105 - 120</td>
</tr>
<tr>
<td>Tungsten</td>
<td>400 - 410</td>
</tr>
<tr>
<td>Tungsten Carbide</td>
<td>450 - 650</td>
</tr>
<tr>
<td>Wrought Iron</td>
<td>190 - 210</td>
</tr>
<tr>
<td>Nylon</td>
<td>2 - 4</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.01 - 0.1</td>
</tr>
<tr>
<td>Gels</td>
<td>10^-6 - 10^-3</td>
</tr>
</tbody>
</table>

**Elastic**
- Gaskets
- Sealants
- Adhesives
- Skin
- Soft Patterning
- Solid lubricants

**Viscoelastic**
- Prosthetic devices
- Drug delivery devices
- Packaging Materials
- Components of automobile

**Poroelastic**
- Soft robotic components
- Artificial Tissue
- Functional surfaces
- Extracellular Matrix
Bio-inspired Patterned Adhesives:

- Hierarchical structure
- Strong and reusable adhesion
- Adheres to almost all surfaces
- Self-cleaning
- Does not leave any residue
- Easy release during locomotion


Crack arrest, crack initiation
Enhancement of adhesion by ~10 times

Adhesive suitable for dry and wet adhesion and delivery of drugs and nutrients

- Reversible adhesion: Clean adhesion
- Good mechanical strength: ability to sustain large deformation
- Multi-functionality: adhesion and drug delivery
- Biocompatibility and biodegradability
- Resistance against particulate contamination
- Amenable for easy wash or cleaning
Inspiration from naturally occurring Adhesives

- **Release of water repellent protein molecules at interface**
- **Adhesion on dry and wet substrate**
- **van der Waals interaction in dry state**
- **Swelling of keratin protein of setae in moist environment**

A novel adhesive patch satisfies these requirements

![Under water adhesion of mussel](image)

![Gecko inspired Adhesive](image)

![Graph of Vitamin C solution-gelatin volume ratio](image)
Blood Vessels in Climbing Organ of Insects:

Adult Rhodnius could climb the glass walls of the jars ... ability to climb smooth surfaces was due to the existence in the adult insects of a **flashy pad** situated at the lower end of tibia of the first two pair of legs.

Air Pockets at the Adhesive Pads of Insects

Attachment pad of *Tettigonia viridissima*

AS: Air sack
CL: Epidermal cell layer
EXO: Rod containing exo-cuticle of the pad
HM: Haemolymph
TD: Tendon of the claw flexor muscle
TK: Tanned cuticle
Peeling off a Microfluidic Adhesive

Peeling Torque:

\[ G = \int F \cdot d\Delta / A_{peeled} \]

\[ \Delta, \text{ mm} \]

\[ M, \text{ Nm/m} \]

\[ h, \mu m \quad d, \mu m \]

1: smooth adhesive
2: 120 cp
3: 1000 cp
4: 5000 cp
5-10: 380 cp

25-30 times enhancement in adhesion
Asymmetry Induced by Pair of Embedded Channels: Differently filled with wetting liquid

$t = 50 \mu m, s_1 = 15 \mu m, d = 550 \mu m$

Majumder et al *Soft Mat.*, 2012
Dynamic Change in Surface Profile During Separation of Adherent:
Adhesion Between a Flexible Plate on a Layer of Adhesive Bonded to a Rigid Substrate:

Adhesive: Elastic, Incompressible,Thin
Adherent: Thin, Flexible & Rigid Plate
Stress Equilibrium Relations:

\[
p_x = \mu (u_{xx} + u_{yy} + u_{zz})
\]

\[
p_y = \mu (v_{xx} + v_{yy} + v_{zz})
\]

\[
p_z = \mu (w_{xx} + w_{yy} + w_{zz})
\]

\[u, v, w\] are displacements in the \[x, y\] and \[z\] direction respectively

Incompressibility relation:

\[u_x + v_y + w_z = 0\]
Plane Strain Approximation:

\[ e_{yy} = e_{xy} = e_{zy} = 0 \]

Stress equilibrium relation

\[ \frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \]

\[ \frac{\partial p}{\partial z} = \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) \]

Incompressibility relation

\[ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \]
**Dimensionless Quantities:**

\[ x = X \cdot L, \quad z = Z \cdot h, \quad u = U \cdot L, \quad w = W \cdot h \]

\[ \varepsilon = h / L \]

\[ p = \mu / \varepsilon^2 \]

\[
\frac{\mu}{(h/L)^2} \frac{1}{L} \frac{\partial P}{\partial X} = \mu \left( \frac{1}{L} \frac{\partial^2 U}{\partial X^2} + \frac{L}{h^2} \frac{\partial^2 U}{\partial Z^2} \right)
\]

\[
\frac{\mu}{(h/L)^2} \frac{1}{h} \frac{\partial P}{\partial Z} = \mu \left( \frac{h}{L^2} \frac{\partial^2 W}{\partial X^2} + \frac{1}{h} \frac{\partial^2 W}{\partial Z^2} \right)
\]

**Dimensionless Stress-Equilibrium relation:**

**Lubrication Approximation:**

\[ \varepsilon \ll 1 \]

\[
\frac{\partial P}{\partial X} = \varepsilon^2 \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Z^2} \approx \frac{\partial^2 U}{\partial Z^2}
\]

\[
\frac{\partial P}{\partial Z} = \varepsilon^4 \frac{\partial^2 W}{\partial X^2} + \varepsilon^2 \frac{\partial^2 W}{\partial Z^2} \approx 0
\]
Boundary Conditions for Film:

\[ u(z = 0) = w(z = 0) = 0 \]
\[ u(z = h) = 0, \quad w(z = h) = \psi \]

at \( 0 < x < a \)

\[ \sigma_{xz} \bigg|_{z=h} = \sigma_{zz} \bigg|_{z=h} = 0 \]

at \( x < 0 \)

\[ p(z = h) = D \frac{d^4 \psi}{dx^4} \]
\[
\begin{align*}
\frac{\partial p}{\partial x} &= \mu \frac{\partial^2 u}{\partial z^2} \\
\frac{\partial p}{\partial z} &= 0 \\
\frac{1}{\mu} \frac{\partial p}{\partial x} &= 2z^2 + A z + B \\
B &= 0, \quad A = -\frac{h}{2\mu} \frac{\partial p}{\partial x} \\
\frac{\partial w}{\partial z} &= -\frac{\partial u}{\partial x} = -\frac{1}{2\mu} \frac{\partial^2 p}{\partial x^2} (z^2 - zh) \\
w &= -\frac{1}{2\mu} \frac{\partial^2 p}{\partial x^2} \left( \frac{z^3}{3} - \frac{z^2 h}{2} \right) + C \\
\psi &= \frac{Dh^3}{12\mu} \frac{\partial^6 \psi}{\partial x^6}
\end{align*}
\]
Equation for Plate:

\[
\psi = \frac{Dh^3}{12\mu} \frac{\partial^6 \psi}{\partial x^6} \quad x < 0
\]

\[
0 = \frac{\partial^4 \psi}{\partial x^4} \quad 0 < x < a
\]
Boundary Conditions for Plate:

(i) \( \psi \bigg|_{x=0^-} = \psi \bigg|_{x=0^+} \)

(ii) \( \frac{\partial \psi}{\partial x} \bigg|_{x=0^-} = \frac{\partial \psi}{\partial x} \bigg|_{x=0^+} \)

(iii) \( \frac{\partial^2 \psi}{\partial x^2} \bigg|_{x=0^-} = \frac{\partial^2 \psi}{\partial x^2} \bigg|_{x=0^+} \)

(iv) \( \frac{\partial^3 \psi}{\partial x^3} \bigg|_{x=0^-} = \frac{\partial^3 \psi}{\partial x^3} \bigg|_{x=0^+} \)

(v) \( \frac{\partial p}{\partial x} \bigg|_{x=0} = 0 \)

(vi) \( \frac{\partial^2 \psi}{\partial x^2} \bigg|_{x=a} = 0 \)

(vii) \( -D \frac{d^3 \psi}{dx^3} \bigg|_{x=a} = F \)

or

(viii-x) \( \psi \bigg|_{x=-\infty} = \psi_x \bigg|_{x=-\infty} = \psi_{xx} \bigg|_{x=-\infty} = 0 \)

\( \psi \bigg|_{x=a} = \Delta \)
Displacement Field in Adhesive Film:

\[ u(x, z) = \frac{6z(z-h)F''}{kh^3} \phi_1(x) \]

\[ w(x, z) = \frac{z^2(3h-2z)F''}{h^3} \phi_2(x) \]

\[ \phi_1(x) = e^{\frac{kx}{2}} \left( ak e^{\frac{kx}{2}} + \frac{3ak + 4}{\sqrt{3}} \sin \left( \frac{\sqrt{3}kx}{2} \right) - ak \cos \left( \frac{\sqrt{3}kx}{2} \right) \right) \]

\[ \phi_2(x) = e^{\frac{kx}{2}} \left( ak e^{\frac{kx}{2}} + \frac{3ak + 2}{\sqrt{3}} \sin \left( \frac{\sqrt{3}kx}{2} \right) + (ak + 2) \cos \left( \frac{\sqrt{3}kx}{2} \right) \right) \]

\( k^{-1} \) and \( F'' \) have dimension of length and are constants
Displacement of Plate and Normal Stress at Interface:

\[
\frac{\sigma_{zz}}{D k^4 F'} = e^{kx/2} \left( a k e^{kx/2} + \frac{3ak + 2}{\sqrt{3}} \sin\left(\frac{\sqrt{3kx}}{2}\right) + (ak + 2) \cos\left(\frac{\sqrt{3kx}}{2}\right) \right)
\]

\[
F' = \frac{3\Delta}{6 + 12ak + 9(ak)^2 + 2(ak)^3}, \quad k^{-1} = \left(\frac{Dh^3}{12\mu}\right)^{1/6}
\]
Adhesive with Spatially Varying Thickness and Modulus

In-phase variation of thickness and modulus

\[ h(x) = h_0 \phi(x) = h_0 (1 + \delta_h \sin(k_h x)) \]
\[ \mu(x) = \mu_0 f(x) = \mu_0 (1 + \delta_\mu \sin(k_\mu x)) \]

Out-of-phase variation of thickness and modulus

Ghatak, Phys. Rev. E, 2010
Adhesive with Phase Lag Between Thickness and Modulus Variation:

\[ k_\mu = k_h = 1.25q, \ \delta_h = 0.005, \ \delta_\mu = 0.9 \]

Non-monotonic variation in torque with phase lag.
Typical Content of Course:

1. Brief Introduction: Total 3
   - Definition of strain, strain tensor, stress, stress tensor, Saint Venant’s principle 1
   - Hooke’s law, stress equilibrium relations 1
   - One dimensional stretching of a rod 1

2. Solid bodies in contact with and without interactions: Total 9
   - Line loading of an elastic half space, distributed loading 1
   - Axisymmetric loading of an elastic half space 1
   - Normal contact of elastic solids: 2
     - Hertzian theory: 1
     - Contact with adhesion, JKR theory 3
   - Compression of an elastic layer between two parallel plates 1

3. Equilibrium of rods and plates: Total 12
   - Equations of equilibrium of rods 2
   - Euler’s buckling instability 1
   - Twisting instability of rods 1
   - Equation of equilibrium for a thin bent plate 1
   - Longitudinal deformation of plates 1
   - Large deflection of plates 1
   - Contact of two rigid or flexible Adherents 3
   - Analysis of wrinkling instability 1
   - Elasticity of an interfacial particle raft 1
4. Nonlinear elasticity: Total 8
   Molecular approach to rubber elasticity 1
   Neo-Hookean elasticity 1
   Analysis of large deformation of an incompressible elastic material 4
   Inflation of a balloon 1
   Cavitation in crosslinked networks 1

5. Mechanics of cell wall: Total 8
   Entropic elasticity-stretching, bending and twisting, persistence length 2
   Mechanics of cellular filaments; 2D and 3D networks in cell 2
   Polymerization and the generated force 2
   Biomembranes, membrane undulations 2.

Text book:

- A treatise on the mathematical theory of elasticity by A. E. H. Love.
- Contact Mechanics by K. L. Johnson.
- Mechanics of the cell by David Boal.
- Research Papers published in variety of journals.
Thank You