MTH666A: Category Theory

First Course Sheet

1. Objectives: The course is an introduction to category theory which is a language for many branches of modern mathematics, giving the student an exposure to the concept of abstraction, and emphasizing on the common ideas in various areas like algebra, topology, analysis and geometry.
2. Prerequisites: Basic understanding of the concepts from naïve set theory, and familiarity with the basic notions of linear algebra/group theory.
3. Course Contents:
    1. Structure vs. property: monoids, groups, preorders, partial orders; structure preserving maps: homomorphisms, continuous maps; category; hom-sets and duality; functor-covariant and contravariant; natural transformations. (3 lectures)
    2. Small, locally small and large categories; set theory vs category theory; Russell's paradox and the category of all categories; skeletons and axiom of choice. (1 lecture)
    3. Isomorphism; groupoid; monomorphism and epimorphism; full, faithful, essentially surjective functors, equivalence of categories; building even more categories from the old ones: slice categories and local property, comma categories, functor categories, congruence and quotients. (3 lectures)
    4. Representable functors; Yoneda lemma and Yoneda embedding; separating and detecting families; injective and projective objects; representables are projective. (3 lectures)
    5. Adjunctions: definition and examples; initial and terminal objects; relation to comma categories; composition of adjoint functors; units, counits and characterization of adjoint functors; equivalence gives adjoint functors. (4 lectures)
    6. Categorical properties: initial and terminal objects, products, coproducts; diagrams and (co)cones; (co)limits of a given shape; (co)equalizers, regular mono(epi)morphisms; pullbacks and pushouts; direct and inverse limits. (2 lectures)
    7. Constructing all (co)limits from some of them; (co)complete categories; absolute (co)limits; preservation, creation and reflection of (co)limits; right adjoints preserve limits. (2 lectures)
    8. General adjoint-functor theorem; well-powered categories; special adjoint functor theorem; examples. (3 lectures)
    9. Definition of monads; monads arising from adjunctions; algebras for a monad; Eilenberg-Moore category; Kleisli category; Kleisli is initial while Eilenberg-Moore is terminal. (2 lectures)
    10. Monadic functors; Beck's monadicity theorem; Crude monadicity theorem; examples. (3 lectures)
    11. Special categories and applications: toposes, additive and abelian categories, symmetric monoidal categories, model categories. (6 lectures)
    12. Solutions to example sheets. (8 lectures)

4. Lecture & Venue: M 6-7:15 PM, Th 5-6:15 PM
5. Office Hours: Mondays 4-5PM
6. Evaluation Components & Policies: Mid-Sem Exam (20%), End-Sem Exam (30%), 4 Assignments (25% in total), Presentation (25%); Grading is relative.

7. Course Policies: Attendance in all lectures is recommended. If an exam is missed for medical reasons, the instructor must be informed immediately along with appropriate medical certificate.
8. Books & References:

1. S. MacLane, Categories for the working mathematician, Vol. 5, Springer Science & Business Media, 2013.
2. S. Awodey, Category theory, Oxford University Press, 2010.
3. M. Kashiwara and P. Schapira, Categories and sheaves, Vol. 332, Springer Science & Business Media, 2005.
4. J. Ada'mek, H. Herrlich and G. E. Strecker, Abstract and concrete categories– The joy of cats, 2004.
5. F. Borceaux, Handbook of categorical algebra 1: Basic category theory, Vol. 50, Encyclopedia of Mathematics and its Applications, 1994.