MTH 613A Rings and Modules
Instructor: Preena Samuel

Schedule: M 1600-1730hrs, T 1430-1600hrs, FB557

Course Objectives: Ring theory is a fairly vast area of mathematics that encompasses almost every realm of research in modern mathematics. Though the study of rings is interesting in its own right, its progress has been driven by its applicability to various other advanced mathematical topics. For example, most of the progress made in the study of commutative rings can be attributed to the progress made in geometry and number theory. As these fields required more and more advanced machinery, along the way commutative rings have grown. On the other hand, the large class of non-commutative rings have developed over the years essentially owing to representation theory. Non-commutative rings have gained lot of interest in the recent years. The study of quantum groups is an active area of research which draws much of its foundation from the theory of non-commutative rings. The theory of non-commutative rings, in turn, owes much of its progress to the understanding of representations of groups. This motivates the study of modules. In fact, it would be right to say that the study of modules is an integral and most significant part in the study of non-commutative rings

The main objective of this course is to introduce the theory of modules and thereby facilitate the study of non-commutative rings.

Contents:
1. Introduction to basic definitions and concepts in ring theory: notion of subrings, ideals, homomorphisms, quotients, direct sums, products.
2. Interesting examples of rings: a) commutative rings - fields, domains, E.D, PID, UFD
   b) Matrix rings, Endomorphism rings, group rings, algebras
3. Module theory: definitions, submodules, homomorphisms, new from old (quotients/ direct sums/ products),
4. Noetherian, Artinian, finite length modules, Jordan-Holder series, Free modules, torsion modules, compare and contrast to vector spaces, modules over PID, structure of f.g. modules over PIDs, artinian rings, noetherian rings, Cohen's theorem.
5. Categorical framework for modules and associated constructions, Tensor product, Hom-tensor adjointness, extension of scalars and restrictions.
6. Simple modules, semi-simple modules, Schur's lemma, Jacobson's density theorem, commutant, bi-commutant, double centraliser theorem, structure theorem for semi-simple rings.
7. Group algebras, modules over group algebras as representations, Maschke's theorem.
8. Module theory for arbitrary rings, socle, radical, isotypical components.
9. Consequences of some of the above theory drawn in the case of specific group algebras.

References:
Rings and Modules, C Musili.
Representation theory of finite groups (Chapters1,2)- C. Musili,
Abstract algebra - Dummit and Foote,
Algebra I,II - Bourbaki,
Basic Algebra - Jacobson,
Groups and representations, Alperin.

Evaluation: There will be a mid-semester and end-semester exam. There would be 2-3 quizzes, one of which would be scheduled before the mid-semester exam. (Some of the quizzes may be take-home assignments.) The weightage for (quizzes/assignments)-midsem-endsem would be 35:30:35.

Course policy:

A. Attendance: It is strongly recommended that the students attend all the classes but there is no formal attendance requirement. (It may be noted here that SOME of the quizzes may be conducted without prior notice. Absence from class will not be accepted as an excuse unless informed formally, in advance.)

B. Honesty Practices: Any dishonest practice will be dealt with strictly and reported to DOAA office for appropriate action.

C. Course drop: <u>After the add-drop period (i.e. after 8th August 2017) requests for course-drop will not be encouraged.</u>