Department of Mathematics and Statistics, IIT Kanpur
MTH 612A (An Introduction to Commutative Algebra),
Monsoon Semester 2017-18
Instructor: Somnath Jha, jhasom@iitk.ac.in
Schedule - Wednesday, Friday 9:00 -10:30, T207
Office hours 10:30 -11:30 Wednesday & Friday

Objective:

Commutative Algebra is the study of commutative rings. A typical example of this is the polynomial ring in *n*-variables. When the coefficients ring of this polynomials is ***Z***, the ring of integers, the polynomial ring encodes number theoretic properties. On the other hand, polynomial ring over fields is a rich source of study in algebraic geometry and in turn also contributes to the understanding of algebraic number theory. The study of commutative algebra is also deeply related to the understanding of homological algebra, which is important in algebraic topology.

The objective of this course is to get familiar with the basic objects in commutative algebra at the level of the classic book of Atiyah-Macdonald on the subject.

Prerequisites:

Instructor's consent; however it will be assumed that the student is comfortable in abstract algebra at a level of MTH204. In particular, it will be assumed that the students are familiar with: ideals and quotient rings, UFD, PID, ED, Gauss lemma and Chinese Remainder Theorem.

Contents:

I. Review of some basics notion of ring theory: notion of ideals, homomorphisms, quotients, direct sums, products, nilradical and Jacobson radical, various operations on ideals.
II. Modules: definitions, submodules, homomorphisms, quotients, direct sums and products, finitely generated modules, review of modules over PID. Exact sequence of modules, tensor product of modules. Algebras, tensor product of algebras. Rings and modules of fraction, localization.
III. Noetherian rings: Chain condition, noetherian modules, primary decomposition, artinian modules, length of a module.
IV. Integral extensions: Integral dependence, going up theorem, integrally closed domains, going down theorem, Noether's normalization lemma, Hilbert Nullstellensatz.
V. Valuation: valuation rings, Dedekind domain, Discrete Valuation Rings.
VI. If time permits Completions: Topologies and completions, Filtration, *I*-adic completion, graded rings and modules, associated graded rings.
VII. If time permits Homology: Complexes, derived functors, Ext and Tor.

Evaluation:

There will be

1. one mid-semester exam of 35 points.
2. (one) end-semester exam of 35 points.
3. 2 quizzes of 15 points each; one before the mid-semester exam and one after the mid semester exam. One of the quiz may be a surprise quiz.
4. In addition, periodically assignment problems will be given. Students are strongly recommended to solve all of them at home, but it is not required to submit them for correction to the instructor.

Course Policies:

A. Attendance: It is strongly recommended that the students attend all the classes but there is no formal attendance requirement.

B. Honesty Practices: Any dishonest practice will be reported to DOAA office for appropriate action.

C. Withdrawal: As per DOAA guideline.

References:

- *Introduction to Commutative Algebra*, M. Atiyah & I. Macdonald, Addison-Wesley.
- *Commutative Algebra*, N.S. Gopalakrishnan, Oxonian Press.
- *Undergraduate Commutative Algebra*, M. Reid, Cambridge University Press.
- *Abstract Algebra,* D. Dummit & R. Foote, John Wiley & Sons.
- *Algebra*, S. Lang, Springer GTM 211.
- *Commutative Ring Theory,* H. Matsumura, Cambridge University Press.
- *Commutative Rings,* I. Kaplansky, The University of Chicago Press.
- *Commutative Algebra with a view toward Algebraic Geometry,* D. Eisenbud, Springer GTM 50.