MTH302A: Set Theory and Logic

First Course Sheet

1. Objectives: The course is an introduction to set theory and mathematical logic, giving the student an exposure to the foundations of mathematics, and indicating how various mathematical theories dealt with in other courses are examples of formal logical systems. A part on Boolean algebras introduces the student to lattice theory as well.
2. Prerequisites: MTH102, MTH202A for M.Sc. 2 yr
3. Course Contents: Set Theory: Some basics of set theory. Functions and sets of functions. Families of sets. Schroder-Bernstein theorem. Cartesian products of families. Relations. Partitions. Countable and uncountable sets. Cantor's theorem. Boolean algebras. Order relations. Boolean algebras as partially ordered sets. Atoms, Homomorphism, sub-algebra. Filters. Stone's representation (sketch). Principles of weak and strong mathematical induction and their equivalence. Axiom of choice, Well-ordering theorem, Zorn's lemma and their equivalence; illustrations of their use. Well-ordering principle and its equivalence with principles of weak and strong induction. (14 lectures)

   Classical propositional calculus (PC): Syntax. Valuations and truth tables, Truth functions, Logical equivalence relation. Semantic consequence and satisfiability. Compactness theorem with application. Adequacy of connectives. Normal forms. Applications to Circuit design. Axiomatic approach to PC: soundness, consistency, completeness. Other proof techniques: Sequent calculus, Computer assisted formal proofs: Tableaux. Decidability of PC, Completeness of PC with respect to the class of all Boolean algebras. (12 lectures)

   Classical first order logic (FOL): First order theories, Syntax. Satisfaction. truth, validity in FOL. Axiomatic approach, soundness. Computer assisted formal proofs: Tableaux. Consistency of FOL and completeness (sketch). Equality. Examples of first order theories with equality. Peano's arithmetic. Zermelo-Fraenkel axioms of Set theory Elementary model theory: Compactness theorem, Löwenheim-Skolem theorems. Completeness of first order theories, Isomorphism of models, Categoricity-illustrations through theories such as those of finite Abelian groups, dense linear orders without end points and Peano's arithmetic. Statements of Gödel's incompleteness theorems and undecidability of FOL. (optional) (12 lectures)
4. Lecture, Tutorial & Venue: WThF (Lec) 12PM; M (Tut) 12 PM in L9
5. Office Hours: Mondays 4-5PM
6. Evaluation Components & Policies: Mid-Sem Exam (30%), End-Sem Exam (40%), 2 Quizzes (12.5% each), Attendance in Tutorials (5%); Grading is relative.
7. Course Policies: Attendance in all lectures and tutorials is recommended. If a quiz/exam is missed for medical reasons, the instructor must be informed immediately along with appropriate medical certificate.
8. Books & References:

   1. J. Bridge: Beginning Model Theory: The Completeness Theorem and Some Consequences. Oxford Logic Guides, 1977.
   2. I. Chiswell and W. Hodges: Mathematical Logic. Oxford, 2007.
   3. R. Cori and D. Lascar: Mathematical Logic, Oxford, 2001.

4. J. Goubalt-Larrecq and J. Mackie: Proof Theory and Automated Deduction, Kluwer, 1997.
5. P. R. Halmos: Naive Set Theory, Springer, 1974.
6. J. Kelly: The Essence of Logic, Pearson, 2011.
7. A. Margaris, First Order Mathematical Logic, Dover, 1990.